Coordinate Conversion

Polar $\rightarrow$ Rectangular: $x = r\cos\theta; \quad y = r\sin\theta$

Rectangular $\rightarrow$ Polar:

$$x^2 + y^2 = r^2; \quad r = \pm\sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}; \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Plot polar point $(r,\theta)=\left(4,\frac{\pi}{2}\right)$.

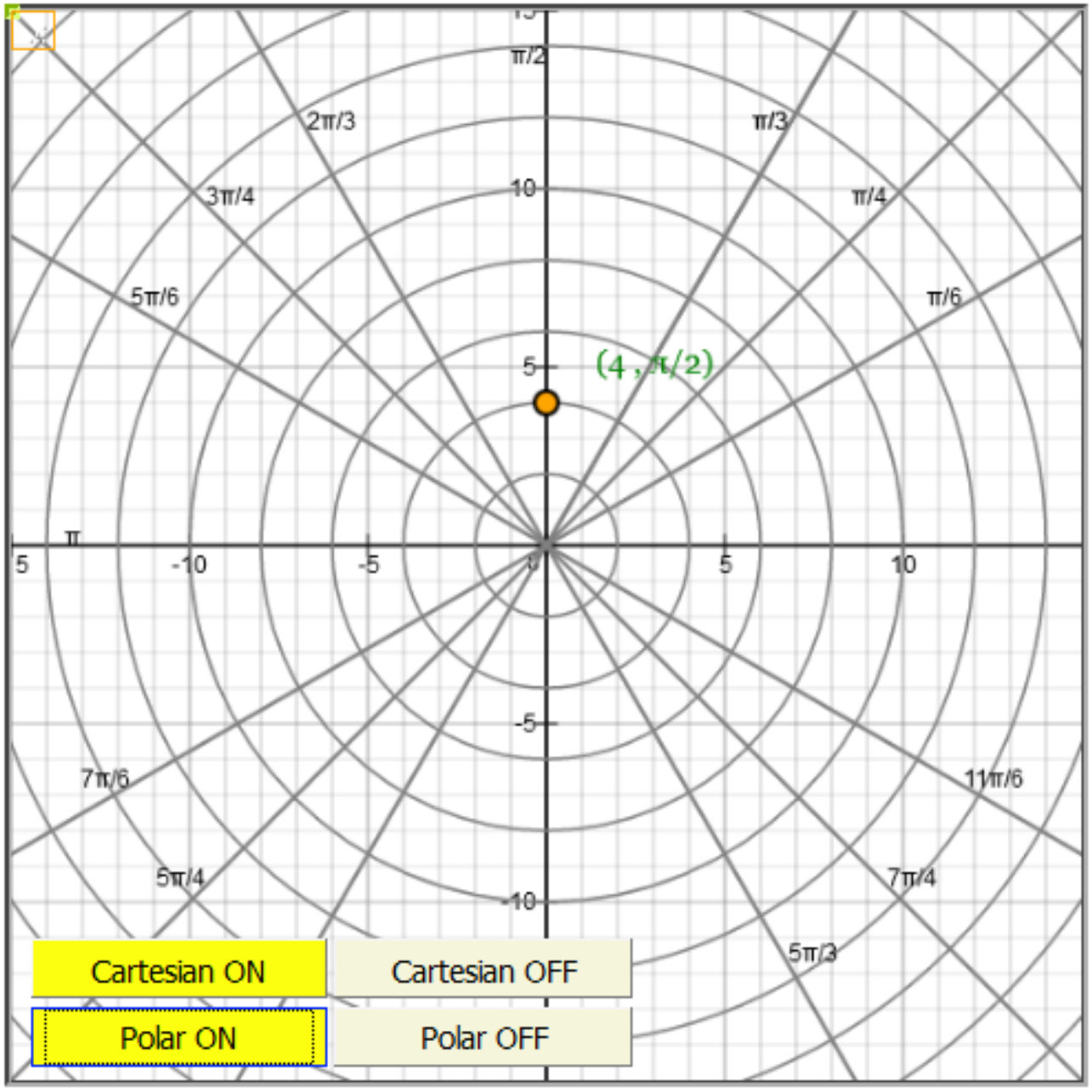
Write Corresponding Rectangular Coordinates:

$r=4; \quad \theta=\frac{\pi}{2}$

$x=r\cos\theta=4\cos\frac{\pi}{2}=(4)(0)=0$

$y=r\sin\theta=4\sin\frac{\pi}{2}=(4)(1)=4$

Rectangular Coordinates $=(0,4)$


π/2
2π/3
π/3
3π/4
π/4
10
5π/6
π/6
(4 , π/2)
5
π
5
-10
-5
0
5
10
-5
7π/6
11π/6
5π/4
7π/4
-10
5π/3
Cartesian ON
Cartesian OFF
Polar ON
Polar OFF

Trigonometry Review:

$\cos(-\theta) = \cos(\theta)$

$\cos(-3\pi/4) = \cos(3\pi/4)$

$\sin(-\theta) = -\sin(\theta)$

$\sin(-3\pi/4) = -\sin(3\pi/4)$

Plot polar point $\left(-2,-\frac{3\pi}{4}\right)$.

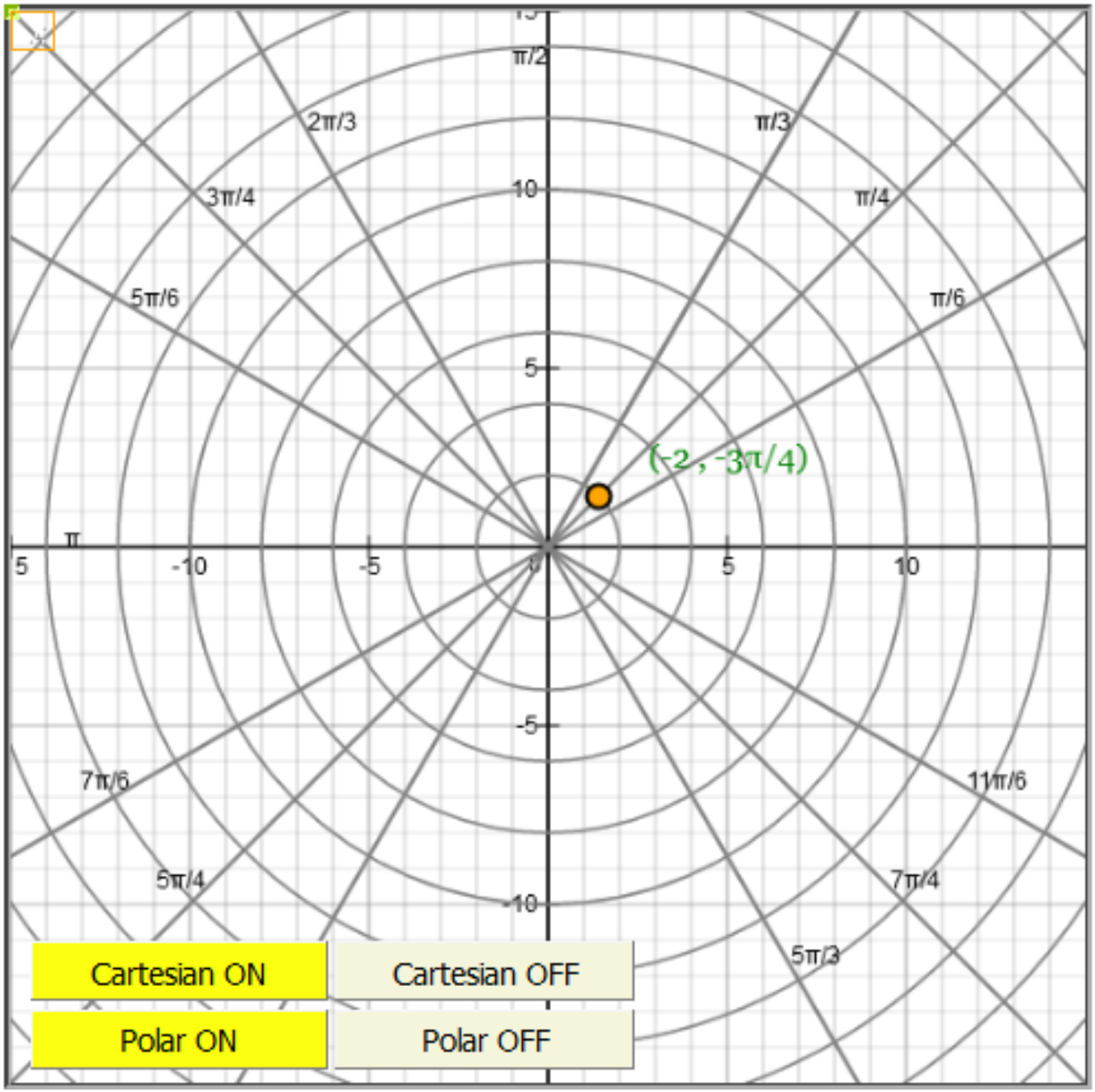
Write Corresponding Rectangular Coordinates:

$r = -2; \quad \theta = -\frac{3\pi}{4}$

$$x = r\cos\theta = -2\cos\left(\frac{-3\pi}{4}\right) = (-2)\left(-\frac{\sqrt{2}}{2}\right) = \sqrt{2}$$

$$y = r\sin\theta = -2\sin\left(\frac{-3\pi}{4}\right) = (-2)\left(-\frac{\sqrt{2}}{2}\right) = \sqrt{2}$$

Rectangular Coordinates $= \left(\sqrt{2},\sqrt{2}\right)$


π/2
2π/3
π/3
3π/4
π/4
10
5π/6
π/6
5
(-2, -3π/4)
π
5
-10
-5
0
5
10
-5
7π/6
11π/6
5π/4
7π/4
-10
5π/3
Cartesian ON
Cartesian OFF
Polar ON
Polar OFF

Plot polar point $\left(\sqrt{3},\ 4.16\right)$

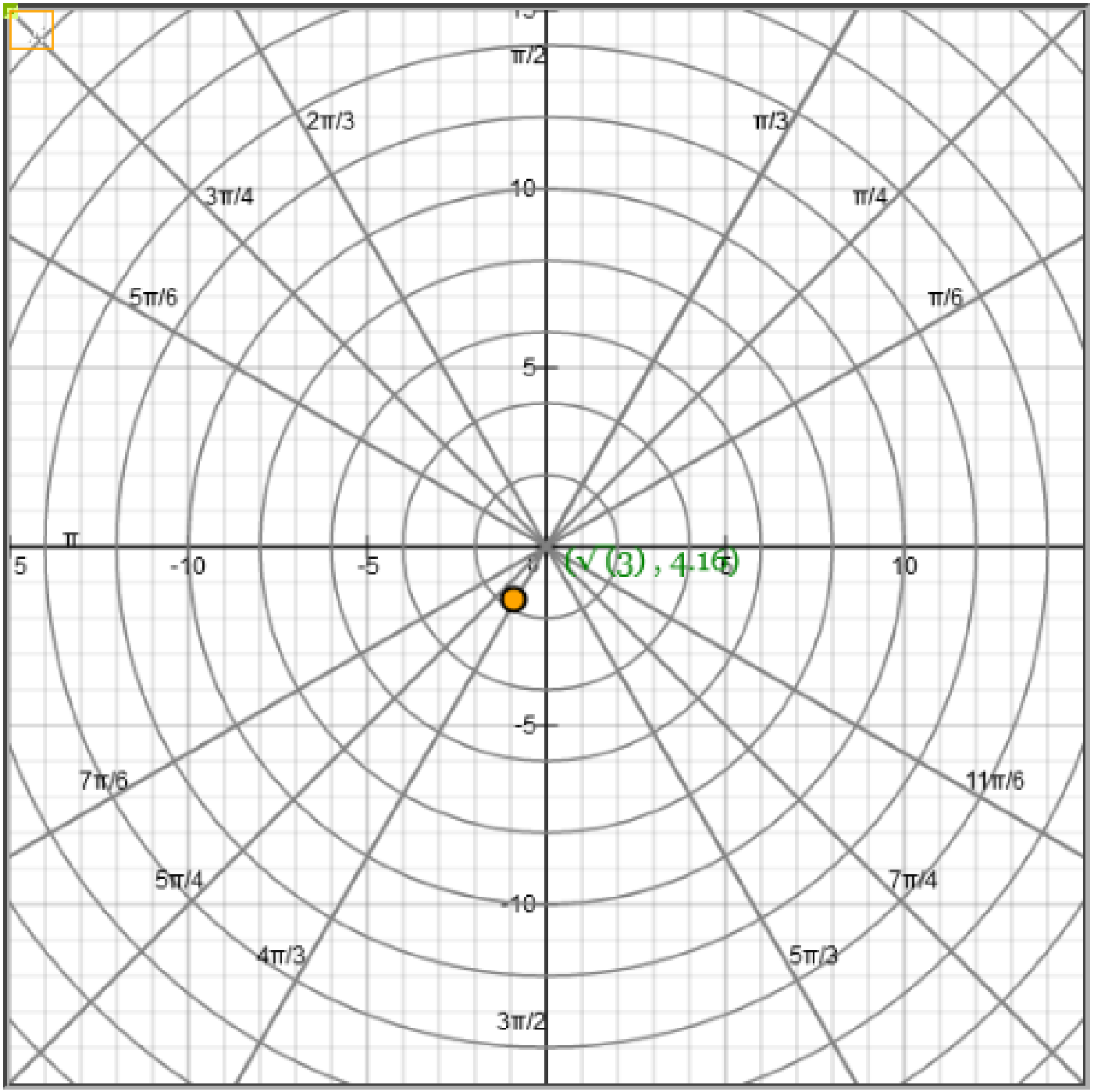
Write Corresponding Rectangular Coordinates:

$r=\sqrt{3};\quad \theta$=4.16 radians

$x=r\cos\theta=\sqrt{3}\cos\left(4.16\right)$ = -0.9088

$y=r\sin\theta=\sqrt{3}\sin\left(4.16\right)$ = -1.4744

Rectangular Coordinates = $\left(-0.9088,-1.4744\right)$


π/2
2π/3
π/3
3π/4
π/4
5π/6
π/6
π
5
-10
-5
10
(√(3) , 4.16)
7π/6
11π/6
5π/4
7π/4
4π/3
5π/3
3π/2

Rectangular Coordinates $(x, y) = (2, 3)$.

Write two sets of corresponding polar coordinates for $(2, 3)$.

Assume: $0 \le \theta \le 2\pi$

Note: $(2, 3)$ is a point in Quadrant I;

hence, θ is between 0 and $\pi/2$

$$\tan\theta = \frac{y}{x} \quad \Leftrightarrow \quad \tan\theta = \frac{3}{2} \quad \Leftrightarrow \quad \theta = \tan^{-1}\left(\frac{3}{2}\right)$$

$$\Leftrightarrow \quad \theta = 0.982793723$$

$$x^2 + y^2 = r^2 \quad \Leftrightarrow \quad r^2 = 4+9 \quad \Leftrightarrow \quad r^2 = 13 \quad \Leftrightarrow \quad r = \sqrt{13}$$

Corresponding Polar Coordinates:

$$\left(\sqrt{13},\ 0.982793723\right) = \left(-\sqrt{13},\ -0.982793723\right)$$

Rectangular Coordinates $(x, y) = (-5, 3)$.

Write two sets of corresponding polar coordinates for $(-5, 3)$.

Assume: $0 \le \theta \le 2\pi$

Note: $(-5, 3)$ is a point in Quadrant II; hence θ is between $\pi/2$ and π.

or θ is between 1.57 and 3.14.

$$\tan\theta = \frac{y}{x} \quad \Leftrightarrow \quad \tan\theta = -\frac{3}{5} \quad \Leftrightarrow \quad \theta = \tan^{-1}\left(-\frac{3}{5}\right) = \text{-0.5404195}$$

But -0.5404195 is not between 1.57 and 3.14.

So we need to use $\theta = \tan^{-1}\left(-\frac{3}{5}\right) = \pi$ - 0.5404195 = 2.6011731

$$x^2 + y^2 = r^2 \quad \Leftrightarrow \quad r^2 = 25+9 \quad \Leftrightarrow \quad r^2 = 34 \quad \Leftrightarrow \quad r = \sqrt{34}$$

Corresponding Polar Coordinates:

$$\left(\sqrt{34},\ 2.6011731\ \right) = \left(\sqrt{34},\ 2.6011731 + 2\pi\right)$$

Rectangular Coordinates $(x, y) = \left(4, -\sqrt{5}\right)$.

Write two sets of corresponding polar coordinates for $\left(4, -\sqrt{5}\right)$.

Assume: $0 \le \theta \le 2\pi$

Note: $\left(4, -\sqrt{5}\right)$ is a point in Quadrant IV; hence θ is between $3\pi/2$ and 2

$$\tan\theta = \frac{y}{x} \quad \Leftrightarrow \quad \tan\theta = -\frac{\sqrt{5}}{4} \quad \Leftrightarrow \quad \theta = \tan^{-1}\left(-\frac{\sqrt{5}}{4}\right) = \text{-0.5097396}$$

But -0.5097396 is not between $3\pi/2$ and 2π.

We will use $\theta = \tan^{-1}\left(-\frac{\sqrt{5}}{4}\right)$ = 2π-0.5097396 = 5.7734456

$$x^2 + y^2 = r^2 \quad \Leftrightarrow \quad r^2 = 16+5 \quad \Leftrightarrow \quad r^2 = 21 \quad \Leftrightarrow \quad r = \sqrt{21}$$

Corresponding Polar Coordinates:

$$\left(\sqrt{21}\ ,\ 5.773445\right) = \left(\sqrt{21}\ ,\ 5.773445+2\pi\right)$$

Rectangular Equation: $x^2 + y^2 = 49$.

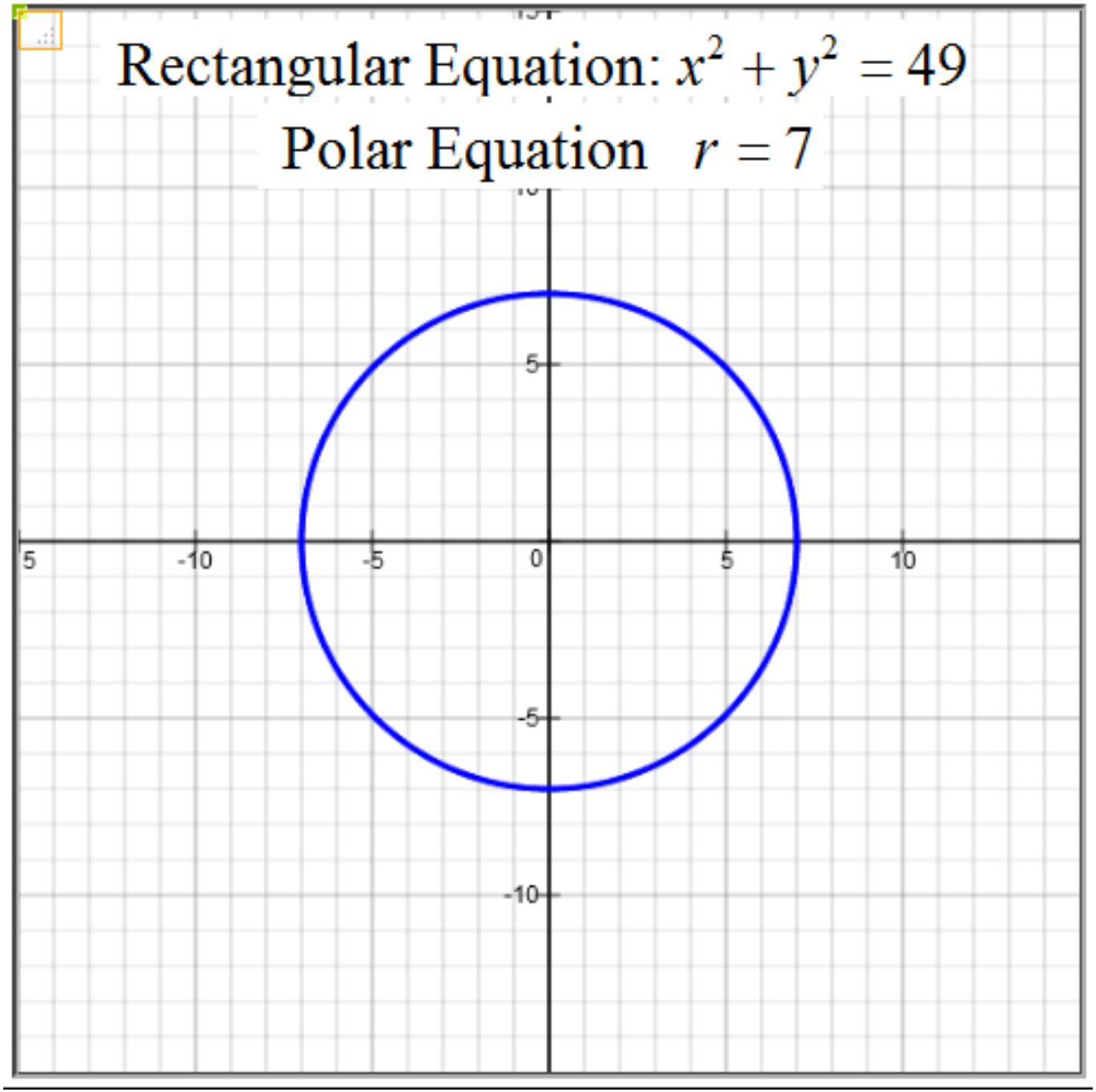
Write Corresponding Polar Equation:

$$x^2 + y^2 = 49 \quad \Leftrightarrow \quad \left(r\cos\theta\right)^2 + \left(r\sin\theta\right)^2 = 49$$

$$\Leftrightarrow \quad r^2\left[\left(\cos\theta\right)^2 + \left(\sin\theta\right)^2\right] = 49$$

$$\Leftrightarrow \quad r^2\left[1\right] = 49$$

$$\Leftrightarrow \quad r = 7 \qquad \text{Polar Equation}$$


Rectangular Equation: $x^2 + y^2 = 49$
Polar Equation $r = 7$
5
-10
-5
0
5
10
5
-5
-10

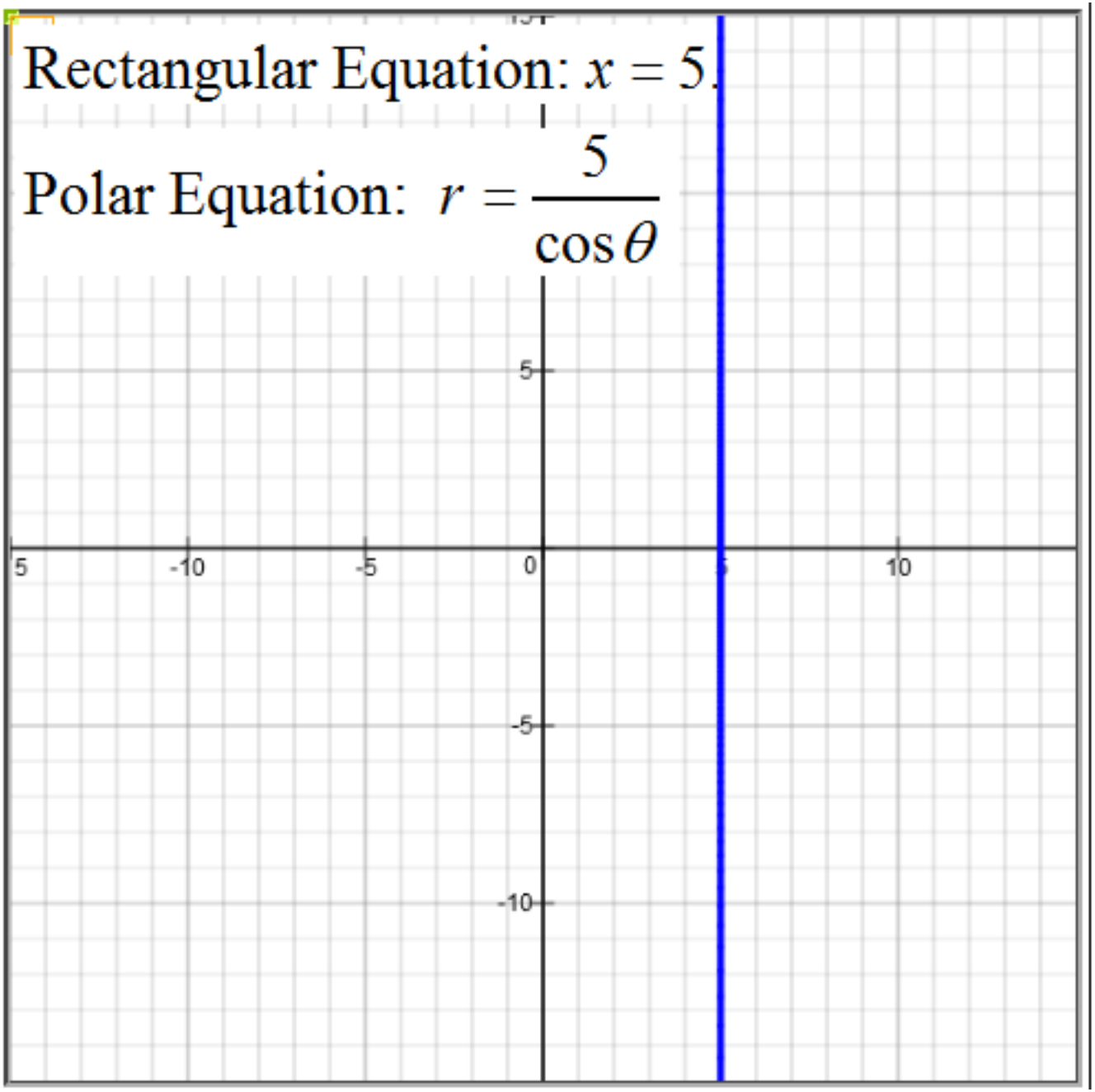
Rectangular Equation: $x = 5$.

a) Write Corresponding Polar Equation:

$$x = 5 \quad \Leftrightarrow \quad r\cos\theta = 5 \quad \Leftrightarrow \quad r = \frac{5}{\cos\theta}$$

Polar Equation: $r = \dfrac{5}{\cos\theta}$

b) Graph the polar equation.


Rectangular Equation: $x = 5$
Polar Equation: $r = \frac{5}{\cos\theta}$
5
-10
-5
0
10
5
-5
-10

Rectangular Equation: $x^2 = y + 5$.

Write Corresponding Polar Equation:

$x^2 = y + 5$

$(r\cos\theta)^2 = r\sin\theta + 5$

Polar Equation: $(r\cos\theta)^2 - r\sin\theta = 5$

Polar Equation: $r = 2$.

Write Corresponding Rectangular Equation:

$r = 2$

$r^2 = 4$

$x^2 + y^2 = 4$

Rectangular Equation: $x^2 + y^2 = 4$

Polar Equation: $r = 5\theta$.

Write Corresponding Rectangular Equation:

$$r = 5\theta$$

$$r^2 = (5\theta)^2$$

$$x^2 + y^2 = 25(\theta)^2$$

$$x^2 + y^2 = 25\left(\tan^{-1}\left(\frac{y}{x}\right)\right)^2$$

Polar Equation: $r = 4\sin\theta$.

Write Corresponding Rectangular Equation:

$r = 4\sin\theta$

$r \cdot r = \left(4\sin\theta\right) \cdot r$

$r^2 = 4r\sin\theta$

$x^2 + y^2 = 4y$

Rectangular Equation: $x^2 + y^2 = 4y$

Polar Equation: $r = 4 - 2\sin\theta$.

Write Corresponding Rectangular Equation:

$$r = 4 - 2\sin\theta$$

$$r \cdot r = \left(4 - 2\sin\theta\right) \cdot r$$

$$r^2 = 4r - 2r\sin\theta$$

$$x^2 + y^2 = 4\left(\pm\sqrt{x^2 + y^2}\right) - 2y$$

Polar Equation: $r = \dfrac{5}{1+2\sin\theta}$.

Write Corresponding Rectangular Equation:

$$r = \frac{5}{1+2\sin\theta}$$

$$r + 2r\sin\theta = 5$$

$$\pm\sqrt{x^2+y^2} + 2y = 5$$

Corresponding Rectangular Equation: $\pm\sqrt{x^2+y^2} + 2y = 5$