

Coordinate Conversion

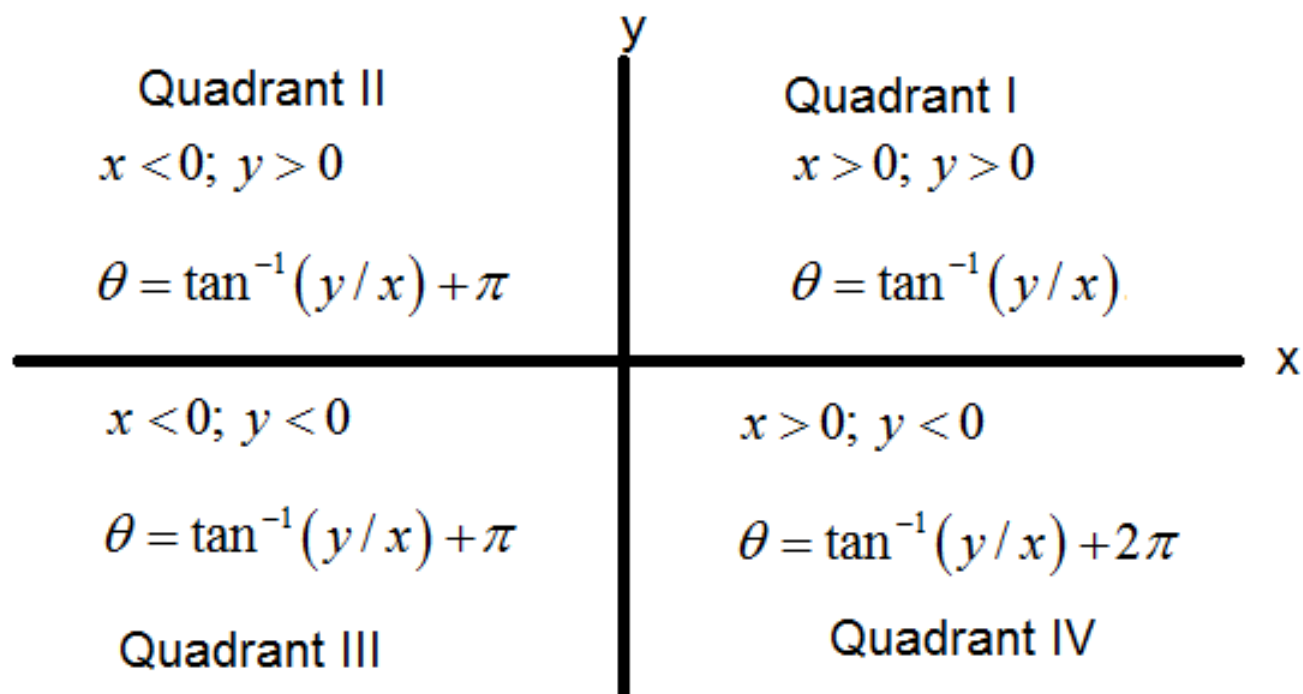
Polar \rightarrow Rectangular: $x = r \cos \theta$; $y = r \sin \theta$

Rectangular \rightarrow Polar:

$$x^2 + y^2 = r^2; \quad r = \pm \sqrt{x^2 + y^2}$$

$$\tan \theta = \frac{y}{x}; \quad \theta = \tan^{-1} \left(\frac{y}{x} \right)$$

How to use calculator to figure out θ :



Also:

If (x, y) is on positive y – axis ($x = 0$ and $y > 0$): $\theta = \pi / 2$.

If (x, y) is on negative y – axis ($x = 0$ and $y < 0$): $\theta = 3\pi / 2$.

If (x, y) is on positive x – axis ($x > 0$ and $y = 0$): $\theta = 0$.

If (x, y) is on negative x – axis ($x < 0$ and $y = 0$): $\theta = \pi$.

Example 1:

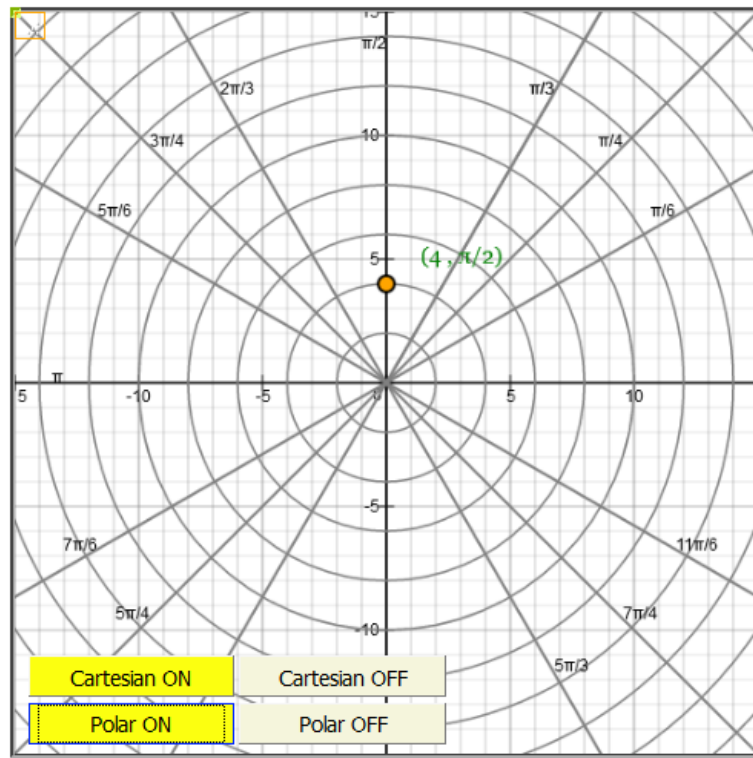
Plot polar point $(r, \theta) = \left(4, \frac{\pi}{2}\right)$.

Write Corresponding Rectangular Coordinates:

$$r = 4; \quad \theta = \frac{\pi}{2}$$

$$x = r \cos \theta = 4 \cos \frac{\pi}{2} = (4)(0) = 0; \quad y = r \sin \theta = 4 \sin \frac{\pi}{2} = (4)(1) = 4$$

Rectangular Coordinates = $(0, 4)$



Example 2:

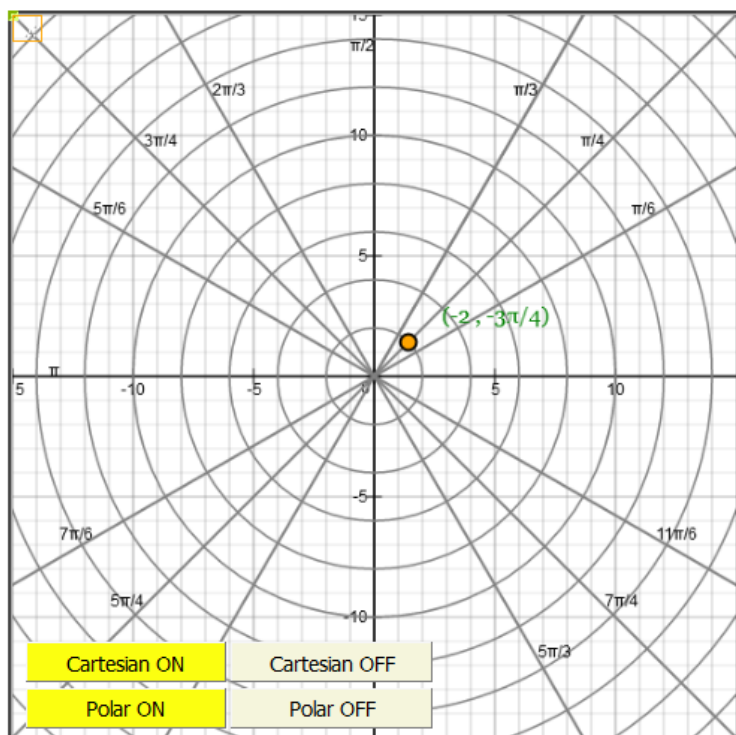
Plot polar point $\left(-2, -\frac{3\pi}{4}\right)$.

Write Corresponding Rectangular Coordinates:

$$r = -2; \quad \theta = -\frac{3\pi}{4}$$

$$x = r \cos \theta = -2 \cos\left(-\frac{3\pi}{4}\right) = (-2)\left(-\frac{\sqrt{2}}{2}\right) = \sqrt{2}; \quad y = r \sin \theta = -2 \sin\left(-\frac{3\pi}{4}\right) = (-2)\left(-\frac{\sqrt{2}}{2}\right) = \sqrt{2}$$

Rectangular Coordinates = $(\sqrt{2}, \sqrt{2})$



Example 3:

Plot polar point $(\sqrt{3}, 4.16)$

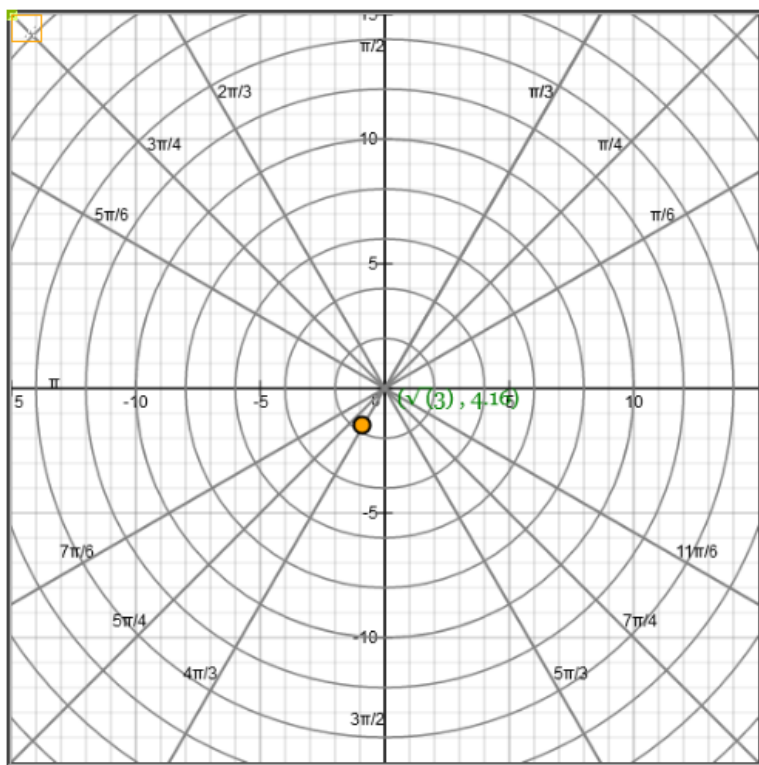
Write Corresponding Rectangular Coordinates:

$$r = \sqrt{3}; \quad \theta = 4.16 \text{ radians}$$

$$x = r \cos \theta = \sqrt{3} \cos(4.16) = -0.9088$$

$$y = r \sin \theta = \sqrt{3} \sin(4.16) = -1.4744$$

Rectangular Coordinates = $(-0.9088, -1.4744)$



Example 4:

Rectangular Coordinates $(x, y) = (2, 3)$.

Write two sets of corresponding polar coordinates for $(2, 3)$.

Note: $(2, 3)$ is a point in Quadrant I; hence, θ is between 0 and $\pi/2$

$$\tan \theta = \frac{y}{x} \quad \Leftrightarrow \quad \tan \theta = \frac{3}{2}$$

$$\text{Using Calculator: } \theta = \tan^{-1}\left(\frac{3}{2}\right) = 0.982793723$$

$$x^2 + y^2 = r^2 \quad \Leftrightarrow \quad r^2 = 4 + 9 \quad \Leftrightarrow \quad r^2 = 13 \quad \Leftrightarrow \quad r = \sqrt{13}$$

Two sets of corresponding Polar Coordinates:

$$\left(\sqrt{13}, 0.982793723\right)$$

$$\left(\sqrt{13}, 0.982793723 + 2\pi\right)$$

Example 5:

Rectangular Coordinates $(x, y) = (-5, 3)$.

Write two sets of corresponding polar coordinates for $(-5, 3)$.

Note: $(-5, 3)$ is a point in Quadrant II; hence θ is between $\pi/2$ and π .

or θ is between 1.57 and 3.14.

$$\tan \theta = \frac{y}{x} \quad \Leftrightarrow \quad \tan \theta = -\frac{3}{5}$$

Using Calculator: $\tan^{-1}\left(-\frac{3}{5}\right) = -0.5404195$

Since $(-5, 3)$ is in Quadrant II, $\theta = \tan^{-1}\left(-\frac{3}{5}\right) + \pi = -0.5404195 + \pi = 2.6011731$

$$x^2 + y^2 = r^2 \quad \Leftrightarrow \quad r^2 = 25 + 9 \quad \Leftrightarrow \quad r^2 = 34 \quad \Leftrightarrow \quad r = \sqrt{34}$$

Two sets of corresponding Polar Coordinates:

$$\left(\sqrt{34}, 2.6011731\right)$$

$$\left(\sqrt{34}, 2.6011731 + 2\pi\right)$$

Example 6:

Rectangular Coordinates $(x, y) = (4, -\sqrt{5})$.

Write two sets of corresponding polar coordinates for $(4, -\sqrt{5})$.

Note: $(4, -\sqrt{5})$ is a point in Quadrant IV; hence θ is between $3\pi/2$ and 2π .

$$\tan \theta = \frac{y}{x} \quad \Leftrightarrow \quad \tan \theta = -\frac{\sqrt{5}}{4}$$

Using Calculator: $\tan^{-1}\left(-\frac{\sqrt{5}}{4}\right) = -0.5097396$

Since $(4, -\sqrt{5})$ is in Quadrant IV, $\theta = \tan^{-1}\left(-\frac{\sqrt{5}}{4}\right) + 2\pi = -0.5097396 + 2\pi = 5.7734456$

$$x^2 + y^2 = r^2 \quad \Leftrightarrow \quad r^2 = 16 + 5 \quad \Leftrightarrow \quad r^2 = 21 \quad \Leftrightarrow \quad r = \sqrt{21}$$

Two sets of corresponding Polar Coordinates:

$$(\sqrt{21}, 5.773445)$$

$$(\sqrt{21}, 5.773445 + 2\pi)$$

Example 7:

Rectangular Equation: $x^2 + y^2 = 49$.

Write Corresponding Polar Equation:

$$x^2 + y^2 = 49$$

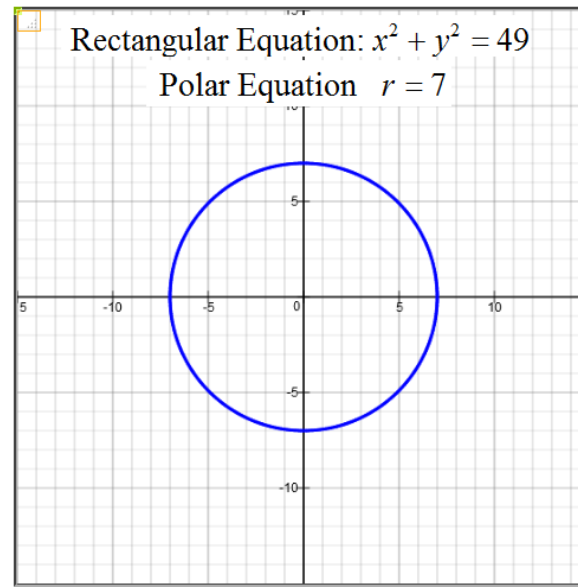
$$(r \cos \theta)^2 + (r \sin \theta)^2 = 49$$

$$r^2 [(\cos \theta)^2 + (\sin \theta)^2] = 49$$

$$r^2 [1] = 49$$

$$r = 7$$

Polar Equation



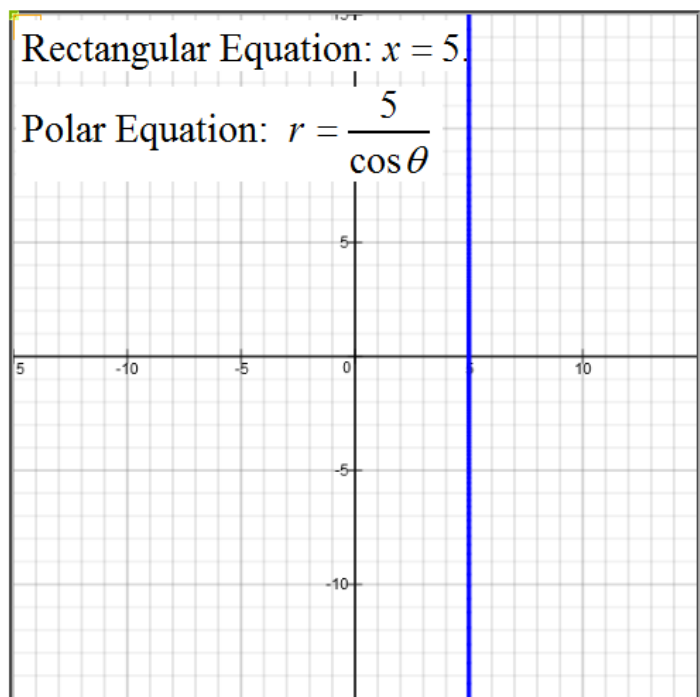
Example 8:

Rectangular Equation: $x = 5$.

a) Write Corresponding Polar Equation:

$$x = 5 \Leftrightarrow r \cos \theta = 5 \Leftrightarrow r = \frac{5}{\cos \theta}$$

Polar Equation: $r = \frac{5}{\cos \theta}$



Example 9:

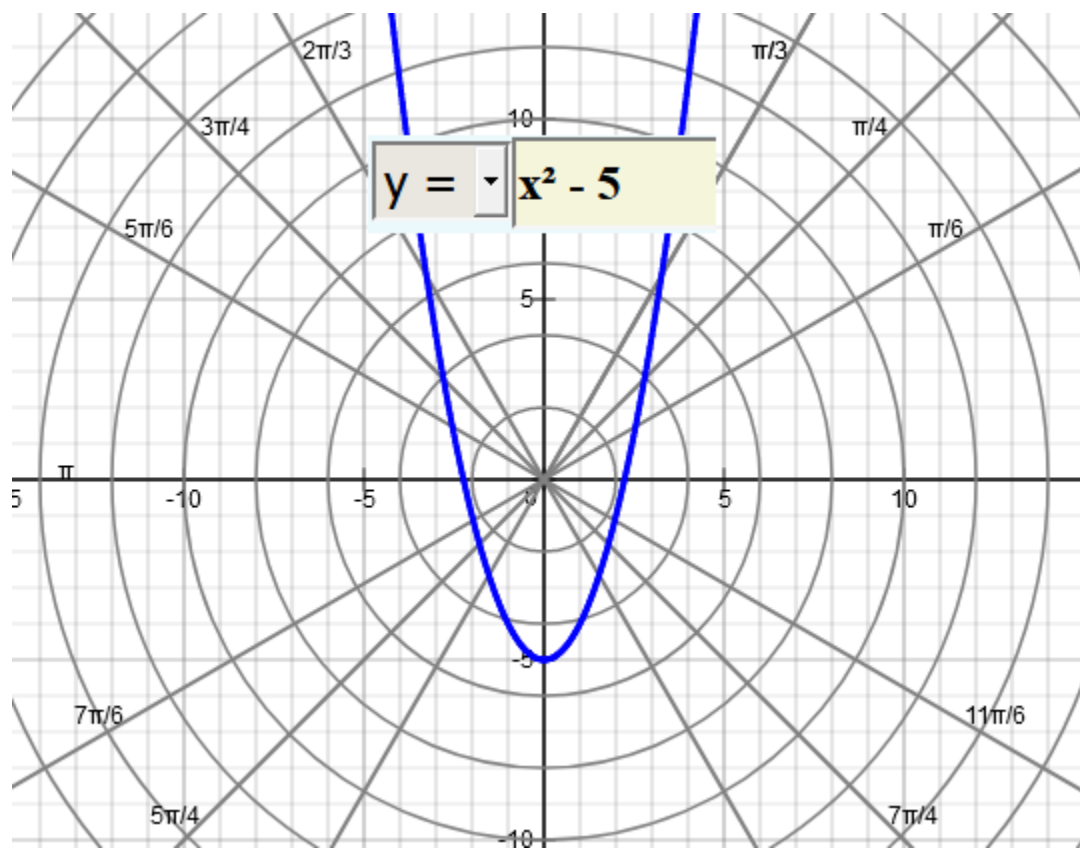
Rectangular Equation: $x^2 = y + 5$.

Write Corresponding Polar Equation:

$$x^2 = y + 5$$

$$(r \cos \theta)^2 = r \sin \theta + 5$$

$$\text{Polar Equation: } (r \cos \theta)^2 - r \sin \theta = 5$$



Example 10:

Polar Equation: $r = 2$.

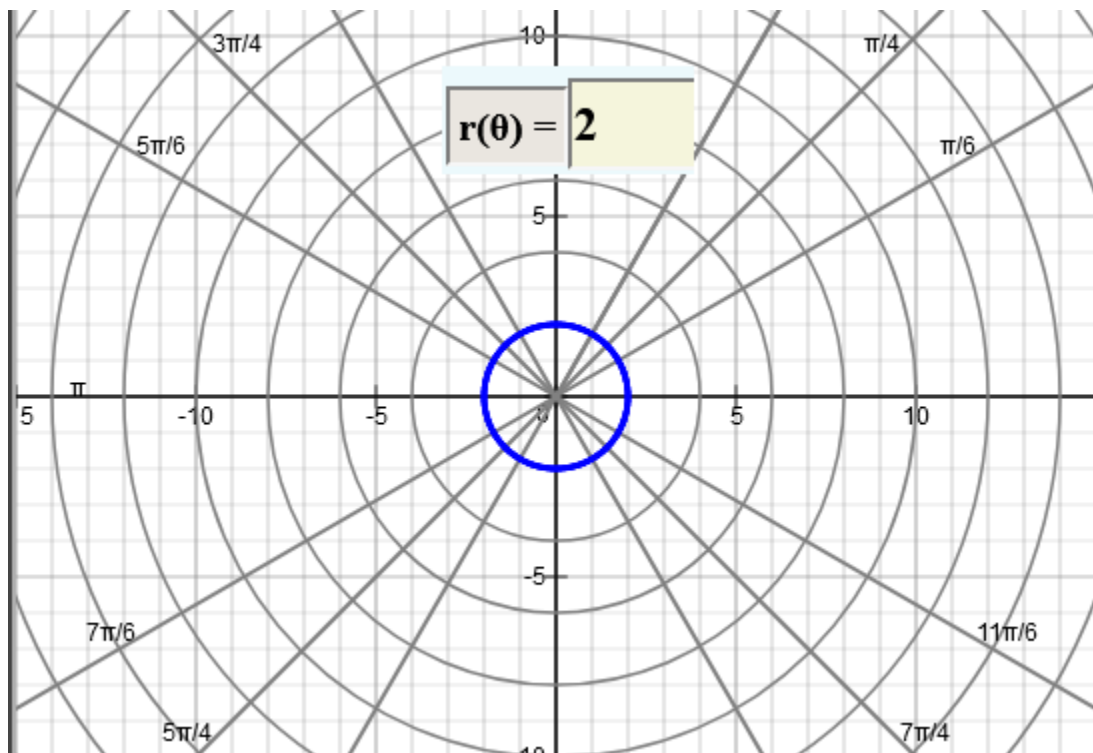
Write Corresponding Rectangular Equation:

$$r = 2$$

$$r^2 = 4$$

$$x^2 + y^2 = 4$$

Rectangular Equation: $x^2 + y^2 = 4$



Example 11:

Polar Equation: $r = 5\theta$.

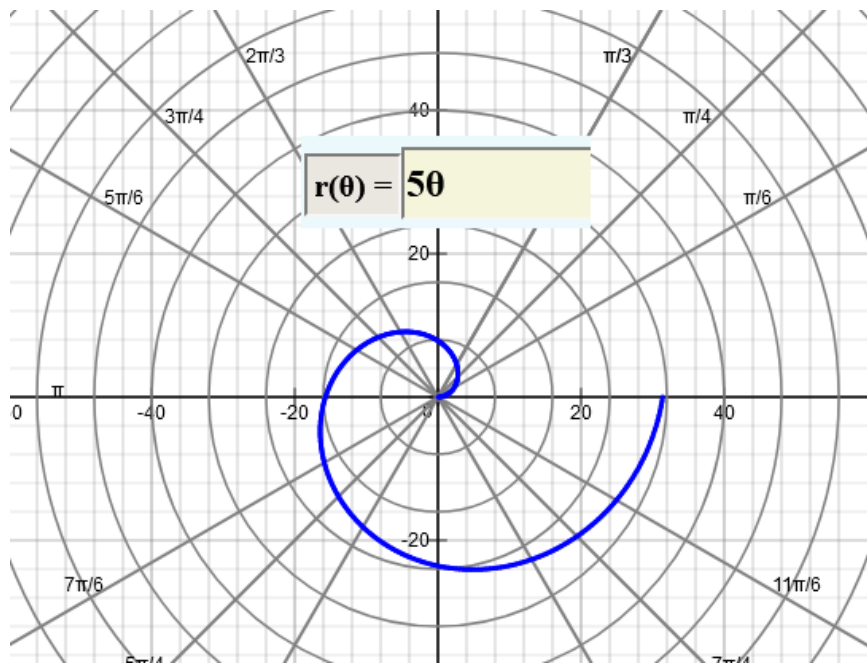
Write Corresponding Rectangular Equation:

$$r = 5\theta$$

$$r^2 = (5\theta)^2$$

$$x^2 + y^2 = 25(\theta)^2$$

$$x^2 + y^2 = 25\left(\tan^{-1}\left(\frac{y}{x}\right)\right)^2$$



Example 12:

Polar Equation: $r = 4\sin\theta$.

Write Corresponding Rectangular Equation:

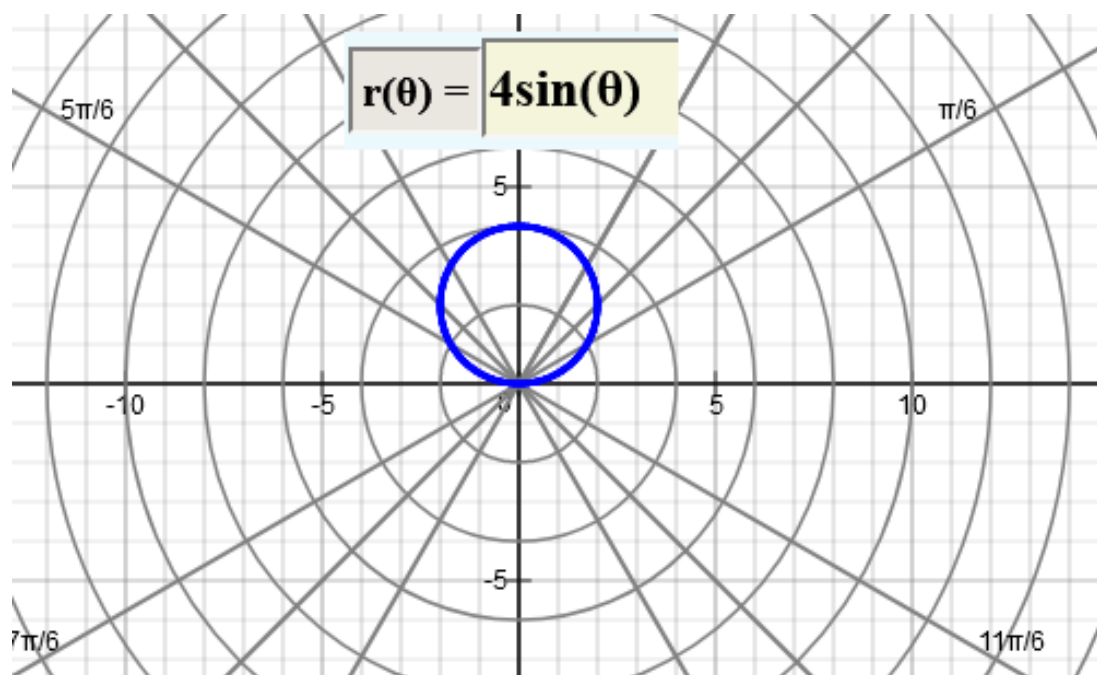
$$r = 4\sin\theta$$

$$r \cdot r = (4\sin\theta) \cdot r$$

$$r^2 = 4r\sin\theta$$

$$x^2 + y^2 = 4y$$

Rectangular Equation: $x^2 + y^2 = 4y$



Example 13:

Polar Equation: $r = 4 - 2\sin \theta$.

Write Corresponding Rectangular Equation:

$$r = 4 - 2\sin \theta$$

$$r \cdot r = (4 - 2\sin \theta) \cdot r$$

$$r^2 = 4r - 2r \sin \theta$$

$$x^2 + y^2 = 4\left(\pm\sqrt{x^2 + y^2}\right) - 2y$$

