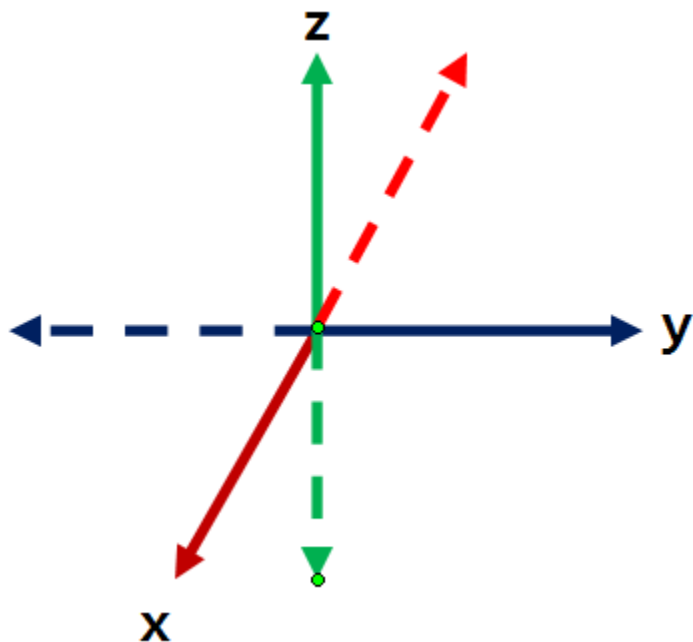


## Space Coordinates and Vectors in Space



Points in  $xy$ -plane:

Points:  $(2, 1, 0)$ ,  $(-2, 4, 0)$ ,  $(-2, 2, 0)$ ,  $(-3, 4, 0)$ ,  $(-1, 5, 0)$ ,  
 $(2, 6, 0)$ , ...

Hence,  $xy$ -plane is the set of points with  $z=0$ .

Describe the set of points with  $z = 2$ :

Points:  $(2, 1, 2)$ ,  $(-2, 4, 2)$ ,  $(-2, 2, 2)$ ,  $(-3, 4, 2)$ ,  $(-1, 5, 2)$ ,  
 $(2, 6, 2)$ , ...

This is the set of points in a plane parallel to  $xy$ -plane and 2 units above  $xy$ -plane.

Points in yz-plane:

Points:  $(0, 1, 2)$ ,  $(0, 4, 2)$ ,  $(0, 2, 2)$ ,  $(0, 4, 2)$ ,  $(0, 5, 2)$ ,  
 $(0, 6, 2)$ , ...

Hence, xz-plane is the set of points with  $x=0$ .

Describe the points with  $x = 4$ :

Points:  $(4, 1, 2)$ ,  $(4, 4, 2)$ ,  $(4, 2, 2)$ ,  $(4, 4, 2)$ ,  $(4, 5, 2)$ ,  
 $(4, 6, 2)$ , ...

This is the set of points in a plane parallel to yz-plane and 4 units from yz-plane.

Points in xz-plane:

Points:  $(4, 0, 2)$ ,  $(4, 0, 2)$ ,  $(4, 0, 2)$ ,  $(4, 0, 2)$ ,  $(4, 0, 2)$ ,  
 $(4, 0, 2)$ , ...

Hence, xz-plane is the set of points with  $y=0$ .

Describe the points with  $y = 4$ :

Points:  $(4, 4, 2), (4, 4, 2), (4, 4, 2), (4, 4, 2), (4, 4, 2),$   
 $(4, 4, 2), \dots$

This is the set of points in a plane parallel to  $xz$ -plane and 4 units from  $xz$ -plane.

Describe the set of points with  $z < 0$ :

This is the set of points below the  $xy$ -plane.

Describe the points with  $x > 0$ :

This is the set of points behind the  $yz$ -plane.

Describe the points with  $z > 4$ :

This is the set of points that are 4 units above the  $yz$ -plane.

Distance Between Points:

Distance Between the points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$ :

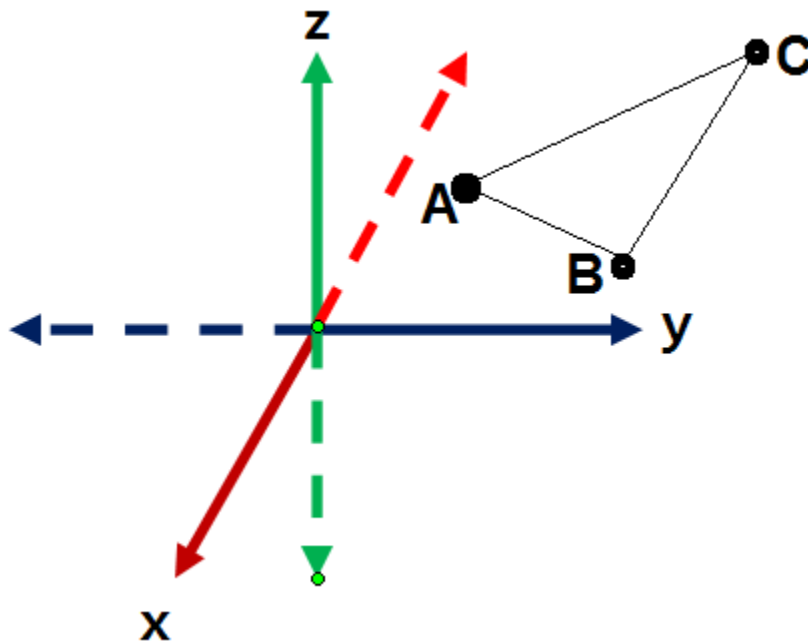
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Distance Between the points (2,1,4) and (7,8,12):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$d = \sqrt{(7-2)^2 + (8-1)^2 + (12-4)^2} = \sqrt{138}$$

Find length of sides of triangle:



Find length of sides of triangle with the following vertices:

$$A(1,2,3), \quad B(4, 3, 3), \quad C(4,7,8)$$

$$A(1,2,3), \quad B(4, 3, 3), \quad C(4,7,8)$$

$$|AB| = \sqrt{(4-1)^2 + (3-2)^2 + (3-3)^2} = \sqrt{10}$$

$$|AC| = \sqrt{(4-1)^2 + (7-2)^2 + (8-3)^2} = \sqrt{59}$$

$$|BC| = \sqrt{(4-4)^2 + (7-3)^2 + (8-3)^2} = \sqrt{41}$$

Is triangle ABC a right triangle?

$$|AC|^2 = |AB|^2 + |BC|^2 \quad \text{True or False?}$$

$$(\sqrt{59})^2 = (\sqrt{10})^2 + (\sqrt{41})^2$$

$$59 = 10 + 41 \quad \text{False.}$$

Triangle is not a right triangle.

Is triangle an isosceles triangle?

No, since lengths of all sides are different.

**Sphere:**

Let  $(x_0, y_0, z_0)$  be center of sphere and  $r$  be radius of sphere.

$$\text{Then equation of sphere: } (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

Find equation of sphere with center  $(1,2,3)$  and radius 5.

Then equation of sphere:

$$(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 5^2$$

$$(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 25$$

Find center and radius sphere.

$$x^2 + y^2 + z^2 - 10x - 4y - 6z + 22 = 0$$

$$(x^2 - 10x) + (y^2 - 4y) + (z^2 - 6z) + 22 = 0$$

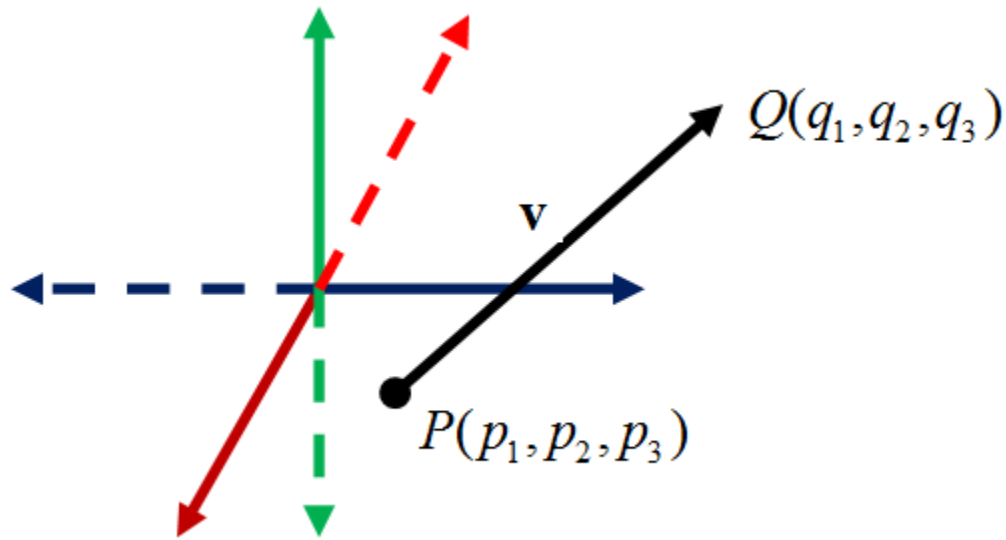
$$(x^2 - 10x + 25) + (y^2 - 4y + 4) + (z^2 - 6z + 9) + 22 = 0 + 25 + 4 + 9$$

$$(x - 5)(x - 5) + (y - 2)(y - 2) + (z - 3)(z - 3) = 16$$

$$(x - 5)^2 + (y - 2)^2 + (z - 3)^2 = 4^2$$

Center(5, 2, 3) and radius = 4

## Vectors in Space



$$\mathbf{v} = \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle = \langle v_1, v_2, v_3 \rangle$$

$$\text{Length of } \mathbf{v} = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2 + (q_3 - p_3)^2} = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

$$\text{Unit Vector in the direction of } \mathbf{v} = \frac{1}{\|\mathbf{v}\|} \mathbf{v}$$

Find the components of the vector with initial point  $P(1, 2, 3)$

and terminal point  $Q(6, 9, 11)$

$$\mathbf{v} = \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle = \langle v_1, v_2, v_3 \rangle$$

$$\mathbf{v} = \langle 6 - 1, 9 - 2, 11 - 3 \rangle = \langle 5, 7, 8 \rangle$$

$$\|\mathbf{v}\| = \text{Length of } \mathbf{v} = \sqrt{(5)^2 + (7)^2 + (8)^2} = \sqrt{138}$$

$$\text{Unit Vector in the direction of } \mathbf{v} = \frac{1}{\|\mathbf{v}\|} \mathbf{v}$$

$$= \frac{1}{\sqrt{138}} \langle 5, 7, 8 \rangle = \left\langle \frac{5}{\sqrt{138}}, \frac{7}{\sqrt{138}}, \frac{8}{\sqrt{138}} \right\rangle$$

Let  $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ . Find the magnitude of  $\mathbf{v}$ .

$$\text{Note: } \mathbf{i} = \langle 1, 0, 0 \rangle \quad \mathbf{j} = \langle 0, 1, 0 \rangle \quad \mathbf{k} = \langle 0, 0, 1 \rangle$$

$$\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k} = \langle 3, 4, 5 \rangle$$

$$\|\mathbf{v}\| = \text{Length of } \mathbf{v} = \sqrt{(3)^2 + (4)^2 + (5)^2} = \sqrt{50}$$

$$\text{Unit Vector in the direction of } \mathbf{v} = \frac{1}{\|\mathbf{v}\|} \mathbf{v}$$

$$= \frac{1}{\sqrt{50}} \langle 3, 4, 5 \rangle = \left\langle \frac{3}{\sqrt{50}}, \frac{4}{\sqrt{50}}, \frac{5}{\sqrt{50}} \right\rangle$$