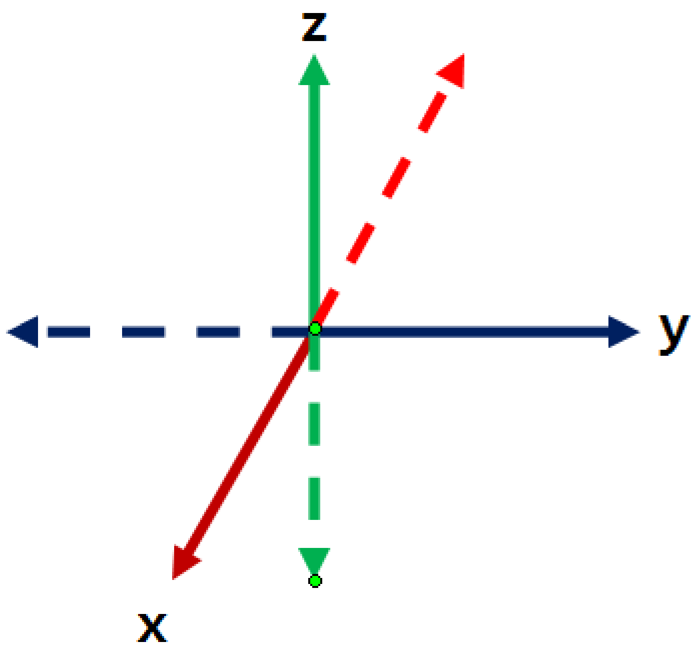
Space Coordinates and Vectors in Space



Plot (2, 1, 3),   (-2, 4, 1), (-2, 2, 1), (-3, 4, -2)

Planes: xy-plane, yz-plane, xz-plane
http://www.mathworks.com/help/phased/ug/rectangular-coordinates.html?requestedDomain=www.mathworks.com

Points in xy-plane:

Points: (2, 1, 0),   (-2, 4, 0), (-2, 2, 0), (-3, 4, 0), (-1, 5, 0),

(2, 6, 0), ...

Hence, xy-plane is the set of points with z=0.

Describe the points with z = 2:

Points: (2, 1, 2), (-2, 4, 2), (-2, 2, 2), (-3, 4, 2), (-1, 5, 2),

(2, 6, 2), ...

This is the set of points in a plane parallel to xy-plane and 2 units above xy-plane.

Points in yz-plane:

Points: (0, 1, 2), (0, 4, 2), (0, 2, 2), (0, 4, 2), (0, 5, 2),

(0, 6, 2), ...

Hence, xz-plane is the set of points with x=0.

Describe the points with x = 4:

Points: (4,1, 2), (4, 4, 2), (4, 2, 2), (4, 4, 2), (4, 5, 2),

(4, 6, 2), ...

This is the set of points in a plane parallel to yz-plane and 4 units from yz-plane.

Points in xz-plane:

Points: (4,0, 2),   (4, 0, 2), (4, 0, 2), (4, 0, 2), (4, 0, 2),

(4, 0, 2), ...

Hence, xz-plane is the set of points with y=0.

Describe the points with y = 4:

Points: (4,4, 2),   (4, 4, 2), (4, 4, 2), (4, 4, 2), (4, 4, 2),

(4, 4, 2), ...

This is the set of points in a plane parallel to xz-plane and 4 units from xz-plane.

Describe the points with z < 0:

This is the set of points below the xy-plane.

Describe the points with x > 0:

This is the set of points behind he yz-plane.

Describe the points with z > 4:

This is the set of points that are 4 units above the yz-plane.

Distance Between Points:

Distance Between the points (x_1, y_1, z_1) and (x_2, y_2, z_2):

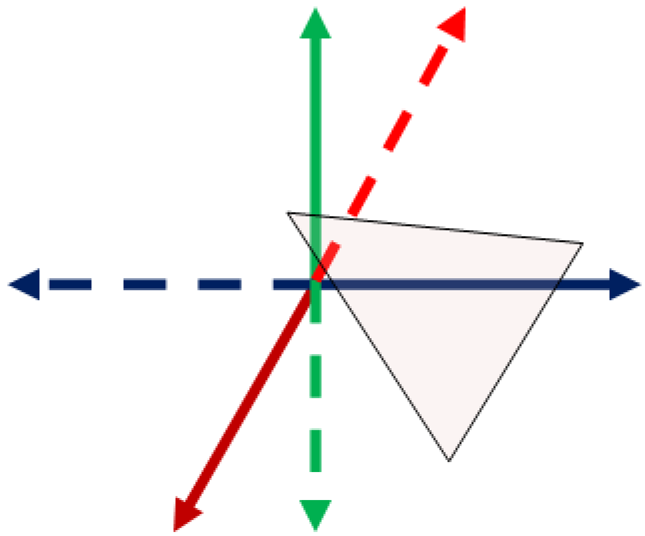
$$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2}$$

Distance Between the points $(2,1,4)$ and $(7,8,12)$:

$$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2}$$

$$d = \sqrt{\left(7-2\right)^2 + \left(8-1\right)^2 + \left(12-4\right)^2} = \sqrt{138}$$

Find length of sides of triangle:



Find length of sides of triangle with the following vertices:

A(1,2,3),   B(4, 3, 3),   C(4,7,8)

A(1,2,3),   B(4, 3, 3),   C(4,7,8)

$$|AB| = \sqrt{(4-1)^2 + (3-2)^2 + (3-3)^2} = \sqrt{10}$$

$$|AC| = \sqrt{(4-1)^2 + (7-2)^2 + (8-3)^2} = \sqrt{59}$$

$$|BC| = \sqrt{(4-4)^2 + (7-3)^2 + (8-3)^2} = \sqrt{41}$$

Is triangle a right triangle?

$|AC|^2 = |AB|^2 + |BC|^2$ True or False?

$\left(\sqrt{59}\right)^2 = \left(\sqrt{10}\right)^2 + \left(\sqrt{41}\right)^2$

$59 = 10 + 41$ False.

Triangle is not a right triangle.

Is triangle an isosceles triangle?

No, since lengths of all sides are different.

Sphere:

http://www.euclideanspace.com/threed/solidmodel/geospatial/ellipsoid/earthCoords.gif

Let (x_0, y_0, z_0) be center of sphere and r be radius of sphere.

Then equation of sphere: $\left(x - x_0\right)^2 + \left(y - y_0\right)^2 + \left(z - z_0\right)^2 = r^2$

Find equation of sphere with center (1,2,3) and radius 5.

Then equation of sphere:

$$(x-1)^2+(y-2)^2+(z-3)^2=5^2$$

$$(x-1)^2+(y-2)^2+(z-3)^2=25$$

Find center and radius sphere.

$$x^2+y^2+z^2-10x-4y-6z+22=0$$

$$(x^2-10x)+(y^2-4y)+(z^2-6z)+22=0$$

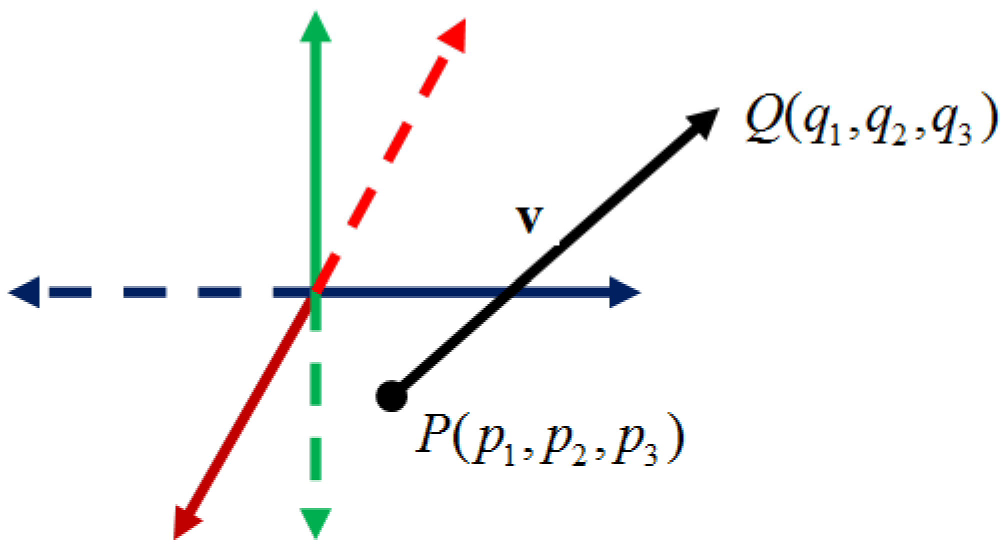
$$(x^2-10x+25)+(y^2-4y+4)+(z^2-6z+9)+22=0+25+4+9$$

$$(x-5)(x-5)+(y-2)(y-2)+(z-3)(z-3)=16$$

$$(x-5)^2+(y-2)^2+(z-3)^2=4^2$$

Center(5, 2, 3) and radius = 4

Vectors in Space



$$\mathbf{v} = \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle = \langle v_1, v_2, v_3 \rangle$$

$$\text{Length of } \mathbf{v} = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2 + (q_3 - p_3)^2} = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

$$\text{Unit Vector in the direction of } \mathbf{v} = \frac{1}{\|\mathbf{v}\|}\mathbf{v}$$

Find the components of the vector with initial point $P(1,2,3)$

and terminal point $Q(6,9,11)$

$$\mathbf{v} = \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle = \langle v_1, v_2, v_3 \rangle$$

$$\mathbf{v} = \langle 6-1, 9-2, 11-3 \rangle = \langle 5, 7, 8 \rangle$$

$$\|\mathbf{v}\| = \text{Length of } \mathbf{v} = \sqrt{(5)^2 + (7)^2 + (8)^2} = \sqrt{138}$$

$$\text{Unit Vector in the direction of } \mathbf{v} = \frac{1}{\|\mathbf{v}\|}\mathbf{v}$$

$$= \frac{1}{\sqrt{138}}\langle 5, 7, 8 \rangle = \left\langle \frac{5}{\sqrt{138}}, \frac{7}{\sqrt{138}}, \frac{8}{\sqrt{138}} \right\rangle$$

Let $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$

Find the magnitude of $\mathbf{v}$.

Note: $\mathbf{i} = \langle 1, 0, 0 \rangle$ $\quad$ $\mathbf{j} = \langle 0, 1, 0 \rangle$ $\quad$ $\mathbf{k} = \langle 0, 0, 1 \rangle$

$\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k} = \langle 3, 4, 5 \rangle$

$\|\mathbf{v}\|$=Length of $\mathbf{v} = \sqrt{(3)^2 + (4)^2 + (5)^2} = \sqrt{50}$

Unit Vector in the direction of $\mathbf{v} = \dfrac{1}{\|\mathbf{v}\|}\mathbf{v}$

$$= \frac{1}{\sqrt{50}}\langle 3, 4, 5 \rangle = \left\langle \frac{3}{\sqrt{50}}, \frac{4}{\sqrt{50}}, \frac{5}{\sqrt{50}} \right\rangle$$