Vector-Valued Functions

Example 1: Let $\mathbf{r}(\mathrm{t}) = t\mathbf{i} + (t^2 - 1)\mathbf{j}$

$\mathbf{r}(0) = (0)\mathbf{i} + (0^2 - 1)\mathbf{j} = -\mathbf{j} = \langle 0,1 \rangle$

$\mathbf{r}(1) = (1)\mathbf{i} + (1^2 - 1)\mathbf{j} = \mathbf{i} = \langle 1,0 \rangle$

$\mathbf{r}(3) = (3)\mathbf{i} + (3^2 - 1)\mathbf{j} = 3\mathbf{i} + 8\mathbf{j} = \langle 3,8 \rangle$

Corresponding Parametric Equations: $x = t;\quad y = (t^2 - 1)$

Example 2: Let $\mathbf{r}(\mathrm{t}) = 3\sin t\mathbf{i} + 4\cos t\mathbf{j}$

$\mathbf{r}(\pi/2) = 3\sin(\pi/2)\mathbf{i} + 4\cos(\pi/2)\mathbf{j} = 3\mathbf{i} + 0\,\mathbf{j} = \langle 3,0 \rangle$

Corresponding Parametric Equations: $x = 3\sin t;\quad y = 4\cos t$

Example 3: Find parametric equations for the line passing through P(0,2,-1) and Q(4,7,2).

Direction Vector $= \langle a,b,c \rangle = \overrightarrow{\mathrm{PQ}} = \langle 4-0, 7-2, 2--1 \rangle = \langle 4,5,3 \rangle$

Use P(0,2,-1) for (x_1, y_1, z_1)

parametric equations for the line:

$x = x_1 + at;\quad y = y_1 + bt;\quad z = z_1 + ct$

$x = 0 + 4t;\quad y = 2 + 5t;\quad z = -1 + 3t$

Corresponding Vector-Valued Function representing the line passing through P(0,2,-1) and Q(4,7,2).

$\mathbf{r}(\mathrm{t}) = 4t\mathbf{i} + (2 + 5t)\,\mathbf{j} + (-1 + 3t)\mathbf{k}$

Example 4: Product of Vector-Valued Functions

Let $\mathbf{r}(\mathrm{t}) = 3\cos t\mathbf{i} + 2\sin t\mathbf{j} + (t-2)\mathbf{k} = \langle 3\cos t, 2\sin t, t-2\rangle$

Let $\mathbf{u}(\mathrm{t}) = 4\sin t\mathbf{i} - 6\cos t\mathbf{j} + t^2\mathbf{k} = \langle 4\sin t, -6\cos t, t^2\rangle$

$\mathbf{r}(\mathrm{t})\cdot\mathbf{u}(\mathrm{t}) = (3\cos t)(4\sin t) + (2\sin t)(-6\cos t) + (t-2)(t^2)$

$\mathbf{r}(\mathrm{t})\cdot\mathbf{u}(\mathrm{t}) = 12\cos t\sin t - 12\cos t\sin t + t^3 - 2t^2 = t^3 - 2t^2$

Note: $\mathbf{r}(\mathrm{t})\cdot\mathbf{u}(\mathrm{t})$ is a scalor.

Example 5: Product of Vector-Valued Functions

Let $\mathbf{r}(\mathrm{t}) = 3\cos t\mathbf{i} + 2\sin t\mathbf{j} + (t-2)\mathbf{k} = \langle 3\cos t, 2\sin t, t-2\rangle$

Let $\mathbf{u}(\mathrm{t}) = 4\sin t\mathbf{i} - 6\cos t\mathbf{j} + t^2\mathbf{k} = \langle 4\sin t, -6\cos t, t^2\rangle$

$$\begin{aligned}\mathbf{r}(\mathrm{t})\times\mathbf{u}(\mathrm{t}) = &\left[(2\sin t)(t^2) - (t-2)(-6\cos t)\right]\mathbf{i}\\ &-\left[(3\cos t)(t^2) - (t-2)(4\sin t)\right]\mathbf{j}\\ &+\left[(3\cos t)(-6\cos t) - (2\sin t)(4\sin t)\right]\mathbf{k}\end{aligned}$$

Example 6: For the plane $2x + 3y + 5 = 0$, find a vector-valued function representing the plane.

$2x + 3y + 5 = 0$

$3y = -2x - 5$

$y = \frac{-2}{3}x - \frac{5}{3}$

Let $x = t$; $\quad y = \frac{-2}{3}x - \frac{5}{3} = \frac{-2}{3}t - \frac{5}{3}$

vector-valued function: $\mathbf{r}(t) = t\mathbf{i} + \left(\frac{-2}{3}t - \frac{5}{3}\right)\mathbf{j}$

Example 7: For the equation $(x-2)^2 + y^2 = 4,$ find a vector-valued function representing the plane.

$$(x-2)^2 + y^2 = 4$$

$$\frac{(x-2)^2}{4} + \frac{y^2}{4} = 1$$

$$\left(\frac{x-2}{2}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$

Recall: $\cos^2 t + \sin^2 t = 1$

So let $\cos t = \dfrac{x-2}{2} \Leftrightarrow x = 2\cos t + 2$

and let $\sin t = \dfrac{y}{2} \Leftrightarrow \quad y = 2\sin t$

vector-valued function: $\mathbf{r}(t) = \left(2\cos t + 2\right)\mathbf{i} + \left(2\sin t\right)\mathbf{j}$