13.1 Functions of Several Variables

$$f(x,y) = \sqrt{16-4x^2-y^2} = \sqrt{16-\underbrace{(4x^2+y^2)}}$$

$$z = \sqrt{16-4x^2-y^2}$$

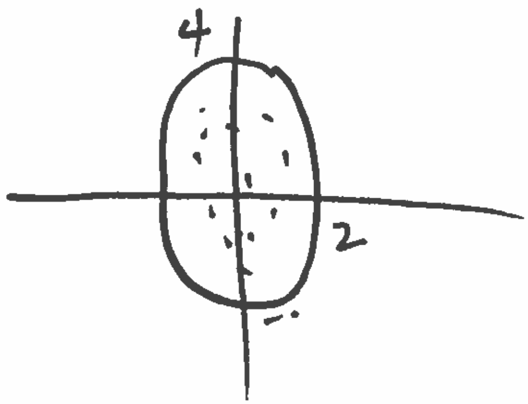
Domain of $f(x,y)$: what can (x,y) be?

$$16-4x^2-y^2 \geq 0$$

$$16 \geq 4x^2+y^2$$

$$4x^2+y^2 \leq 16$$

$$\frac{x^2}{4}+\frac{y^2}{16} \leq 1$$



Range of $f(x,y)$: $0 \leq z \leq 4$

$g(x, y) = x\sqrt{y}$

Domain: $y \geq 0$ $\{(x,y) : y \geq 0\}$

Range: $(-\infty, \infty)$ = All real numbers

$f(x, y) = \ln(4 - x - y)$

Domain: $4 - x - y > 0$

$D : \{(x,y) : 4 - x - y > 0\}$

R : All real numbers

$$z = \frac{x+y}{x \cdot y}$$

D : $\{(x, y) : x \neq 0 \text{ and } y \neq 0\}$

R : All real numbers

$$f(x, y) = 2x + y^2$$

$$f(1, 2) = 2(1) + (2)^2 = 6$$

$$f(-2, 4) = 2(-2) + (4)^2 = 12$$

$$\text{x-Difference Quotient} = \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

$$= \frac{\left[2(x + \Delta x) + y^2\right] - \left[2x + y^2\right]}{\Delta x}$$

$$= \frac{2 \cdot \Delta x}{\Delta x} = 2$$

$$\text{Y-Difference Quotient} = \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}$$

$$= \frac{\left[2x + (y+\Delta y)^2\right] - \left[2x + y^2\right]}{\Delta y}$$

$$= \frac{\left[\cancel{2x} + \cancel{y^2} + 2 \cdot y \cdot \Delta y + (\Delta y)^2\right] - \left[\cancel{2x} + \cancel{y^2}\right]}{\Delta y}$$

$$= 2y + \Delta y$$

$$f(x, y) = 3x^2 - 2y$$

$$x\text{-}DQ = \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

$$= \frac{\left[3\cdot(x + \Delta x)^2 - 2y\right] - \left[3x^2 - 2y\right]}{\Delta x}$$

$$= \frac{\left[3\left(x^2 + 2\cdot x\cdot\Delta x + (\Delta x)^2\right) - 2y\right] - \left[3x^2 - 2y\right]}{\Delta x}$$

$$= \frac{\left[\cancel{3x^2} + 6\cdot x\cdot\Delta x + 3\cdot(\Delta x)^2 \cancel{-2y}\right] - \cancel{\left[3x^2 - 2y\right]}}{\Delta x}$$

$$= 6x + 3\cdot\Delta x$$

Draw Countour Map or Level Curves

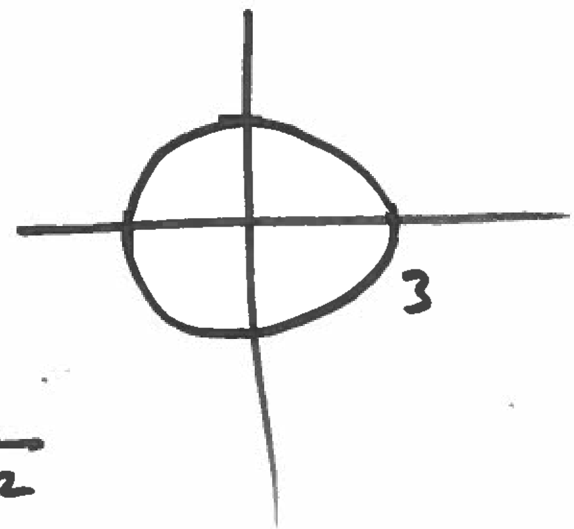
$$z = f(x, y) = \sqrt{9 - x^2 - y^2}$$

Draw Level curve corresponding to $c = 0$

$$0 = \sqrt{9 - x^2 - y^2}$$

$$0 = 9 - x^2 - y^2$$

$$x^2 + y^2 = 9$$



For $c = 1$:

$$1 = \sqrt{9 - x^2 - y^2}$$

$$1 = 9 - x^2 - y^2$$

$$x^2 + y^2 = 8$$