Section 13.1 Notes

Example 1: Let $f(x, y) = xy$

a) $f(1,5) = \quad (1)(5) = 5$

b) $f(a,b) = (a)(b) = ab$

c) Domain of $f(x, y)$ is $\{(x, y) :\ x$ is a real number; y is a real number$\}$

d) Range of $f(x, y)$ is {All real numbers}.

Example 2: Let $f(x, y) = \ln(3x + y)$.

a) $f(-5,15) = \quad \ln(0) =$ undefined

b) $f(-4,1) = \quad \ln(-11) =$ undefined

c) Domain of $f(x, y)$ is $\{(x, y) :\ \ 3x + y > 0\}$

d) Range of $f(x, y)$ is $\{-\infty, \infty\}$

Example 3: Let $f(x,y) = x + 2y^2$

a) $f(x+\Delta x) = (x+\Delta x) + 2y^2$

b) $f(y+\Delta y) = x + 2(y+\Delta y)^2$

c) $\dfrac{f(x+\Delta x) - f(x,y)}{\Delta x} = \dfrac{\left[(x+\Delta x) + 2y^2\right] - \left[x + 2y^2\right]}{\Delta x}$

d) $\dfrac{f(y+\Delta y) - f(x,y)}{\Delta y} = \dfrac{\left[x + 2(y+\Delta y)^2\right] - \left[x + 2y^2\right]}{\Delta y}$

Example 4: Let $f(x,y) = 7x^2 + y^2$

a) Domain of $f(x,y)$ is $\{(x,y):$ x is a real number; y is a real number$\}$

b) Range of $f(x,y)$ is $(-\infty,\infty)$

Example 5: Let $f(x,y) = -2e^{xy}$

a) Domain of $f(x,y)$ is $\{(x,y):$ x is a real number; y is a real number$\}$

b) Range of $f(x,y)$ is $(-\infty,\infty)$.

Example 6: Let $f(x,y) = -2y\sqrt{x+y}$

a) Domain of $f(x,y)$ is $\{(x,y): \ x+y>0\}$

b) Range of $f(x,y)$ is $(-\infty,\infty)$.

Example 7: Let $f(x,y) = \dfrac{x-y}{x+4y}$

a) Domain of $f(x,y)$ is $\{(x,y): \ x+4y \neq 0\}$

b) Range of $f(x,y)$ is $(-\infty,\infty)$.

Example 8: Let $f(x,y) = \ln(4x+3y)$

a) Domain of $f(x,y)$ is $\{(x,y): \ 4x+3y>0\}$

b) Range of $f(x,y)$ is $(-\infty,\infty)$.

Example 9: Let $f(x,y) = x^2 + y^2 + 4$

a) Domain of $f(x,y)$ is $\{(x,y): \ x$ is a real number; y is a real number$\}$

b) Range of $f(x,y)$ is $z \geq 4$.

Example 10 : Let $f(x,y)=\sqrt{x^2-y^2+4}$

a) Domain of $f(x,y)$ is $\{(x,y): x^2-y^2+4\geq 0\}$

b) Range of $f(x,y)$ is $z\geq 0$.

Example 11: $z=1-x+y$

a) Level Curve or Contour Map corresponding to $c=0: 1-x+y=0$

Note: Level curve is intersection of $z=1-x+y$ and the plane $z=0$.

b) Level Curve or Contour Map corresponding to $c=2: 1-x+y=2$

Note: Level curve is intersection of $z=1-x+y$ and the plane $z=2$.

c) Level Curve or Contour Map corresponding to $c=4: 1-x+y=4$

Note: Level curve is intersection of $z=1-x+y$ and the plane $z=4$.

Exmaple 12 : $z = x^2 + y^2 + 4$

a) Level Curve or Contour Map corresponding to $c = 4$: $x^2 + y^2 + 4 = 0$

Note: Level curve is intersection of $z = x^2 + y^2 + 4$ and the plane $z = 4$.

b) Level Curve or Contour Map corresponding to $c = 8$: $x^2 + y^2 + 4 = 8$

Note: Level curve is intersection of $z = x^2 + y^2 + 4$ and the plane $z = 8$.

c) Level Curve or Contour Map corresponding to $c = 13$: $x^2 + y^2 + 4 = 13$

Note: Level curve is intersection of $z = x^2 + y^2 + 4$ and the plane $z = 13$.