13.4 Differentials

Example 1

$$z = 2x^4 \cdot y - 8 \cdot x^2 \cdot y^3$$

$$\frac{\partial z}{\partial x} = (2y)(4x^3) - (8y^3)(2x)$$

$$= 8x^3y - 16xy^3$$

$$\frac{\partial z}{\partial y} = (2x^4)(1) - (8x^2)(3y^2)$$

$$= 2x^4 - 24x^2y^2$$

$$dz = \frac{\partial z}{\partial x} \cdot dx + \frac{\partial z}{\partial y} \cdot dy$$

$$dz = (8x^3y - 16xy^3)dx + (2x^4 - 24x^2y^2)dy$$

Example

$$w = \frac{x+y}{z-3y} = \frac{1}{z-3y}\cdot(x+y) = (z-3y)^{-1}\cdot(x+y)$$

②

$$\frac{\partial w}{\partial x} = \left(\frac{1}{z-3y}\right)(1) = \frac{1}{z-3y}$$

$$\frac{\partial w}{\partial y} = \frac{(z-3y)(1) - (x+y)(-3)}{(z-3y)^2} = \frac{z+3x}{(z-3y)^2}$$

$$\frac{\partial w}{\partial z} = (x+y)\cdot\left[-1(z-3y)^{-2}(1)\right] = \frac{-(x+y)}{(z-3y)^2}$$

$$dw = \frac{\partial w}{\partial x}\cdot dx + \frac{\partial w}{\partial y}\cdot dy + \frac{\partial w}{\partial z}\cdot dz$$

$$dw = \left(\frac{1}{z-3y}\right)dx + \left(\frac{z+3x}{(z-3y)^2}\right)dy + \frac{-(x+y)}{(z-3y)^2}dz$$

Example 3

$$w = e^{y} \cdot \cos x + z^{2}$$

$$\frac{\partial w}{\partial x} = e^{y} \cdot (-\sin x) = -e^{y} \sin x$$

$$\frac{\partial w}{\partial y} = (\cos x)(e^{y}) = e^{y} \cos x$$

$$\frac{\partial w}{\partial z} = 2z$$

$$dw = \frac{\partial w}{\partial x} \cdot dx + \frac{\partial w}{\partial y} \cdot dy + \frac{\partial w}{\partial z} \cdot dz$$

$$dw = (-e^{y} \cdot \sin x)dx + (e^{y} \cos x)dy + (2z)dz$$

Example 4

$z = f(x,y) = x^2 + y^2$

Find Δz when $x = 2$ and $y = 1$

$x = 2 \rightarrow x = 2.1$

$y = 1 \rightarrow 1.05$

$f(2,1) = (2)^2 + (1)^2 = 5$

$f(2.1, 1.05) = (2.1)^2 + (1.05)^2 = 5.5125$

$\Delta z = f(2.1, 1.05) - f(2,1) = 5.5125 - 5 = 0.5125$

$$dz = \frac{\partial z}{\partial x} \cdot dx + \frac{\partial z}{\partial y} dy$$

$$dz = (2x) \cdot \Delta x + (2y) \cdot \Delta y$$

$$= (2x)(0.1) + (2y)(0.05)$$

At $x = 2$, $y = 1$: $dz = (2 \cdot 2)(0.1) + (2 \cdot 1)(0.05) = 0.5$

Example 5

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

$R_1 : 10 \to 10.5$

$\Delta R_1 = 0.5$

$R_2 : 15 \to 13$

$\Delta R_2 = 13 - 15 = -2$

Find ΔR.

$$R(10, 15) = \frac{(10)(15)}{10 + 15} = \frac{150}{25} = 6$$

$$R(10.5, 13) = \frac{(10.5)(13)}{(10.5) + (13)} = 5.8085$$

$$\Delta R = R(10.5, 13) - R(10, 15) = 5.8085 - 6$$

$$= -0.191489$$

$$R = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

$$\frac{\partial R}{\partial R_1} = \frac{(R_1 + R_2)(R_2) - (R_1 R_2)(1)}{(R_1 + R_2)^2}$$

$$\left.\frac{\partial R}{\partial R_1}\right|_{\substack{R_1 = 10 \\ R_2 = 15}} = \frac{(10+15)(15) - (10)(15)}{(10+15)^2} = \frac{225}{625}$$

$$\frac{\partial R}{\partial R_2} = \frac{(R_1 + R_2)(R_1) - (R_1 R_2)(1)}{(R_1 + R_2)^2} = \frac{100}{625}$$

$$\Delta R \approx dR = \frac{\partial R}{\partial R_1} \cdot \Delta R_1 + \frac{\partial R}{\partial R_2} \cdot \Delta R_2$$

$$= \left(\frac{225}{625}\right)(0.5) + \left(\frac{100}{625}\right)(-2) = -0.14$$