

## 13.8 Extremum

$$f(x, y) = 2x^2 + y^2 + 8x - 6y + 20$$

$$f_x = 4x + 8$$

$$f_y = 2y - 6$$

$$\text{Set } f_x = 0 \quad ; \quad f_y = 0$$

$$4x + 8 = 0 \quad ; \quad 2y - 6 = 0$$

$$x = -2 \quad ; \quad y = 3$$

$$z = f(-2, 3) = 2(-2)^2 + (3)^2 + 8(-2) - 6(3) + 20 = 3$$

~~Ext~~  $f(x, y)$  has extremum at  $(-2, 3, 3)$

$$f(x, y) = 1 - (x^2 + y^2)^{1/3}$$

$$f_x = 0 - \frac{1}{3}(x^2 + y^2)^{-2/3}(2x)$$

$$f_x = -\frac{2x}{3} \cdot \frac{1}{(x^2 + y^2)^{2/3}} = \frac{-2x}{3(x^2 + y^2)^{2/3}}$$

$$f_y = 0 - \frac{1}{3}(x^2 + y^2)^{-2/3}(2y)$$

$$= \frac{-2y}{3(x^2 + y^2)^{2/3}}$$

$$\text{Set } f_x = 0$$

$$\frac{-2x}{3(x^2 + y^2)^{2/3}} = 0$$

$$f_y = 0$$

$$\frac{-2y}{3(x^2 + y^2)^{2/3}} = 0$$

$$\Rightarrow \begin{array}{l} -2x = 0 \\ x = 0 \end{array} ; \quad \begin{array}{l} -2y = 0 \\ y = 0 \end{array}$$

$$3(x^2 + y^2)^{2/3} = 0$$

$\Rightarrow$  Critical Point =  $(x=0, y=0)$

$$\begin{aligned} z = f(x, y) &= 1 - (x^2 + y^2)^{1/3} \\ &= 1 - (0^2 + 0^2)^{1/3} = 1 \end{aligned}$$

Extremum at  $(0, 0, 1)$

$$f(x, y) = -x^3 + 4xy - 2y^2 + 1$$

$$f_x = -3x^2 + 4y$$

$$f_y = 4x - 4y$$

$$\text{Set } f_x = 0$$

$$f_y = 0$$

$$\begin{array}{r} -3x^2 + 4y = 0 \quad (1) \\ + \quad 4x - 4y = 0 \quad (2) \\ \hline \end{array}$$

$$-3x^2 + 4x = 0$$

$$x \cdot (-3x + 4) = 0$$

$$x = 0$$

$$(2) \quad 4x - 4y = 0$$

$$4(0) - 4y = 0$$

$$y = 0$$

$$(0, 0)$$

$$-3x + 4 = 0$$

$$x = 4/3$$

$$(2) \quad 4x - 4y = 0$$

$$4(4/3) - 4y = 0$$

$$y = 4/3$$

$$\text{Critical points: } (0, 0) \quad ; \quad \left(\frac{4}{3}, \frac{4}{3}\right)$$
$$\Rightarrow (0, 0, 1) \quad \Rightarrow \left(\frac{4}{3}, \frac{4}{3}, \frac{59}{27}\right)$$

Determine if  $(0, 0, 1)$  is relative max, min,  
or saddle point.

$$f_{xx} = -6x \quad f_{xx}(0, 0) = -6(0) = 0$$

$$f_{yy} = -4 \quad f_{yy}(0, 0) = -4$$

$$f_{xy} = 4 \quad f_{xy}(0, 0) = 4$$

$$d = f_{xx}(0, 0) \cdot f_{yy}(0, 0) - [f_{xy}(0, 0)]^2$$

$$d = (0)(-4) - [4]^2 = -16 < 0$$

$\therefore (0, 0, 1)$  is a ~~relative max.~~ saddle point

$$f_{xx} \left( \frac{4}{3}, \frac{4}{3} \right) = -6 \left( \frac{4}{3} \right) = -8$$

$$f_{yy} \left( \frac{4}{3}, \frac{4}{3} \right) = -4$$

$$f_{xy} \left( \frac{4}{3}, \frac{4}{3} \right) = 4$$

$$d = (-8)(-4) - [4]^2 = 16 > 0$$

Note:  $d > 0$  and  $f_{xx} < 0$ .

$\Rightarrow \left( \frac{4}{3}, \frac{4}{3}, \frac{59}{27} \right)$  is a relative max.

Section 13.8 : Find Extremum

$$z = f(x, y) = x^2 y^2 \geq 0$$

$$f_x = 2x y^2$$

$$f_y = 2y x^2$$

$$f_{xx} = 2y^2$$

$$f_{yy} = 2x^2$$

$$f_{xy} = 4xy$$

To find critical point:

$$\text{set } f_x = 0$$

$$f_y = 0$$

$$(2x)(y^2) = 0 \Rightarrow 2x=0 ; y^2=0$$

$$x=0 \quad y=0$$

$$(2y)(x^2) = 0$$

$$2y=0 ; x^2=0$$

$$y=0 ; x=0$$

Critical point  $(0, 0)$

$$x=0, y=0, z = x^2 y^2 = 0$$

$$(0, 0, 0)$$

Determine what is happening at  $(0, 0, 0)$ :

$$f_{xx}(0, 0) = 0$$

$$f_{yy}(0, 0) = 0$$

$$f_{xy}(0, 0) = 0$$

$$d = f_{xx} \cdot f_{yy} - [f_{xy}]^2 = (0)(0) - (0)^2 = 0$$

$(0, 0, 0)$  is a relative minimum because

$$z = f(x, y) = x^2 y^2 \geq 0$$



$$f(x, y) = \sin(xy)$$

$$0 \leq x \leq \pi$$

$$0 \leq y \leq 1$$

$$f_x = \cos(xy) \cdot y = y \cos(xy)$$

$$f_y = \cos(xy) \cdot x = x \cos(xy)$$

To find critical point: set  $f_x = 0$  and  $f_y = 0$

$$\cancel{y} \cos[xy] \cos(xy) = 0$$

$$y = 0 ; \cos(xy) = 0$$

$$y = 0 ; \boxed{xy = \pi/2}$$

$$x \cdot \cos(xy) = 0$$

$$x = 0 \quad | \quad \begin{array}{l} \cos(xy) = 0 \\ xy = \pi/2 \end{array}$$

Critical points:  $(x=0, y=0)$

$$\{(x, y) : xy = \pi/2\}$$

For  $x=0, y=0, z = f(x,y) = \sin(xy) = \sin(0) = 0$   
 $(x=0, y=0, z=0)$

$$z = f(x,y) = \sin(xy) \geq 0 \quad 0 \leq x \leq \pi$$
$$0 \leq y \leq 1$$

Therefore ~~for~~  $(0, 0, 0)$  is a relative min.

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For  $\{(x,y): xy = \pi/2\}$

$$\#. z = f(x,y) = \sin(xy) = \sin(\pi/2) = 1$$

$\{(x,y,1) : xy = \pi/2\}$  is absolute max.