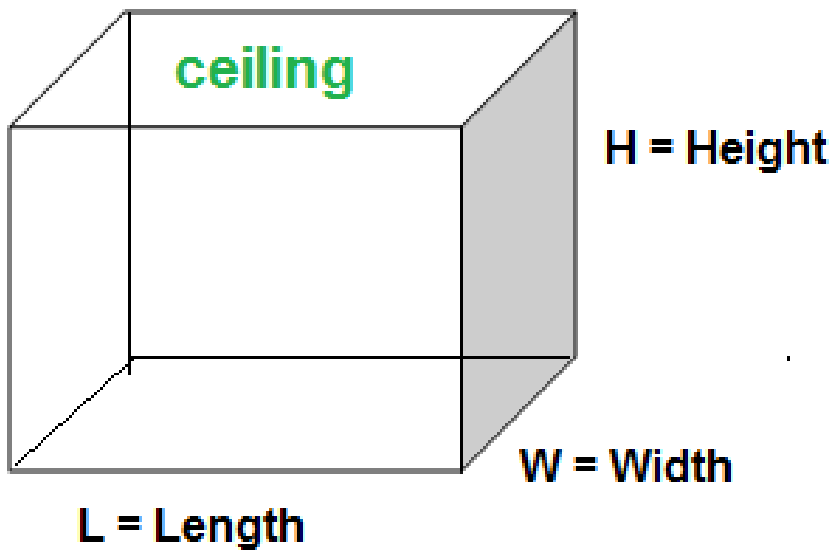
Section 13.9

9)



Area of left wall = $H \cdot W$; Cost of paint for left wall = $(H \cdot W)(0.06)$

Area of right wall = $H \cdot W$; Cost of paint for right wall = $(H \cdot W)(0.06)$

Area of front wall = $H \cdot L$; Cost of paint for front wall = $(H \cdot L)(0.06)$

Area of back wall = $H \cdot L$; Cost of paint for back wall = $(H \cdot L)(0.06)$

Area of ceiling = $L \cdot W$; Cost of paint for ceiling = $(L \cdot W)(0.11)$

C = Cost of paint = $(H \cdot W)(0.06) + (H \cdot W)(0.06) + (H \cdot L)(0.06) + (H \cdot L)(0.06) + (L \cdot W)(0.11) = 0.12H \cdot W + 0.12H \cdot L + (L \cdot W)(0.11)$

Volume = 668.25 = $L \cdot W \cdot H$

$$L = \frac{668.25}{W \cdot H}$$

$$C = 0.12H \cdot W + 0.12H \cdot \frac{668.25}{W \cdot H} + \left(\frac{668.25}{W \cdot H} \cdot W\right)(0.11) = 0.12H \cdot W + \frac{80.19}{W} + \frac{73.5075}{H}$$

Find partial derivatives C_H and C_W; Set $C_H = 0$ and $C_W = 0$ and solve H and W.

13)

Let $x =$ number of pairs of running shoes

Let $y =$ number of pairs of basketball shoes

R = Revenue = $-5x^2 - 8y^2 - 2xy + 43x + 102y$

Find partial derivatives:

$R_x = -10x - 2y + 42$

$R_y = -16y - 2x + 102$

Set $R_x = 0$ and $R_y = 0$:

$R_x = -10x - 2y + 42 = 0$

$R_y = -16y - 2x + 102 = 0$

Solve this system of equations to find x and y that will maximize the Revenue function.

15)

Note: $p+q+r=1 \;\; \Rightarrow r=1-p-q$

$P(p,q,r)=2pq+2pr+2qr$

$P(p,q,r)=2pq+2p\left(1-p-q\right)+2q\left(1-p-q\right)=2p-2p^2+2q-2pq-2q^2$

Find partial derivatives:

$\mathrm{P_p}=2-4p-2q$

$\mathrm{P_q}=2-2p-4q$

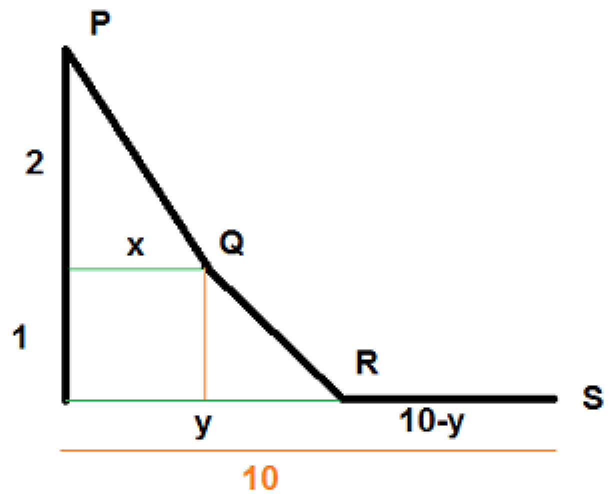
Set $\mathrm{P_p}=0$ and $\mathrm{P_q}=0$:

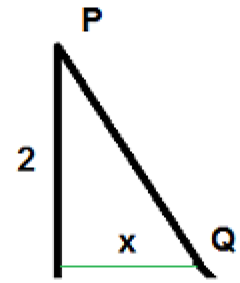
$\mathrm{P_p}=2-4p-2q=0$

$\mathrm{P_q}=2-2p-4q=0$

Solve this system of equations to find p and q.

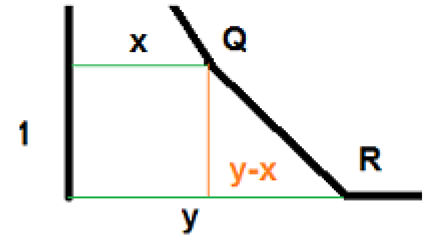
17)





length of PQ = $\sqrt{x^2+2^2}=\sqrt{x^2+4}$

Cost of construction for PQ = $(3k)\sqrt{x^2+4}$



length of QR = $\sqrt{(y-x)^2+1^2}=\sqrt{y^2-2xy+x^2+1}$

Cost of construction for QR = $(2k)\sqrt{y^2-2xy+x^2+1}$

length of RS $=10-y$

Cost of construction for RS = $(k)(10-y)$

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Cost of construction for QR = $(2k)\sqrt{y^2-2xy+x^2+1}$

length of RS $=10-y$

Cost of construction for RS = $(k)(10-y)$

$C=$ total cost of construction = $(3k)\sqrt{x^2+4}+(2k)\sqrt{y^2-2xy+x^2+1}+(k)(10-y)$

Find partial derivatives:

$$C_x=3k\left[\frac{1}{2}\left(x^2+4\right)^{-1/2}(2x)\right]+2k\left(\frac{1}{2}\right)\left(y^2-2xy+x^2+1\right)^{-1/2}(-2y+2x)$$

$$C_x=\frac{3kx}{\left(x^2+4\right)^{1/2}}+\frac{k(-2y+2x)}{\left(y^2-2xy+x^2+1\right)^{1/2}}$$

$$C_y=2k\left[\frac{1}{2}\left(y^2-2xy+x^2+1\right)^{-1/2}(2y-2x)\right]+k(-1)=\frac{k(2y-2x)}{\left(y^2-2xy+x^2+1\right)^{1/2}}-k$$

Set $C_x=0$ and $C_y=0$:

$$C_x=\frac{3kx}{\left(x^2+4\right)^{1/2}}+\frac{k(-2y+2x)}{\left(y^2-2xy+x^2+1\right)^{1/2}}=0 \qquad \text{Equ. 1}$$

$$C_y=\frac{k(2y-2x)}{\left(y^2-2xy+x^2+1\right)^{1/2}}-k=0 \qquad \text{Equ. 2}$$

Set $C_x = 0$ and $C_y = 0$; solve for x and y:

$$C_x = \frac{3kx}{\left(x^2+4\right)^{1/2}} + \frac{k\left(-2y+2x\right)}{\left(y^2-2xy+x^2+1\right)^{1/2}} = 0 \qquad \text{Equ. 1}$$

$$C_y = \frac{k\left(2y-2x\right)}{\left(y^2-2xy+x^2+1\right)^{1/2}} - k = 0 \qquad \text{Equ. 2}$$

Equ. 2: $\dfrac{\left(2y-2x\right)}{\left(y^2-2xy+x^2+1\right)^{1/2}} - 1 = 0$ divide each term by k

Equ. 2: $\dfrac{\left(2y-2x\right)}{\left(y^2-2xy+x^2+1\right)^{1/2}} = 1 \quad \Rightarrow \quad 2y-2x = \left(y^2-2xy+x^2+1\right)^{1/2}$

Equ. 1: $\dfrac{3kx}{\left(x^2+4\right)^{1/2}} + \dfrac{k\left(-2y+2x\right)}{\left(y^2-2xy+x^2+1\right)^{1/2}} = 0$

Equ. 1: $\dfrac{3x}{\left(x^2+4\right)^{1/2}} + \dfrac{\left(-2y+2x\right)}{\left(y^2-2xy+x^2+1\right)^{1/2}} = 0$ divide each term by k

Equ. 1: $\dfrac{3x}{\left(x^2+4\right)^{1/2}} + \dfrac{\left(-2y+2x\right)}{2y-2x} = 0 \quad \Rightarrow \quad \dfrac{3x}{\left(x^2+4\right)^{1/2}} + \dfrac{-\left(2y-2x\right)}{2y-2x} = 0 \quad \Rightarrow \quad \dfrac{3x}{\left(x^2+4\right)^{1/2}} - 1 = 0$

Equ. 1: $\dfrac{3x}{\left(x^2+4\right)^{1/2}} = 1$

Equ. 1: $3x = \left(x^2+4\right)^{1/2} \quad \Rightarrow \quad \left(3x\right)^2 = x^2+4 \quad \Rightarrow \quad 9x^2 = x^2+4 \quad \Rightarrow \quad 8x^2 = 4$

$$\Rightarrow \quad x^2 = \frac{1}{2} \quad \Rightarrow \quad x = \sqrt{\frac{1}{2}}$$

Finding y when $x=\sqrt{\frac{1}{2}}$:

Equ. 2: $$\frac{(2y-2x)}{\left(y^2-2xy+x^2+1\right)^{1/2}}=1 \quad \Rightarrow \quad 2y-2x=\left(y^2-2xy+x^2+1\right)^{1/2}$$

$$2y-2\sqrt{\frac{1}{2}}=\left(y^2-2\sqrt{\frac{1}{2}}y+\left(\sqrt{\frac{1}{2}}\right)^2+1\right)^{1/2}$$

$$\left(2y-2\sqrt{\frac{1}{2}}\right)^2=y^2-2\sqrt{\frac{1}{2}}y+\left(\sqrt{\frac{1}{2}}\right)^2+1$$ square both sides

$$4y^2-8\sqrt{\frac{1}{2}}y+2=y^2-2\sqrt{\frac{1}{2}}y+1.5$$

$$3y^2-6\sqrt{\frac{1}{2}}y+0.5=0$$

$y=1.2844570503761734$ Using Quadratic Formula