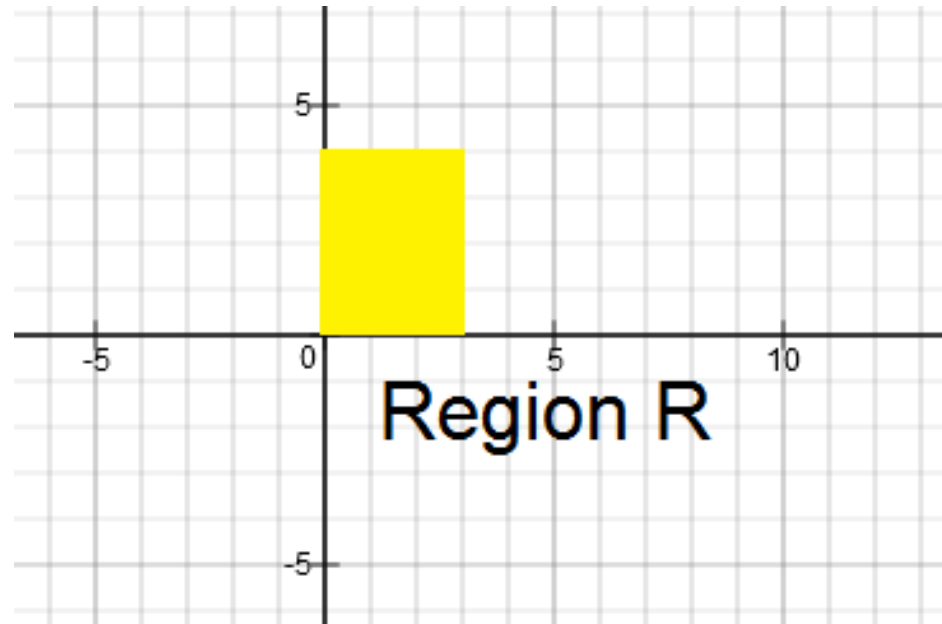
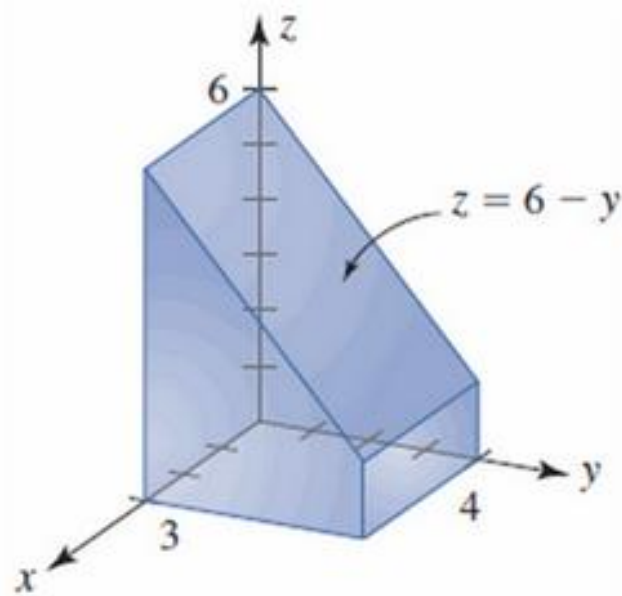


Example 1: Use a double integral to find the volume of the following solid

Surface Function: $f(x, y) = z = 6 - y$



Order of Integration: $dx dy$

$$\text{Volume} = \int_0^4 \int_0^3 f(x+y) dx dy = \int_0^4 \int_0^3 (6-y) dx dy$$

Order of Integration: $dy dx$

$$\text{Volume} = \int_0^3 \int_0^4 f(x+y) dy dx = \int_0^3 \int_0^4 (6-y) dy dx$$

Evaluating the double integral:

$$\text{Volume} = \int_0^4 \int_0^3 f(x+y) dx dy = \int_0^4 \int_0^3 (6-y) dx dy$$

$$\int_0^3 (6-y) dx = \left[(6-y)x \right]_0^3 = (6-y)(3) = 18 - 3y$$

$$\int_0^4 \int_0^3 (6-y) dx dy = \int_0^4 (18 - 3y) dy = \left[18y - 3 \cdot \frac{y^2}{2} \right]_0^4 = 48$$

Evaluating the double integral:

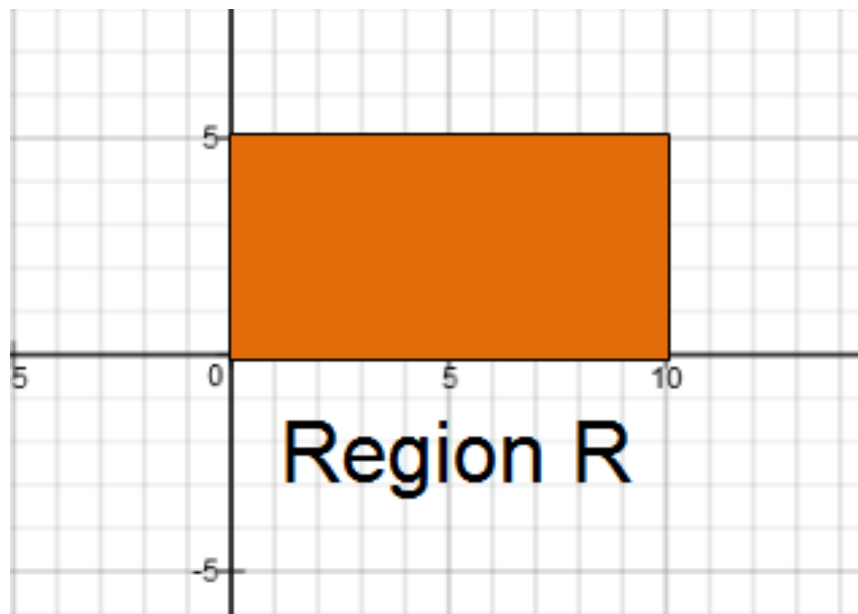
$$\text{Volume} = \int_0^3 \int_0^4 f(x+y) dy dx = \int_0^3 \int_0^4 (6-y) dy dx$$

$$\int_0^4 (6-y) dy = \left[\left(6y - \frac{y^2}{2} \right) \right]_0^4 = 24 - 8 = 16$$

$$\int_0^3 \int_0^4 (6-y) dy dx = \int_0^3 16 dx = 48$$

Example 2: Use a double integral to find the volume of the solid that is formed by the following surface function and region R .

Surface Function: $f(x, y) = 1 + 4y$



Order of Integration: $dx dy$

$$\text{Volume} = \int_0^5 \int_0^{10} f(x+y) dx dy = \int_0^5 \int_0^{10} (1+4y) dx dy$$

Order of Integration: $dy dx$

$$\text{Volume} = \int_0^{10} \int_0^5 f(x+y) dy dx = \int_0^{10} \int_0^5 (1+4y) dy dx$$

Evaluating the double integral:

$$\text{Volume} = \int_0^5 \int_0^{10} f(x+y) dx dy = \int_0^5 \int_0^{10} (1+4y) dx dy$$

$$\int_0^{10} (1+4y) dx = \left[(1+4y)x \right]_0^{10} = (1+4y)(10) = 10 + 40y$$

$$\int_0^5 \int_0^{10} (1+4y) dx dy = \int_0^5 (10+40y) dy = \left[10y + 20y^2 \right]_0^5 = 550$$

Evaluating the double integral:

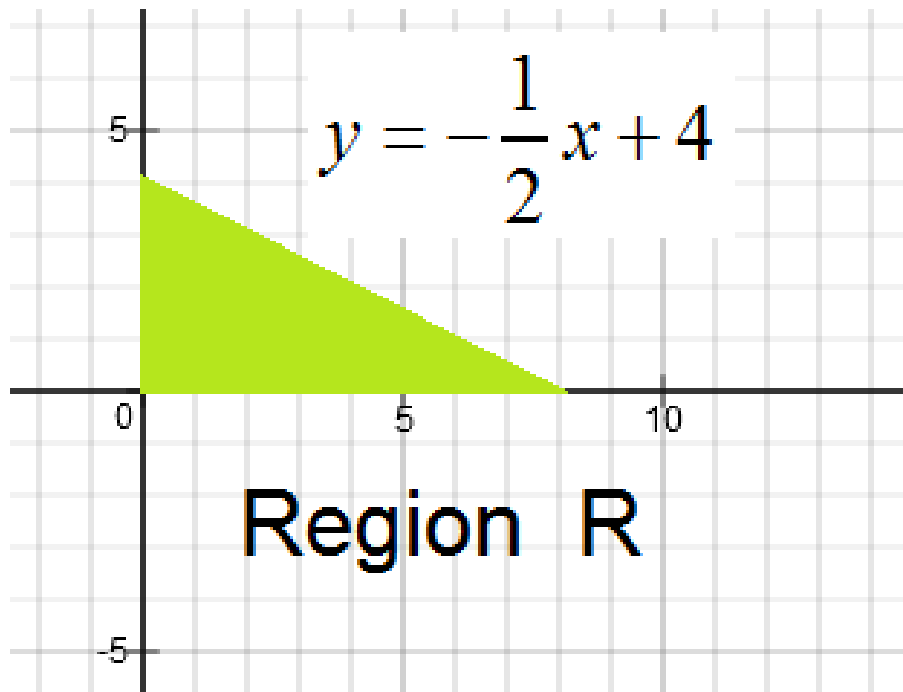
$$\text{Volume} = \int_0^{10} \int_0^5 f(x+y) dy dx = \int_0^{10} \int_0^5 (1+4y) dy dx$$

$$\int_0^5 (1+4y) dy = \left[y + 2y^2 \right]_0^5 = 5 + 50 = 55$$

$$\int_0^{10} \int_0^5 (1+4y) dy dx = \int_0^{10} 55 dx = \left[55x \right]_0^{10} = 550$$

Example 3: Use a double integral to find the volume of the solid formed by the following surface function and region R .

Surface Function: $f(x, y) = 5 - 4xy$



Order of Integration: $dy \cdot dx$

$$\text{Volume} = \int_0^8 \int_{y=0}^{y=-\frac{1}{2}x+4} f(x+y) dy dx = \int_0^8 \int_{y=0}^{y=-\frac{1}{2}x+4} (5 - 4xy) dy dx$$

Order of Integration: $dy \cdot dx$

$$\text{Volume} = \int_0^8 \int_{y=0}^{y=-\frac{1}{2}x+4} f(x+y) dy dx = \int_0^8 \int_{y=0}^{y=-\frac{1}{2}x+4} (5-4xy) dy dx$$

$$\int_{y=0}^{y=-\frac{1}{2}x+4} (5-4xy) dy = \left[5y - 4x \frac{y^2}{2} \right]_0^{-\frac{1}{2}x+4} = \left[5y - 2xy^2 \right]_0^{-\frac{1}{2}x+4}$$

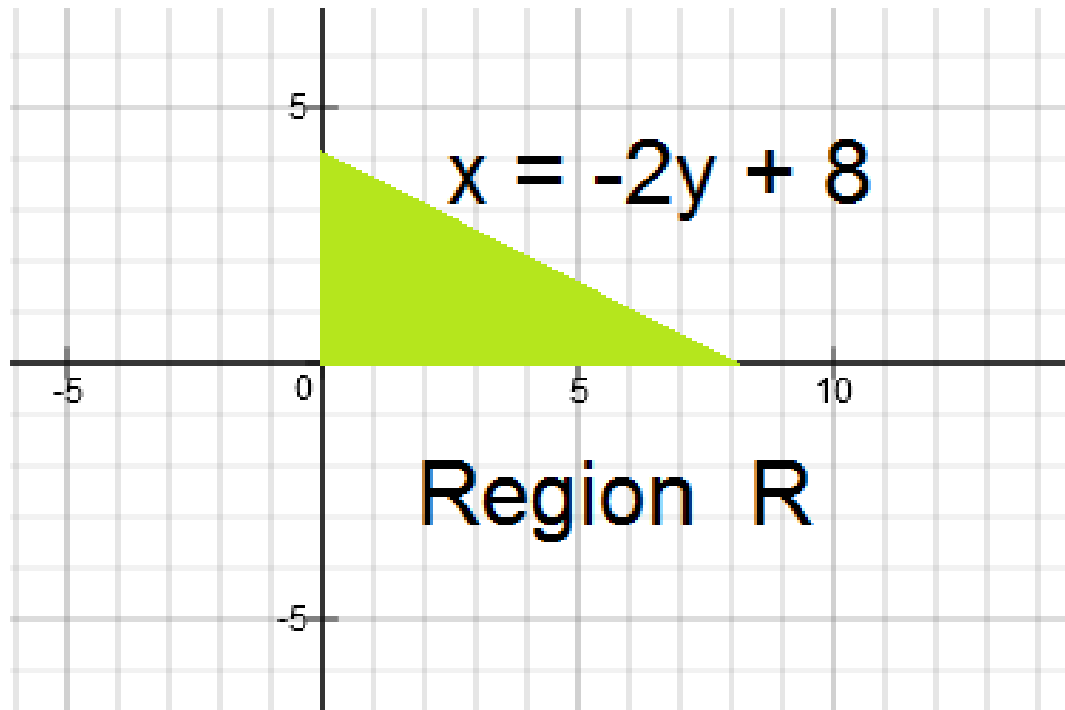
$$= 5 \left(-\frac{1}{2}x + 4 \right) - 2x \left(-\frac{1}{2}x + 4 \right)^2 = -\frac{5}{2}x + 20 - 2x \left(-\frac{1}{2}x + 4 \right)^2$$

$$\int_0^8 \int_{y=0}^{y=-\frac{1}{2}x+4} (5-4xy) dy dx = \int_0^8 \left[-\frac{5}{2}x + 20 - 2x \left(-\frac{1}{2}x + 4 \right)^2 \right] dx = -90 \frac{2}{3}$$

Note: $y = \frac{-1}{2}x + 4$

$-2y = x - 8$ Multiply each term by -2

$x = -2y + 8$



Order of Integration: $dx \cdot dy$

$$\text{Volume} = \int_0^4 \int_{x=0}^{x=-2y+8} f(x+y) dx dy = \int_0^4 \int_{x=0}^{x=-2y+8} (5-4xy) dx dy$$

$$\int_{x=0}^{x=-2y+8} (5-4xy) dx = \left[5x - 4y \frac{x^2}{2} \right]_0^{-2y+8} = \left[5x - 2yx^2 \right]_0^{-2y+8}$$

$$= 5(-2y+8) - 2y(-2y+8)^2$$

$$\int_0^4 \int_{x=0}^{x=-2y+8} (5-4xy) dx dy = \int_0^4 \left[5(-2y+8) - 2y(-2y+8)^2 \right] dy = -90 \frac{2}{3}$$

Example 4: Use a double integral to find the volume of the solid formed by the following surface function and region R .

Surface Function: $f(x, y) = e^{-2x-3y}$

Region $R : \{(x, y) : 0 \leq x \leq \infty; 0 \leq y \leq \infty\}$

$$\text{Volume of solid} = \int_0^{\infty} \int_0^{\infty} f(x, y) dy dx = \int_0^{\infty} \int_0^{\infty} e^{-2x-3y} dy dx$$

$$\text{Volume of solid} = \int_0^{\infty} \int_0^{\infty} f(x, y) dx dy = \int_0^{\infty} \int_0^{\infty} e^{-2x-3y} dx dy$$

Evaluating the double integral with integration order $dydx$:

$$\text{Volume of solid} = \int_0^{\infty} \int_0^{\infty} f(x+y) dydx = \int_0^{\infty} \int_0^{\infty} e^{-2x-3y} dydx$$

Evaluating $\int_0^{\infty} e^{-2x-3y} dy$:

$$\text{Recall: } \int e^{ax} dx = \frac{1}{a} e^{ax}; \quad \int e^{ay} dy = \frac{1}{a} e^{ay}; \quad e^{-\infty} = 0$$

$$\begin{aligned} \int_0^{\infty} e^{-2x-3y} dy &= \int_0^{\infty} e^{-2x} e^{-3y} dy = e^{-2x} \int_0^{\infty} e^{-3y} dy = e^{-2x} \left[\frac{-1}{3} e^{-3y} \right]_0^{\infty} \\ &= \left(e^{-2x} \right) \left(\frac{-1}{3} e^{-\infty} \right) - \left(e^{-2x} \right) \left(\frac{-1}{3} e^0 \right) = \frac{1}{3} e^{-2x} \end{aligned}$$

$$\int_0^{\infty} \int_0^{\infty} e^{-2x-3y} dydx = \int_0^{\infty} \frac{1}{3} e^{-2x} dx = \frac{1}{3} \left[\frac{-1}{2} e^{-2x} \right]_0^{\infty} = \frac{1}{3} \left[\frac{-1}{2} e^{-\infty} \right] - \frac{1}{3} \left[\frac{-1}{2} e^0 \right] = \frac{1}{6}$$

Evaluating the double integral with integration order $dx dy$:

$$\text{Volume of solid} = \int_0^{\infty} \int_0^{\infty} f(x+y) dx dy = \int_0^{\infty} \int_0^{\infty} e^{-2x-3y} dx dy$$

$$\text{Recall: } \int e^{ax} dx = \frac{1}{a} e^{ax}; \quad \int e^{ay} dy = \frac{1}{a} e^{ay}; \quad e^{-\infty} = 0$$

$$\int_0^{\infty} e^{-2x-3y} dx = \int_0^{\infty} e^{-2x} e^{-3y} dx = e^{-3y} \int_0^{\infty} e^{-2x} dx = e^{-3y} \left[\frac{-1}{2} e^{-2x} \right]_0^{\infty}$$

$$= (e^{-3y}) \left(\frac{-1}{2} e^{-\infty} \right) - (e^{-3y}) \left(\frac{-1}{2} e^0 \right) = \frac{1}{2} e^{-3y}$$

$$\int_0^{\infty} \int_0^{\infty} e^{-2x-3y} dx dy = \int_0^{\infty} \frac{1}{2} e^{-3y} dy = \frac{1}{2} \left[\frac{-1}{3} e^{-3y} \right]_0^{\infty} = \frac{1}{2} \left[\frac{-1}{3} e^{-\infty} \right] - \frac{1}{2} \left[\frac{-1}{3} e^0 \right] = \frac{1}{6}$$