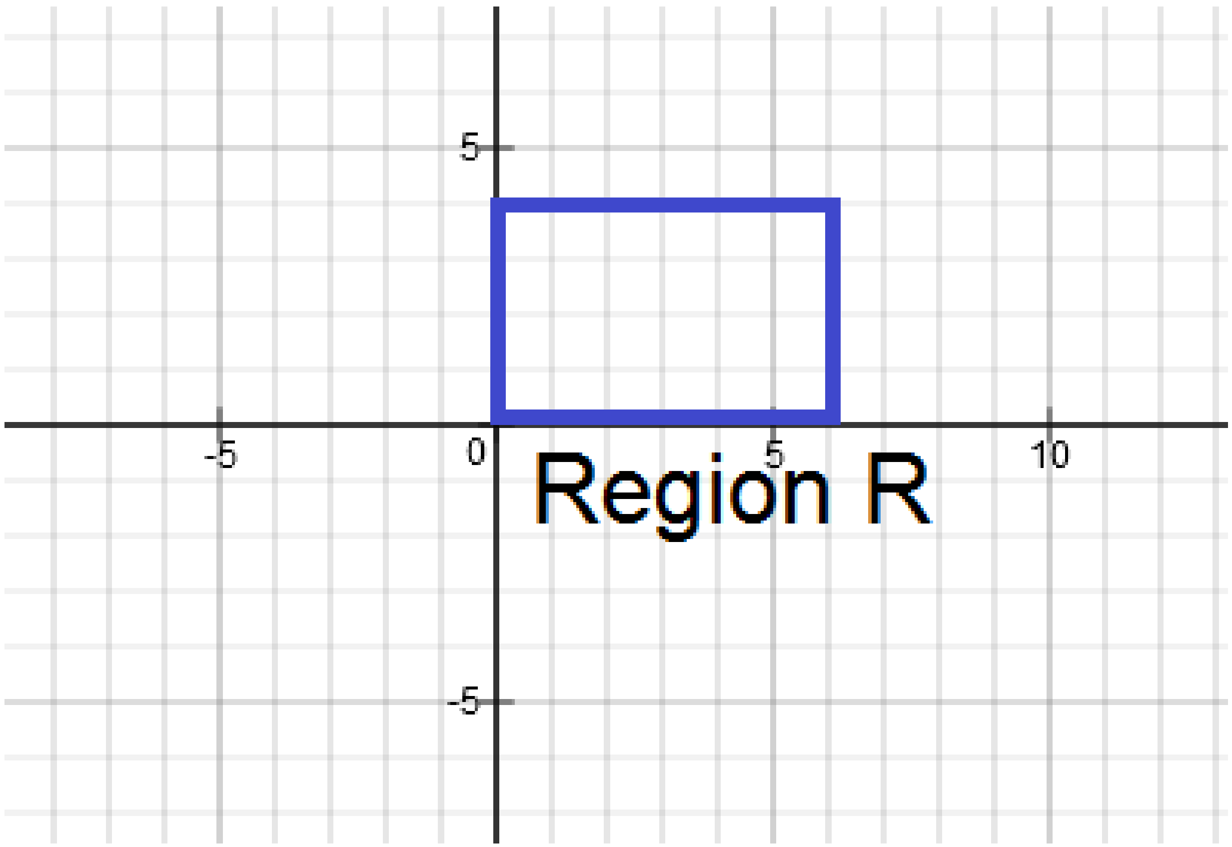
Example 1; Find the area of the surface over the given region R.

Let $f(x, y) = 5x + 2y$



Let $f(x, y) = 5x + 2y$; $\quad f_x = 5$; $\quad f_y = 2$

$$S = \iint_R \sqrt{1 + \left(f_x\right)^2 + \left(f_y\right)^2}\, dA$$

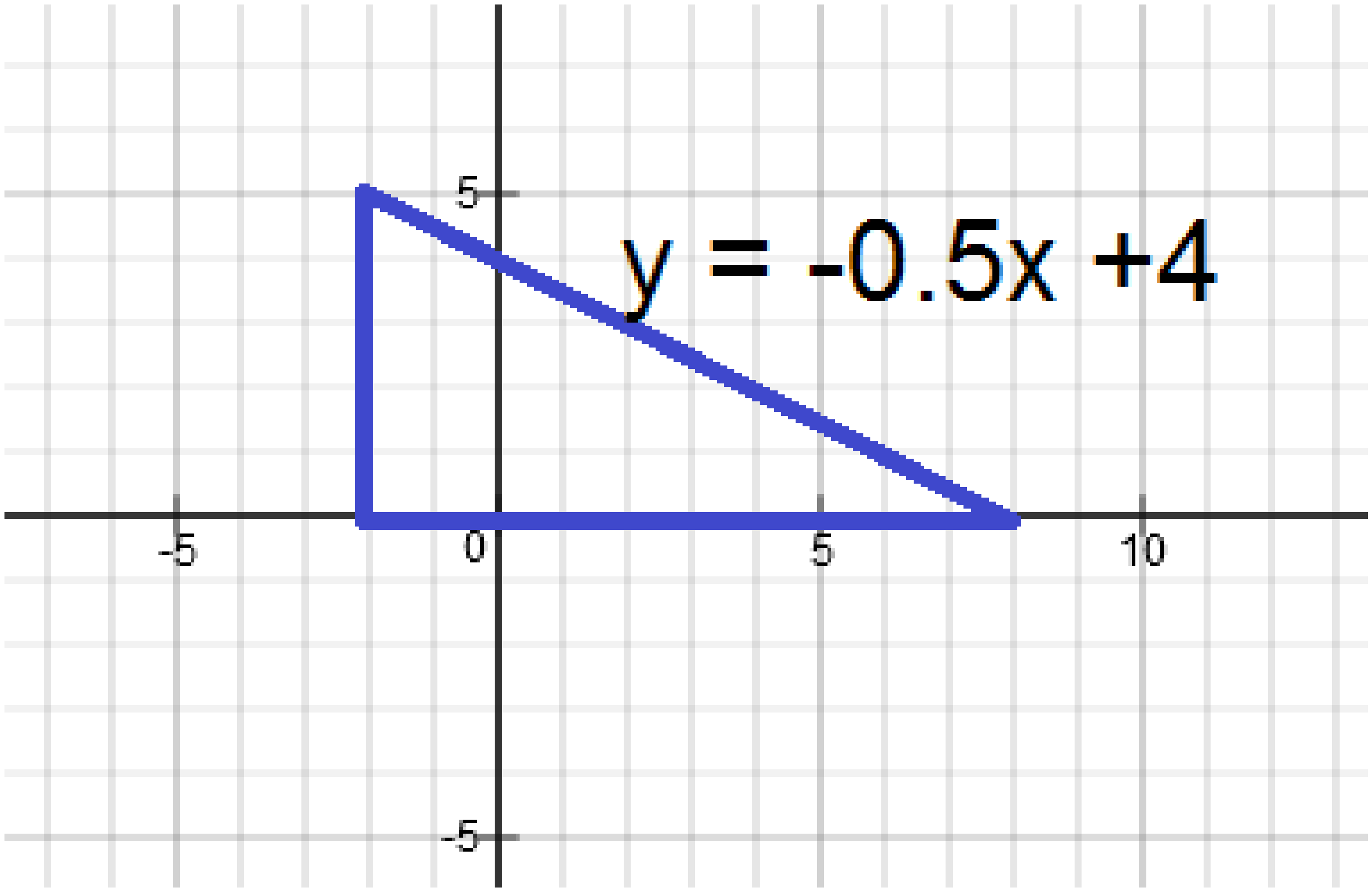
$$= \int_0^6 \int_0^4 \sqrt{1 + (5)^2 + (2)^2}\, dy dx = \int_0^6 \int_0^4 \sqrt{30}\, dy dx$$

Evaluating $\int_0^4 \sqrt{30}\, dy = \left[\sqrt{30}\, y\right]_0^4 = 4\sqrt{30}$

Hence, $S = \int_0^6 \int_0^4 \sqrt{30}\, dy dx = \int_0^6 4\sqrt{30}\, dx = 24\sqrt{30}$

Example 2: Find the area of the surface over the given region *R*.

Let $f(x, y) = 2 - x^2$



Example 2: Find the area of the surface over the given region R.

Let $f(x, y) = 2 - x^2$

$f_x = -2x;$ $\qquad f_y = 0$

$$S = \iint_R \sqrt{1 + \left(f_x\right)^2 + \left(f_y\right)^2}\, dA = \int_{-2}^{8} \int_{y=0}^{y=-0.5x+4} \sqrt{1 + (-2x)^2 + (0)^2}\, dy dx$$

$$= \int_{-2}^{8} \int_{y=0}^{y=-0.5x+4} \sqrt{1 + 4x^2}\, dy dx$$

Evaluating $\int\limits_{y=0}^{y=-0.5x+4} \sqrt{1+4x^2}\,dy = \left[\sqrt{1+4x^2}\,y\right]_0^{-0.5x+4}$

$$= \sqrt{1+4x^2}\left(-0.5x+4\right)$$

$$S = \int\limits_{-2}^{8} \int\limits_{y=0}^{y=-0.5x+4} \sqrt{1+4x^2}\,dy\,dx = \int\limits_{-2}^{8} \sqrt{1+4x^2}\left(-0.5x+4\right)dx$$

$$= 60.90287$$

Example 3: Find the area of the surface over the given region *R*.

$$f(x, y) = e^{2y}$$

Region R: $R = \{(x, y) : 0 \le x \le 1,\ 0 \le y \le 4\}$

$$f_x = 0; \qquad f_y = e^{2y} \cdot 2$$

$$S = \iint_R \sqrt{1 + (f_x)^2 + (f_y)^2}\, dA = \int_0^4 \int_0^1 \sqrt{1 + (0)^2 + \left(e^{2y} \cdot 2\right)^2}\, dx\, dy$$

Evaluating $\int_0^1 \sqrt{1 + 4e^{4y}}\, dx = \left[\sqrt{1 + 4e^{4y}}\, x\right]_0^1 = \sqrt{1 + 4e^{4y}}$

$$S = \int_0^4 \int_0^1 \sqrt{1 + (0)^2 + \left(e^{2y} \cdot 2\right)^2}\, dx\, dy = \int_0^4 \sqrt{1 + 4e^{4y}}\, dy = 2980.08$$

Example 4: Find the area of the surface over the given region R.

$$f(x, y) = x^2 + 3$$

Region R: $R = \{(x, y) : 0 \le x \le 1,\ 0 \le y \le 2x\}$

$$f_x = 2x; \qquad f_y = 0$$

$$S = \iint_R \sqrt{1 + (f_x)^2 + (f_y)^2}\, dA = \int_0^1 \int_0^{2x} \sqrt{1 + (2x)^2 + (0)^2}\, dy dx$$

$$= \int_0^1 \int_0^{2x} \sqrt{1 + 4x^2}\, dy dx$$

Evaluating $\int_0^{2x} \sqrt{1 + 4x^2}\, dy = \left[\sqrt{1 + 4x^2}\, y\right]_0^{2x} = \sqrt{1 + 4x^2}\,(2x)$

$$S = \int_0^1 \int_0^{2x} \sqrt{1 + (2x)^2 + (0)^2}\, dy dx = \int_0^1 \sqrt{1 + 4x^2}\,(2x)\, dx = 1.69672$$