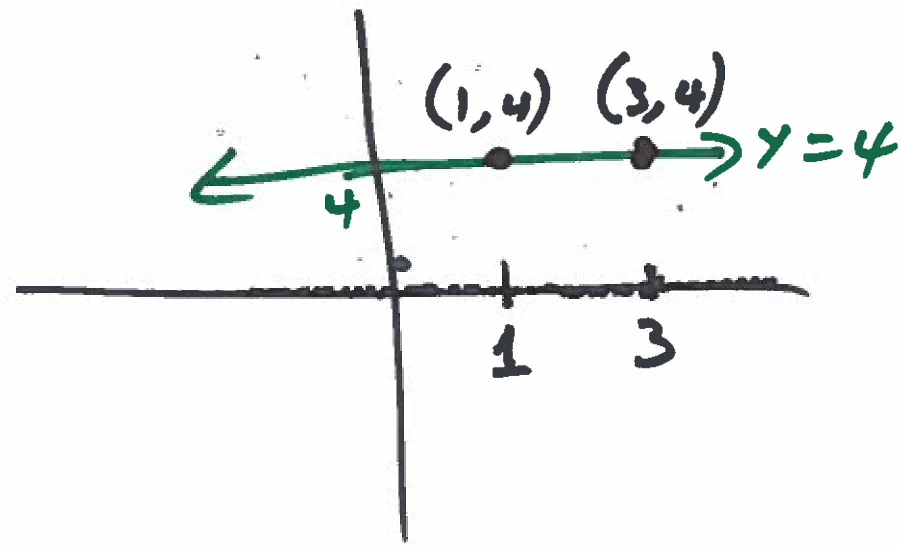
1.3 Evaluating Limits Analytically

$y = f(x) = 4$ constant function

$\lim_{x \to 1} f(x) = \underline{? \quad 4}$



$\lim_{x \to 3} f(x) = \underline{? \quad 4}$

$\lim_{x \to -5} f(x) = 4$

$\lim_{x \to 0} f(x) = 4$

Polynomial Functions (Exponents positive, whole number)

$f(x) = x^2 + 4x + 5$

$\lim_{x\to 0} f(x) = (0)^2 + 4(0) + 5 = 5$

$\lim_{x\to 3} f(x) = (3)^2 + 4(3) + 5 = 26$

$$f(x) = 0.1x^2 + 7$$

$$\lim_{x \to 2} f(x) = 0.1(2)^2 + 7 = 7.4$$

$$\lim_{x \to 0} f(x) = 0.1(0)^2 + 7 = 7$$

$$f(x) = -3x^3 + 4x + 7$$

$$\lim_{x \to -1} f(x) = -3 \cdot (-1)^3 + 4(-1) + 7 = 6$$

$f(x) = \sqrt{x+1}$ Square Root Function

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \sqrt{x+1} = \sqrt{\lim_{x \to 2} (x+1)}$$

$$= \sqrt{3}$$ Answer

$$f(x) = \sqrt{3x - 4}$$

$$\lim_{x \to 5} f(x) = \lim_{x \to 5} \sqrt{3x - 4}$$

$$= \sqrt{\lim_{x \to 5} (3x - 4)}$$

$$= \sqrt{11}$$ Answer

$$f(x) = \sqrt[3]{x+4}$$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \sqrt[3]{x+4}$$

$$= \sqrt[3]{\lim_{x \to 1} (x+4)}$$

$$= \sqrt[3]{5}$$

$$f(x) = \frac{3}{x+4}$$ Rational Function

$$\lim_{x \to 1} f(x) = \frac{3}{1+4} = \frac{3}{5}$$ Answer

$$\lim_{x \to -4} f(x) = \cancel{\frac{3}{-4+4} = \frac{3}{0} = \text{undefined}}$$ DNE

$$\lim_{x \to -4^-} f(x) = \underline{\ ?\quad -\infty\ }$$

$$\lim_{x \to -4^+} f(x) = \underline{\ ?\quad \infty\ }$$

$$f(x) = \frac{\sqrt{x+6}}{x+2}$$

$$\lim_{x \to 3} f(x) = \frac{\sqrt{3+6}}{3+2} = \frac{\sqrt{9}}{5} = \frac{3}{5}$$ Answer

$$f(x) = \sin x$$

$$\lim_{x \to \pi/2} f(x) = \sin(\pi/2) = 1$$

$$f(x) = \cos\left(\frac{\pi x}{3}\right)$$

$$\lim_{x \to 1} f(x) = \cos\left(\frac{\pi \cdot 1}{3}\right) = \cos(\pi/3) = \frac{1}{2}$$

$$f(x) = \frac{2x}{x^2 + 4x} = \frac{2x/x}{x^2/x + 4x/x} = \frac{2}{x+4}$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(\frac{2}{x+4} \right) = \frac{2}{0+4} = \frac{2}{4} = \frac{1}{2}$$

Recall:

$$a^2 - b^2 = (a-b)(a+b)$$

$$x^2 - 4 = (x-2)(x+2)$$

$$x^2 - 25 = (x-5)(x+5)$$

$$a - b = (-1)(b-a)$$

$$x - 5 = (-1)(5-x)$$

$$3 - y = (-1)(y-3)$$

$$f(x) = \frac{5 - x}{x^2 - 25} = \frac{(-1)\cancel{(x-5)}}{\cancel{(x-5)}(x+5)} = \frac{-1}{x+5}$$

$$\text{Find } \lim_{x \to 5} f(x) = \lim_{x \to 5} \left(\frac{-1}{x+5} \right) = -\frac{1}{10} \quad \text{Answer}$$

$$f(x) = \frac{x^2 + 2x - 8}{x^2 - 1x - 2} = \frac{(x+4)\cancel{(x-2)}}{\cancel{(x-2)}(x+1)}$$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \left(\frac{x+4}{x+1} \right) = \frac{6}{3} = 2$$

Recall:

$$(\sqrt{3})^2 = 3$$

$$(\sqrt{4})^2 = 4$$

$$(\sqrt{5})^2 = 5$$

$$(\sqrt{x})^2 = x$$

$$(\sqrt{x+1})^2 = x+1$$

$$\sqrt{x+1} \cdot \sqrt{x+1} = (\sqrt{x+1})^2 = x+1$$

$$\sqrt{x+4} \cdot \sqrt{x+4} = x+4$$

$$\sqrt{x-5} \cdot \sqrt{x-5} = x-5$$

$$\sqrt{3x+1} \cdot \sqrt{3x+1} = 3x+1$$

$$(x+4)(x-4) = x^2 - 4x + 4x - 16 = x^2 - 16$$

$$(x-3)(x+3) = x^2 - 3^2 = x^2 - 9$$

$$(x-2)(x+2) = x^2 - 2^2 = x^2 - 4$$

$$(\sqrt{x} - 2)(\sqrt{x} + 2) = (\sqrt{x})^2 - 2^2 = x - 4$$

$$(\sqrt{x+1} - \sqrt{3})(\sqrt{x+1} + \sqrt{3}) = (\sqrt{x+1})^2 - (\sqrt{3})^2$$

$$= x + 1 - 3$$

conjugate

$$= x - 2$$

$$f(x) = \frac{(\sqrt{x+1} - 2)}{(x-3)} \cdot \frac{(\sqrt{x+1} + 2)}{(\sqrt{x+1} + 2)} = \frac{(\sqrt{x+1})^2 - (2)^2}{(x-3)(\sqrt{x+1} + 2)}$$

$$= \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$$

$$= \frac{\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$$

$$= \frac{1}{(\sqrt{x+1}+2)}$$

$$\lim_{x \to 3} f(x) = \underline{\ ?\ }$$

$$= \lim_{x \to 3} \left(\frac{1}{\sqrt{x+1} + 2} \right)$$

$$= \frac{1}{\sqrt{4} + 2}$$

$$= \frac{1}{2+2}$$

$$= \frac{1}{4}$$

$$f(x) = \frac{\left(\sqrt{2+x} - \sqrt{2}\right)}{(x)} \cdot \frac{\left(\sqrt{2+x} + \sqrt{2}\right)}{\left(\sqrt{2+x} + \sqrt{2}\right)} = \frac{\left(\sqrt{2+x}\right)^2 - \left(\sqrt{2}\right)^2}{(x)\left(\sqrt{2+x} + \sqrt{2}\right)}$$

$$= \frac{(2+x) - 2}{(x)\left(\sqrt{2+x} + \sqrt{2}\right)}$$

$$= \frac{\cancel{(x)}}{\cancel{(x)}\left(\sqrt{2+x} + \sqrt{2}\right)}$$

$$= \frac{1}{\left(\sqrt{2+x} + \sqrt{2}\right)}$$

Find $\lim_{x \to 0} f(x)$

$$= \lim_{x \to 0} \frac{1}{\sqrt{2+x} + \sqrt{2}}$$

$$= \frac{1}{\sqrt{2} + \sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} \text{ Answer}$$

$$\text{Difference Quotient} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Δx = ~~Detal~~ Delta x

Let $f(x) = 3x + 4$

Find the difference quotient for $f(x)$.

$$f(x+\Delta x) = 3\cdot(x+\Delta x) + 4 = 3x + 3\cdot\Delta x + 4$$

$$DQ = \frac{(\cancel{3x} + 3\cdot\Delta x + \cancel{4}) - (\cancel{3x} + \cancel{4})}{\Delta x} = \frac{3\cdot\Delta x}{\Delta x} = 3$$

$$\lim_{\Delta x \to 0} DQ = \lim_{\Delta x \to 0} (3) = 3$$

Recall:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 \cancel{+} - 2ab + b^2$$

$$(\Delta x + 4)^2 = (\Delta x)^2 + 8 \cdot \Delta x + (4)^2$$

$f(x) = x^2 - 4x$ Find the difference quotient.

$$DQ = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) = (x+\Delta x)^2 - 4(x+\Delta x)$$

$$= x^2 + 2\cdot x\cdot \Delta x + (\Delta x)^2 - 4x - 4\Delta x$$

$$DQ = \frac{\left(\cancel{x^2} + 2x\cdot\Delta x + (\Delta x)^2 \cancel{- 4x} - 4\cdot\Delta x\right) - \cancel{(x^2 - 4x)}}{\Delta x}$$

$$= \frac{2x\cdot\Delta x + (\Delta x)^2 - 4\Delta x}{\Delta x} = \frac{2x\cdot\Delta x}{\Delta x} + \frac{(\Delta x)^2}{\Delta x} - \frac{4\Delta x}{\Delta x}$$

$$= 2x + \Delta x - 4$$

$$\lim_{\Delta x\to 0} DQ = \lim_{\Delta x\to 0}(2x + \Delta x - 4) = 2x - 4$$