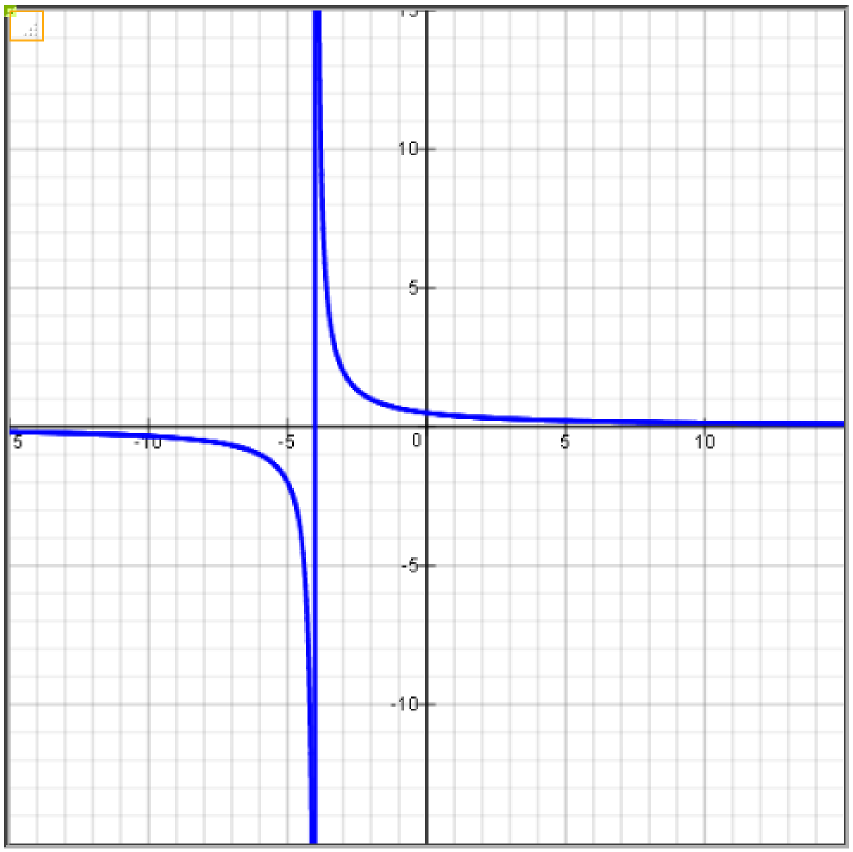
Infinite Limits and Vertical Asymptotes

Example 1:

$$f(x) = \frac{2}{x+4}$$



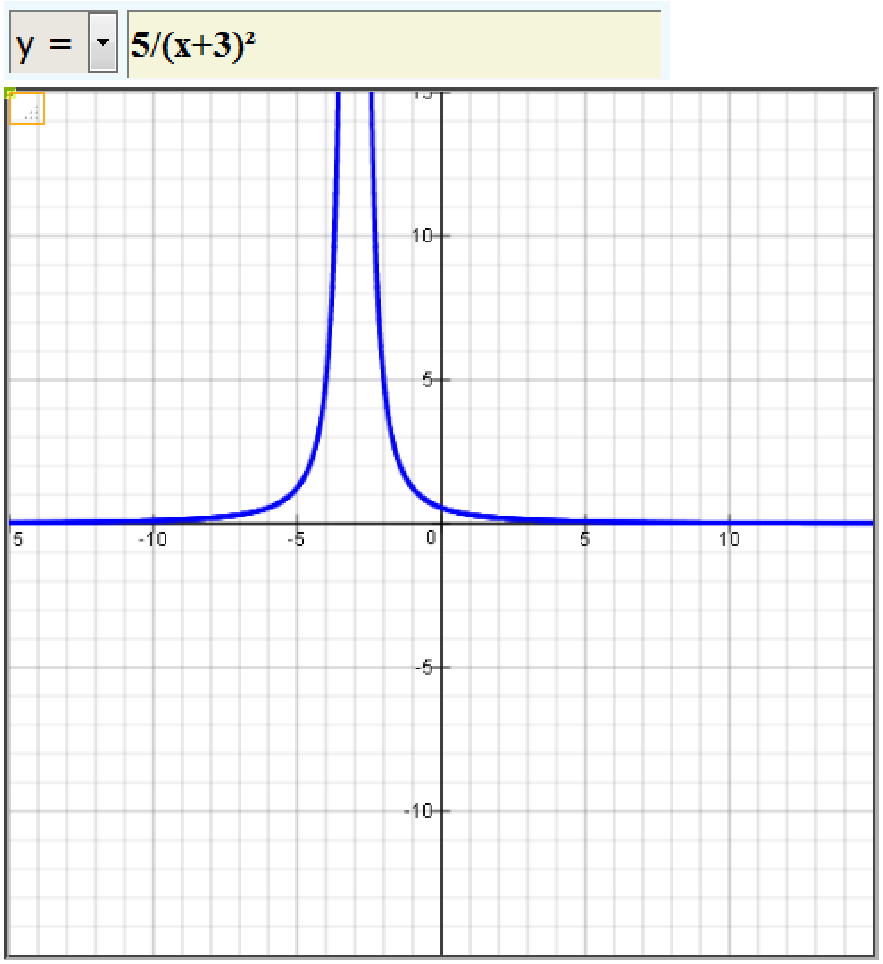
$$\lim_{x \to -4^-} f(x) = -\infty$$

$$\lim_{x \to -4^+} f(x) = \infty$$

$$\lim_{x \to -4} f(x) = \text{Does Not Exist}$$

Example 2:

$$f(x) = \frac{5}{(x+3)^2}$$



$$\lim_{x \to -3^-} f(x) = \infty$$

$$\lim_{x \to -3^+} f(x) = \infty$$

$$\lim_{x \to -3} f(x) = \infty = \text{Does Not Exist}$$

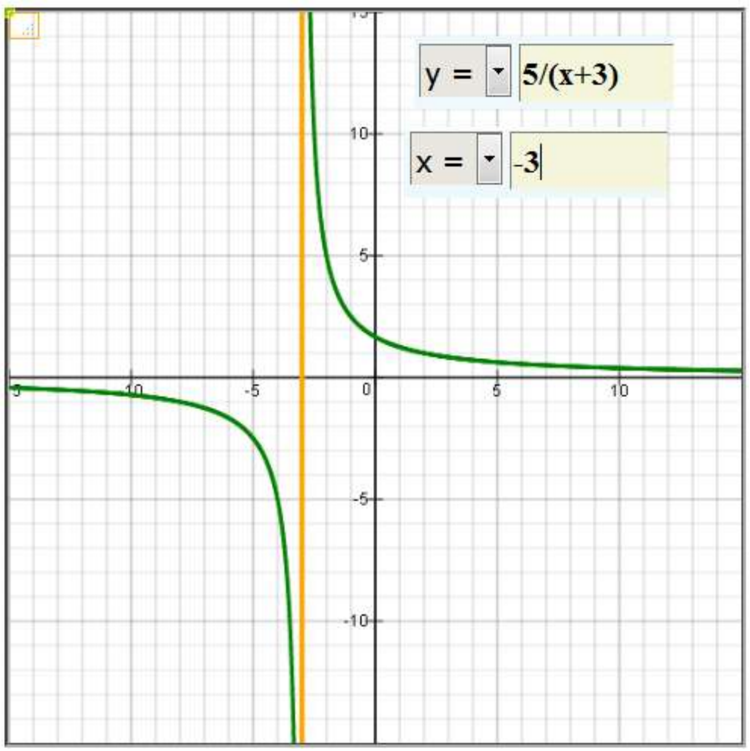
Example 3: Find vertical asymptote

$$f(x) = \frac{5}{x+3}$$

To find vertical asymptote,

set $x+3=0 \Rightarrow x=-3$

Therefore, $f(x) = \dfrac{5}{x+3}$ has a vertical asymptote at $x=-3$



$$\lim_{x\to -3^-} f(x) = -\infty$$

$$\lim_{x\to -3^+} f(x) = \infty$$

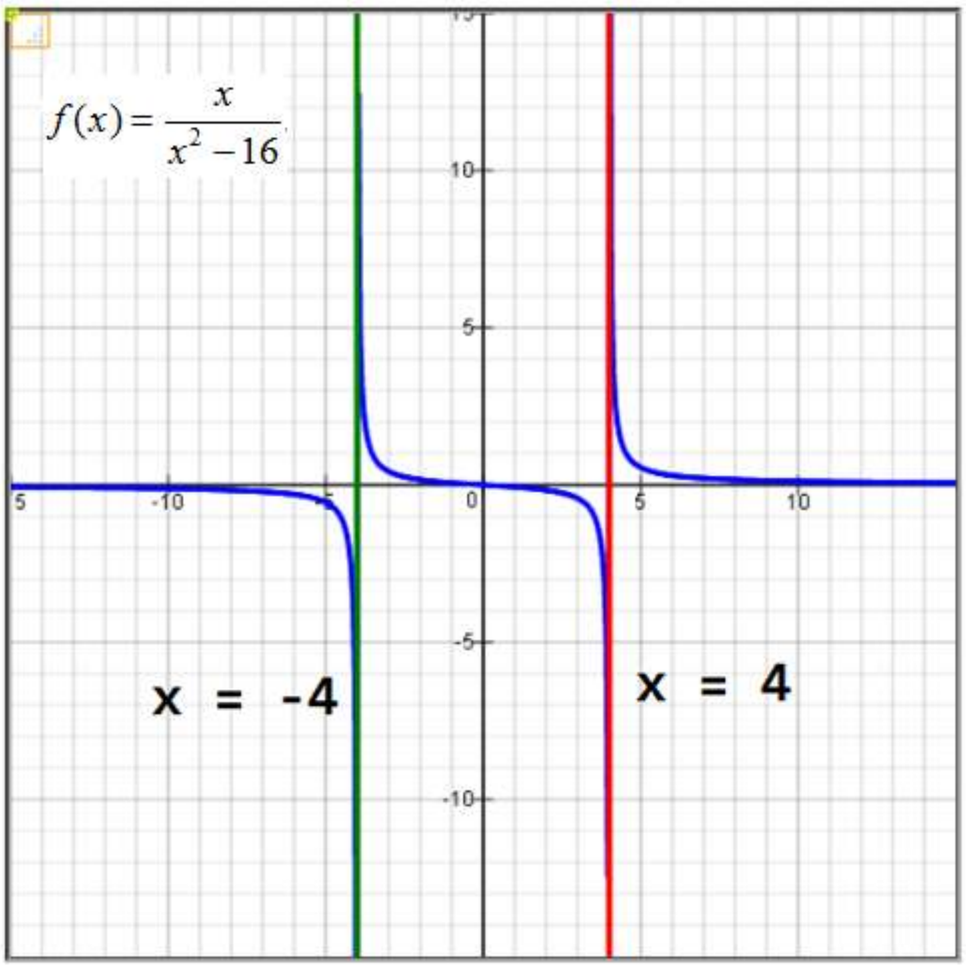
$$\lim_{x\to -3} f(x) = \text{ Does Not Exist}$$

Example 4: Find vertical asymptote of $f(x)=\dfrac{x}{x^2-16}$.

To find vertical asymptote, set $x^2-16=0$ and solve:

$$x^2-16=0 \quad \Rightarrow \quad x^2=16 \quad \Rightarrow \quad \sqrt{x^2}=\pm\sqrt{16} \quad \Rightarrow \quad x=\pm 4$$

Therefore, $f(x)=\dfrac{x}{x^2-16}$ has a vertical asymptote at $x=\pm 4$



$$\lim_{x\to -4^-} f(x) = -\infty$$

$$\lim_{x\to -4^+} f(x) = \infty$$

$$\lim_{x\to -4} f(x) = \text{ Does Not Exist}$$

$$\lim_{x\to 4^-} f(x) = -\infty$$

$$\lim_{x\to 4^+} f(x) = \infty$$

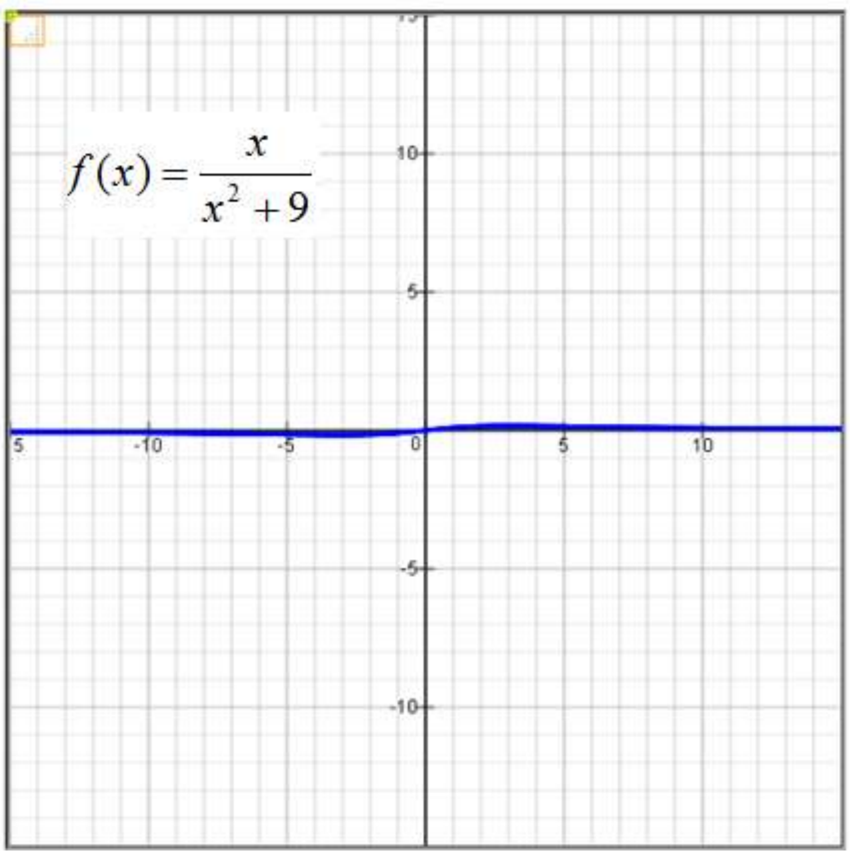
$$\lim_{x\to 4} f(x) = \text{ Does Not Exist}$$

Example 5: Find vertical asymptote of $f(x)=\dfrac{x}{x^2+9}$.

To find vertical asymptote, set $x^2+9=0$ and solve:

$$x^2+9=0 \quad \Rightarrow \quad x^2=-9 \quad \Rightarrow \quad \sqrt{x^2}=\pm\sqrt{-9} \quad \Rightarrow \quad x=\pm 3i$$

Therefore, $f(x)=\dfrac{x}{x^2+9}$ has no vertical asymptote.



Example 6: Find vertical asymptote of $f(x) = \dfrac{x^2 + x - 2}{x^2 + 6x + 8}$.

$Note:\ \ f(x) = \dfrac{x^2 + x - 2}{x^2 + 6x + 8} = \dfrac{(x+2)(x-1)}{(x+2)(x+4)}$

To find vertical asymptote or hole, set $(x+2)(x+4) = 0$ and solve:

$(x+2)(x+4) = 0$

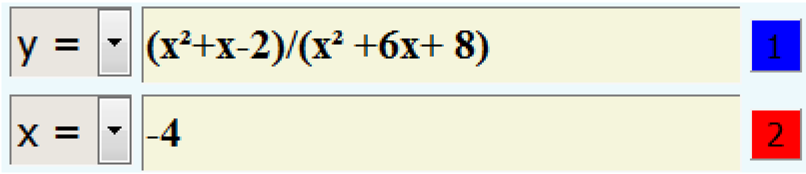
$(x+2) = 0$ or $(x+4) = 0$

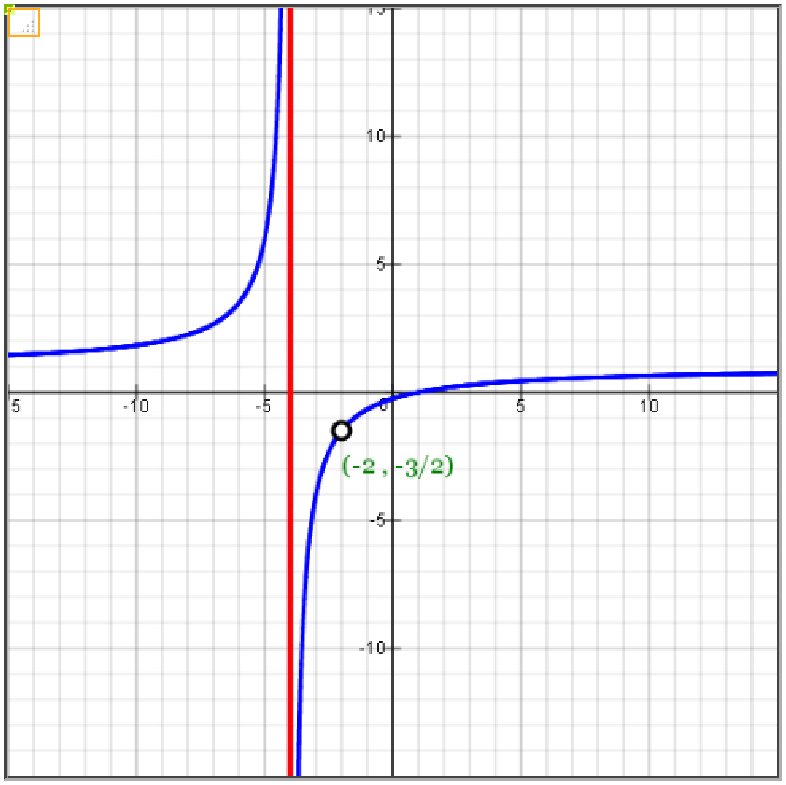
$x = -2$ or $x = -4$

Therefore, $f(x) = \dfrac{x^2 + x - 2}{x^2 + 6x + 8}$ has vertical asymptote at $x = -4;$ and

.$f(x) = \dfrac{x^2 + x - 2}{x^2 + 6x + 8}$ has a hole at $x = -2$.

The reason $x = -2$ is a hole and not a vertical asymptote is because $f(x)$ contains the factor $(x+2)$ in both the numerator and denominator.





Graph has vertical asymptote at x = -4.

Graph has a hole at x = -2.

Graph has a non removable discontinuity at x = - 4.

Graph has a removable discontinuity at x = -2.

$$\lim_{x\to -4^-} f(x) = \infty$$

$$\lim_{x\to -4^+} f(x) = -\infty$$

$$\lim_{x\to -4} f(x) = \text{ Does Not Exist}$$