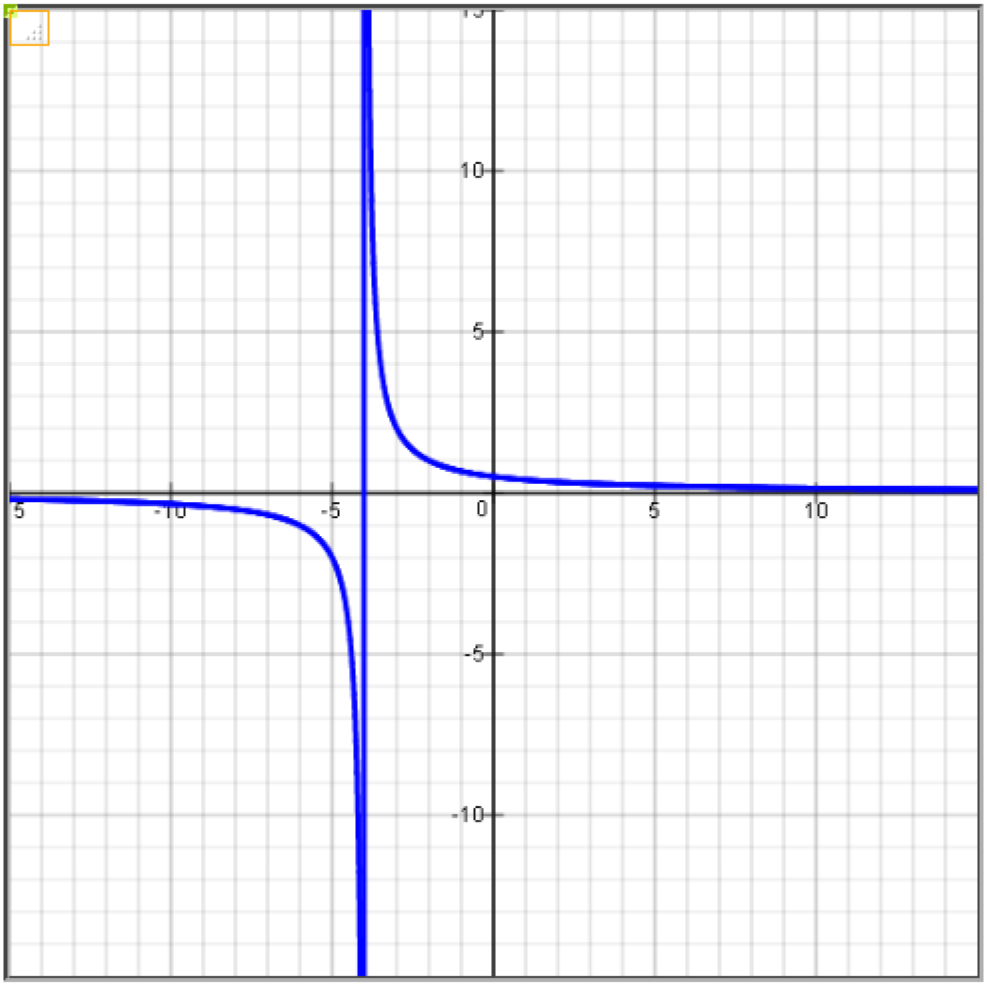
Infinite Limits

Example 1:

$$f(x)=\frac{2}{x+4}$$



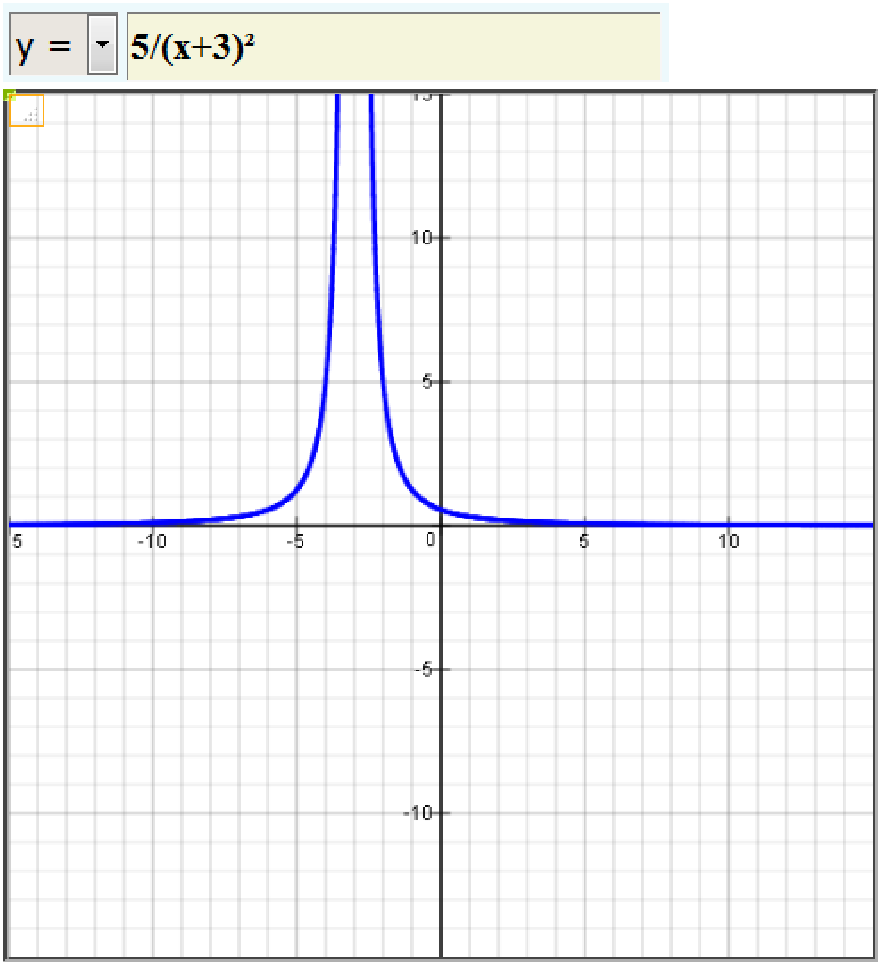
$$\lim_{x\to -4^-} f(x) = -\infty$$

$$\lim_{x\to -4^+} f(x) = \infty$$

$$\lim_{x\to -4} f(x) = \text{Does Not Exist}$$

Example 2:

$$f(x)=\frac{5}{(x+3)^2}$$



$$\lim_{x\to -3^-} f(x)=\infty$$

$$\lim_{x\to -3^+} f(x)=\infty$$

$$\lim_{x\to -3} f(x)=\infty=\text{Does Not Exist}$$

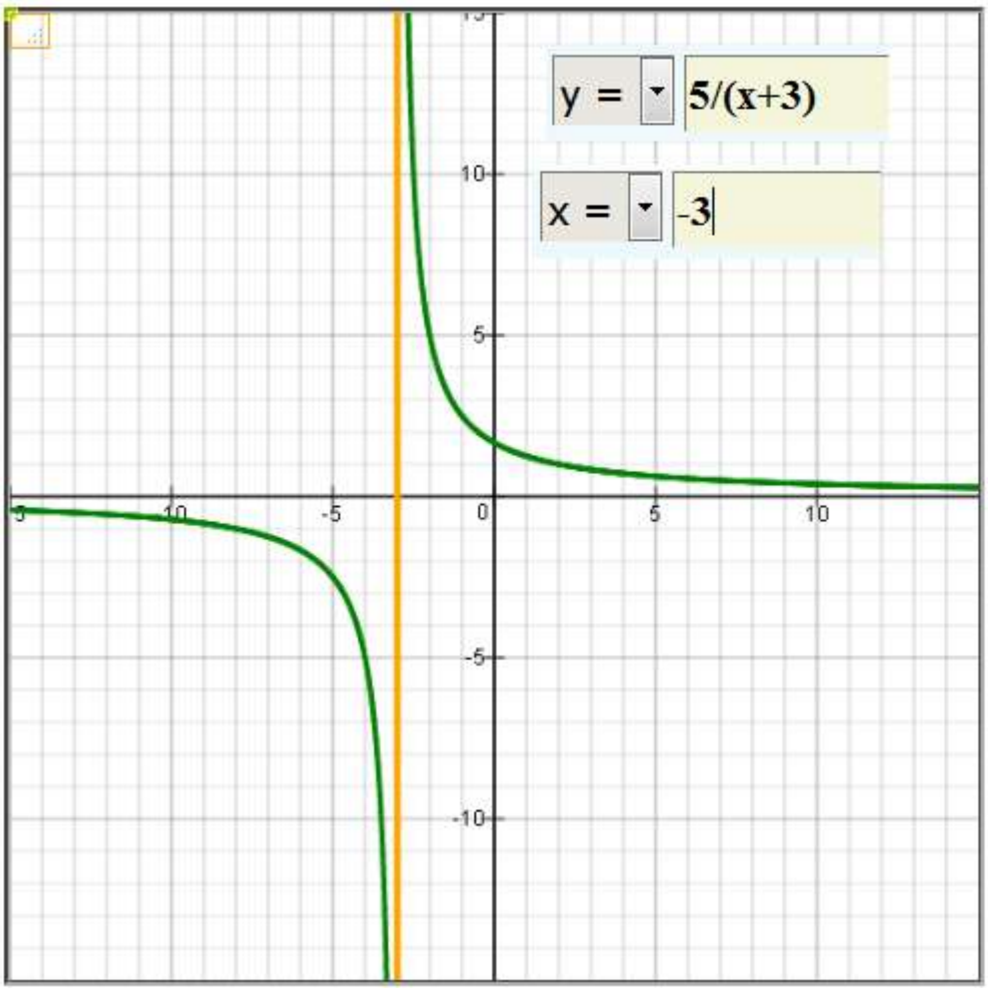
Example 3: Find vertical asymptote

$$f(x) = \frac{5}{x+3}$$

To find vertical asymptote,

set $x + 3 = 0 \Rightarrow x = -3$

Therefore, $f(x) = \dfrac{5}{x+3}$ has a vertical asymptote at $x = -3$



Example 4: Find vertical asymptote of $f(x) = \dfrac{x}{x^2 - 16}$.

To find vertical asymptote, set $x^2 - 16 = 0$ and solve:

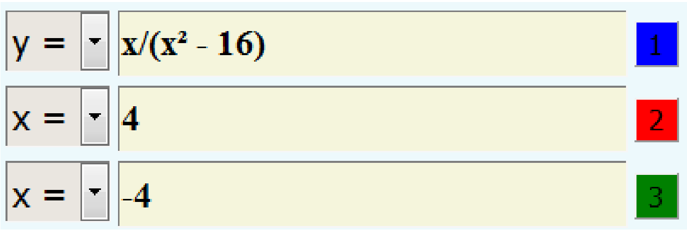
$x^2 - 16 = 0$

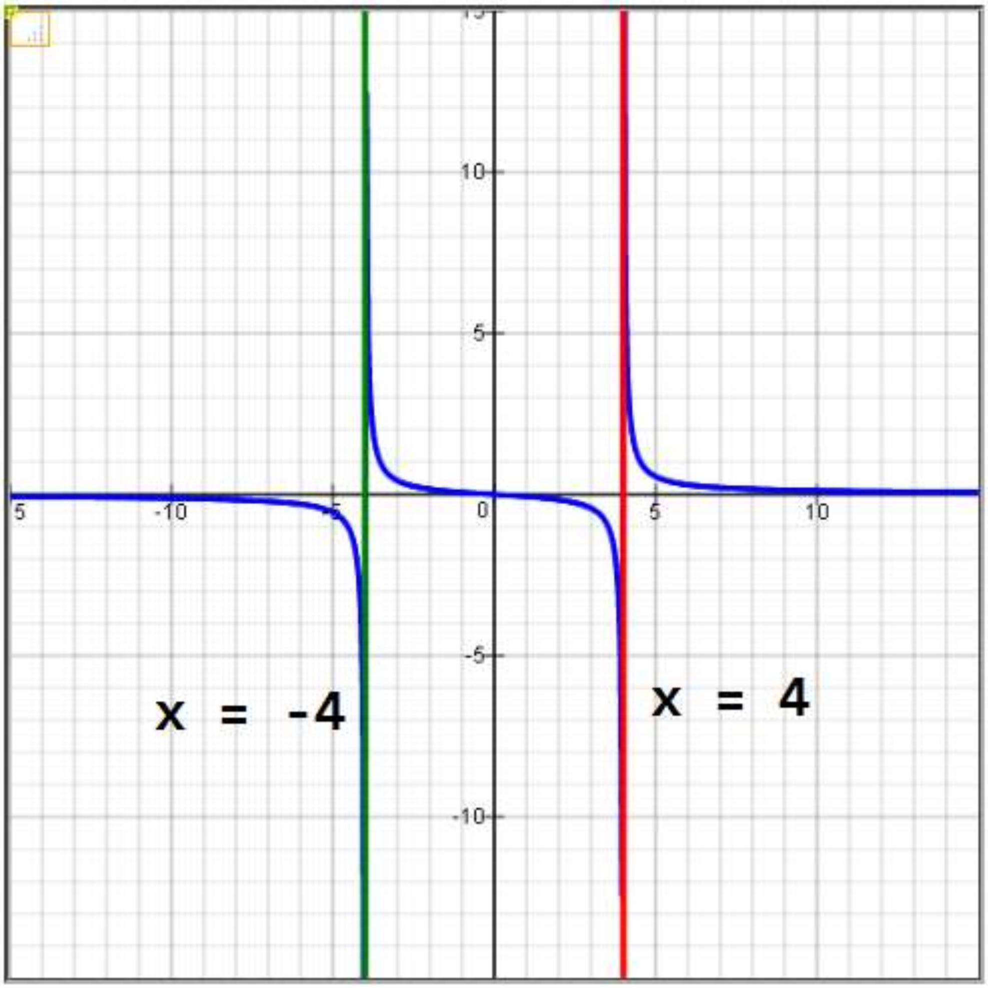
$x^2 = 16$

$\sqrt{x^2} = \pm\sqrt{16}$

$x = \pm 4$

Therefore, $f(x) = \dfrac{x}{x^2 - 16}$ has a vertical asymptote at $x = \pm 4$


y = x/(x² - 16) 1
x = 4 2
x = -4 3


10
5
-5
-10
-10
0
5
10
x = -4
x = 4

Example 5: Find vertical asymptote of $f(x)=\frac{x}{x^2+9}$.

To find vertical asymptote, set $x^2+9=0$ and solve:

$x^2+9=0$

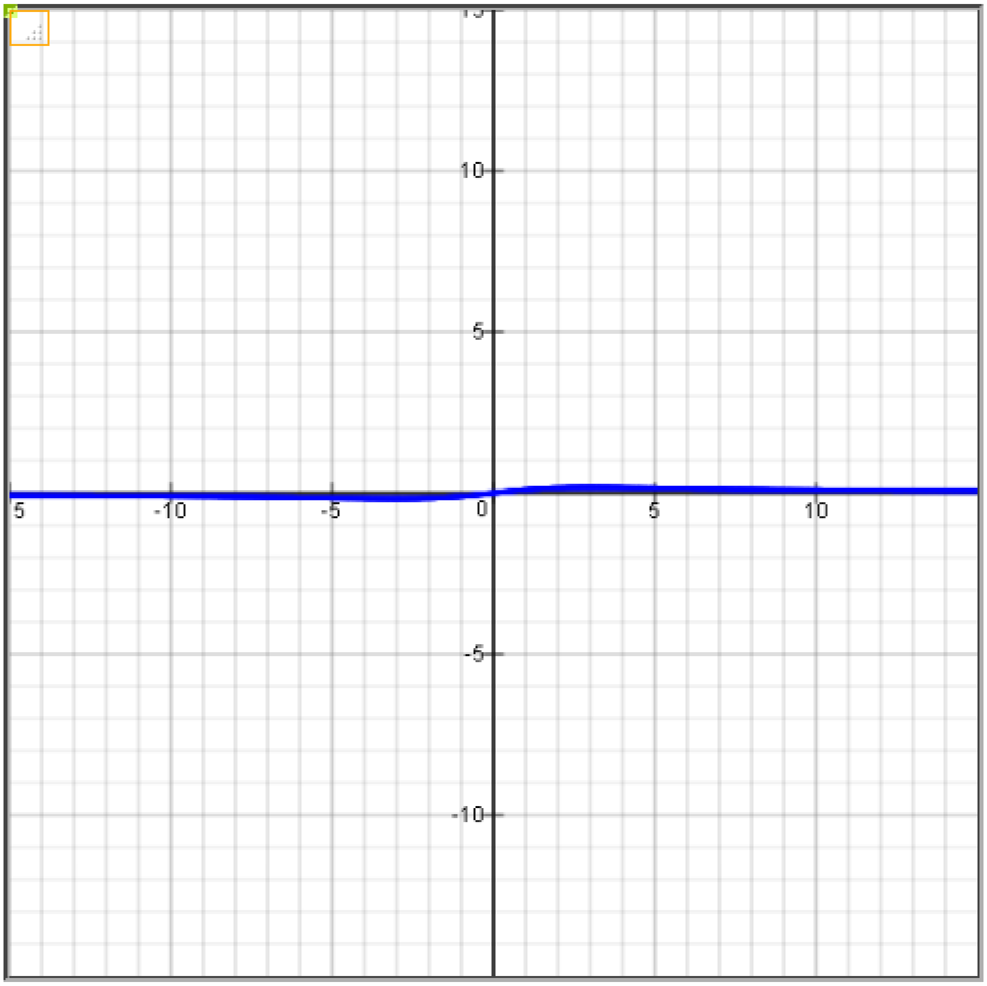
$x^2=-9$

$\sqrt{x^2}=\pm\sqrt{-9}$

$x=\pm 3i$

Therefore, $f(x)=\frac{x}{x^2+9}$ has no vertical asymptote.

y = $x/(x^2 + 9)$ 1



Example 6: Find vertical asymptote of $f(x) = \dfrac{x^2 + x - 2}{x^2 + 6x + 8}$.

$Note$: $f(x) = \dfrac{x^2 + x - 2}{x^2 + 6x + 8} = \dfrac{(x+2)(x-1)}{(x+2)(x+4)}$

To find vertical asymptote or hole, set $(x+2)(x+4) = 0$ and solve:

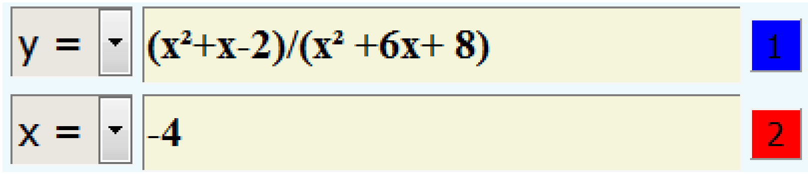
$(x+2)(x+4) = 0$

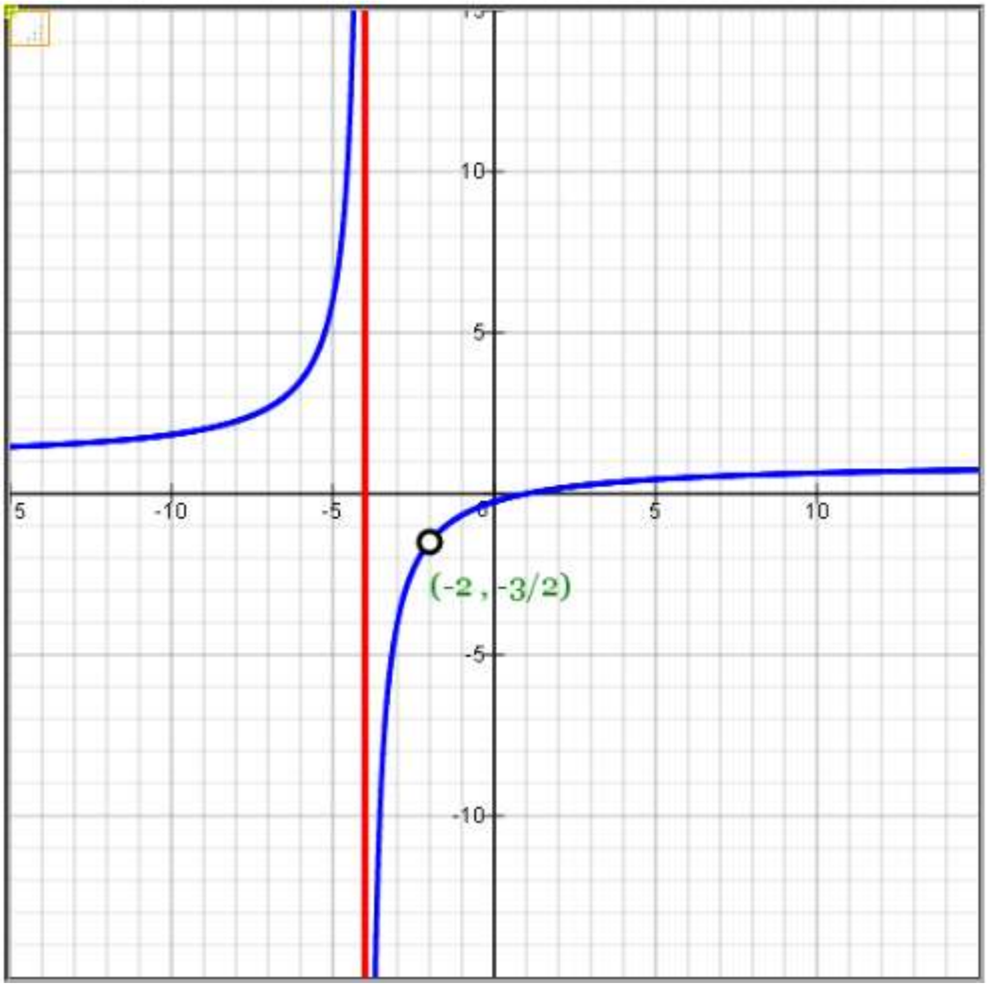
$\Rightarrow (x+2) = 0 \quad$ or $\quad (x+4) = 0$

$\Rightarrow x = -2 \quad$ or $\quad x = -4$

Therefore, $f(x) = \dfrac{x^2 + x - 2}{x^2 + 6x + 8}$ has vertical asymptote at $x = -4$; and

.$f(x) = \dfrac{x^2 + x - 2}{x^2 + 6x + 8}$ has a hole at $x = -2$.





Graph has a non removable discontinuity at x = -4.

Graph has a removable discontinuity at x = -2.