Basic Differentiation Rules

For $f(x) = c$ (constant function), $f'(x) = 0$

For $f(x) = x$ (constant function), $f'(x) = 1$

For $f(x) = x^2$ (constant function), $f'(x) = 2x$

For $f(x) = x^3$ (constant function), $f'(x) = 3x^2$

For $f(x) = x^4$ (constant function), $f'(x) = 4x^3$

For $f(x) = x^5$ (constant function), $f'(x) = 5x^4$

In general:

Let $f(x) = x^n$. Find the derivative of the function $f(x)$.

$f'(x) = nx^{n-1}$

Example 1:

Let $f(x) = 3x$

$f'(x) = 3 \cdot D_x(x) = 3(1) = 3$

Example 2:

Let $f(x) = 5x^2$

$$f'(x) = 5 \cdot D_x(x^2) = 5(2x) = 10x$$

Example 3:

Let $f(x) = 7x^3$

$$f'(x) = 7 \cdot D_x(x^3) = 7(3x^2) = 21x^2$$

Example 4:

Let $f(x) = 7x^3 + 5x - 8$

$$f'(x) = D_x(7x^3) + D_x(5x) - D_x(8)$$

$$f'(x) = 7D_x(x^3) + 5D_x(x) - D_x(8)$$

$$f'(x) = 7 \cdot (3x^2) + 5(1) - 0$$

$$f'(x) = 21x^2 + 5$$

Example 5:

Let $f(x) = \dfrac{7}{x^3}$

$$f(x) = \frac{7}{x^3} = 7x^{-3}$$

$$f'(x) = D_x(7x^{-3})$$

$$f'(x) = 7D_x(x^{-3})$$

$$f'(x) = 7 \cdot (-3x^{-3-1})$$

$$f'(x) = -21x^{-4}$$

Example 6:

Let $f(x) = \sqrt{x}$

$$f(x) = \sqrt{x} = x^{1/2}$$

$$f'(x) = D_x(x^{1/2})$$

$$f'(x) = \frac{1}{2}x^{-1/2}$$

Example 7:

Let $f(x) = 4\cos x$

$$f'(x) = D_x(4\cos x)$$

$$f'(x) = 4(-\sin x)$$

$$f'(x) = -4\sin x$$

Example 7:

Let $f(x) = \dfrac{4}{x^2} + 7\sin x$

$$f'(x) = D_x\left(\frac{4}{x^2}\right) + D_x(7\sin x)$$

$$f'(x) = D_x\left(4x^{-2}\right) + D_x(7\sin x)$$

$$f'(x) = 4D_x\left(x^{-2}\right) + 7D_x(\sin x)$$

$$f'(x) = 4\left(-2x^{-3}\right) + 7(\cos x)$$

$$f'(x) = -8x^{-3} + 7\cos x$$

Example 8:

Let $f(x) = (4x+1)^2$. Find equation of tangent line at (0,1).

$f(x) = 16x^2 + 8x + 1$

$f'(x) = D_x\left(16x^2\right) + D_x(8x) + D_x(1)$

$f'(x) = 16D_x\left(x^2\right) + 8D_x(x) + D_x(1)$

$f'(x) = 16\left(2x\right) + 8(1) + 0$

$f'(x) = 32x + 8$

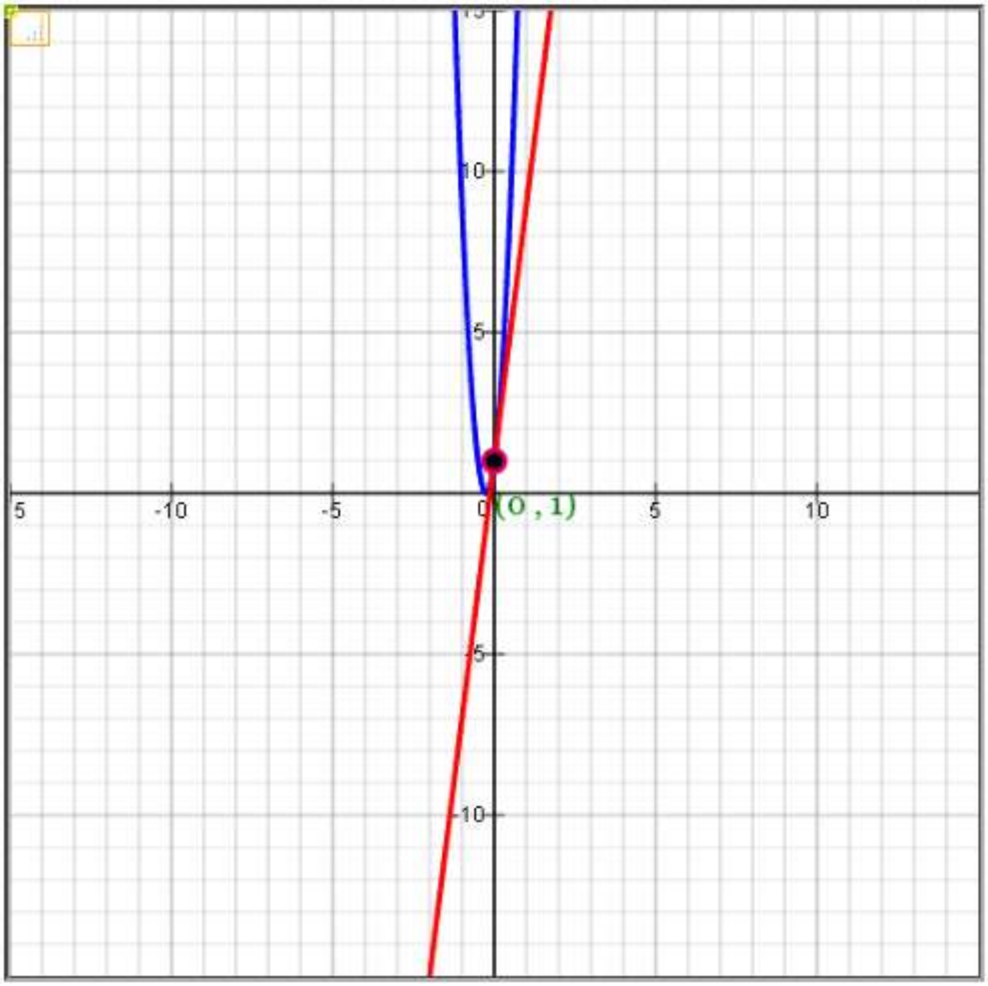
Hence, slope of tangent line at (0,1) = $m = f'(0)$

$m = f'(0) = 32x + 8 = 32(0) + 8 = 8$

Equation of tangent line: $y - y_1 = m(x - x_1)$

Equation of tangent line: $y - 1 = 8(x - 0)$

Equation of tangent line: $y = 8(x - 0) + 1 = 8x + 1$



10
5
-5
-10
5
-10
-5
0
5
10
(0 , 1)

Example 9:

Let $f(x) = 3\cos x$. Find equation of tangent line at ($\pi/2$, 0).

$f(x) = 3\cos x$

$f'(x) = D_x(3\cos x)$

$f'(x) = 3D_x(\cos x)$

$f'(x) = 3(-\sin x)$

$f'(x) = -3\sin x$

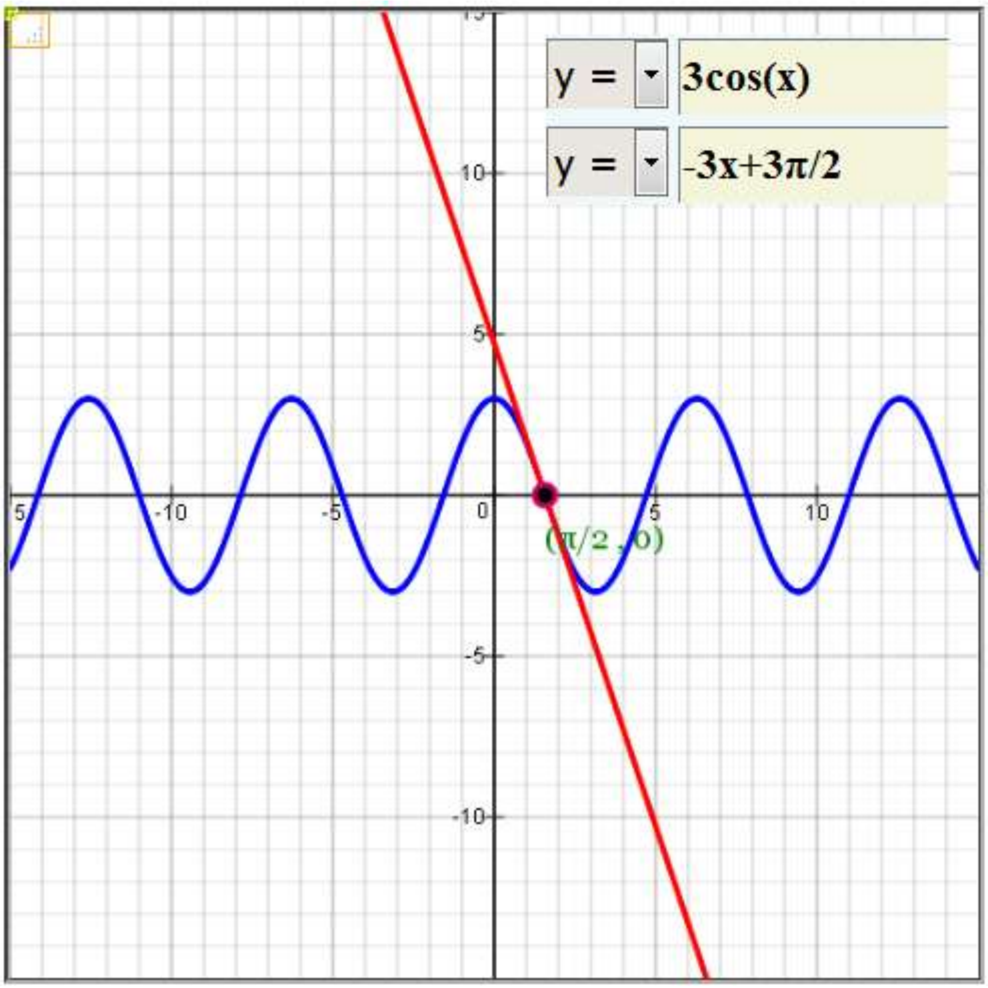
Hence, slope of tangent line at ($\pi/2$, 0) = $m = f'(\pi/2)$

$m = f'(\pi/2) = -3\sin x = -3\sin(\pi/2)$ = -3(1) = -3

Equation of tangent line: $y - y_1 = m(x - x_1)$

Equation of tangent line: $y - 0 = -3(x - \pi/2)$

Equation of tangent line: y = -3$x + 3\pi/2$



y = 3cos(x)
y = -3x+3π/2
(π/2 , 0)

Example 10:

Let $f(x) = \dfrac{4x^4 - 7x}{x^2}$

$$f(x) = \frac{4x^4 - 7x}{x^2} = \frac{4x^4}{x^2} - \frac{7x}{x^2} = 4x^2 - 7x^{-1}$$

$$f'(x) = D_x\left(4x^2\right) - D_x(7x^{-1})$$

$$f'(x) = 4D_x\left(x^2\right) - 7D_x(x^{-1})$$

$$f'(x) = 4\left(2x\right) - 7(-1x^{-2})$$

$$f'(x) = 8x - 7x^{-2}$$

Example 10:

Let $f(x) = \dfrac{5}{\sqrt[3]{x^2}}$

$$f(x) = \frac{5}{\sqrt[3]{x^2}} = \frac{5}{x^{2/3}} = 5x^{-2/3}$$

$$f'(x) = D_x\left(5x^{-2/3}\right)$$

$$f'(x) = 5D_x\left(x^{-2/3}\right)$$

$$f'(x) = 5\left(\frac{-2}{3}x^{-2/3-1}\right)$$

$$f'(x) = \frac{-10}{3}x^{-5/3}$$