Implicit Differentiation

1) Find y' for the implicit function $-3x + y = 5$.

$-3x + y = 5$

$D_x\left(-3x\right) + D_x\left(1y\right) = D_x\left(5\right)$

$-3 + 1 \cdot y' = 0$

$y' = 3$

2) Find y' for the implicit function $3x + 4y = 12$.

$3x + 4y = 12$

$D_x\left(3x\right) + D_x\left(4y\right) = D_x\left(12\right)$

$3 + 4 \cdot y' = 0$

$4y' = -3$

$y' = \dfrac{-3}{4}$

3) Find y' for the implicit function $x^2 + y^2 = 25$.

$x^2 + y^2 = 25$

$D_x\left(x^2\right) + D_x\left(y^2\right) = D_x\left(25\right)$

$2x + 2y \cdot y' = 0$

$2y \cdot y' = -2x$

$\dfrac{2y \cdot y'}{2y} = \dfrac{-2x}{2y}$

$y' = \dfrac{-x}{y}$

4) Find y' for the implicit function $2x^3 + 3y^3 = 64$.

$$2x^3 + 3y^3 = 64$$

$$D_x\left(2x^3\right) + D_x\left(3y^3\right) = D_x\left(64\right)$$

$$2(3x^2) + 3(3y^2) \cdot y' = 0$$

$$6x^2 + 9y^2 \cdot y' = 0$$

$$9y^2 \cdot y' = -6x^2$$

$$\frac{9y^2 \cdot y'}{9y^2} = \frac{-6x^2}{9y^2}$$

$$y' = \frac{-2x^2}{3y^2}$$

5) Find y' for the implicit function $x^2y+y^2x=-2$.

$$x^2y+y^2x=-2$$

$$D_x\left(x^2\cdot y\right)+D_x\left(y^2\cdot x\right)=D_x\left(-2\right)$$

Note: $D_x\left(x^2\cdot y\right)$ is derivative of a product:	Note: $D_x\left(y^2\cdot x\right)$ is derivative of a product:
$D_x\left(x^2\cdot y\right)=x^2\cdot D_x\left(y\right)+y\cdot D_x\left(x^2\right)$	$D_x\left(y^2\cdot x\right)=y^2\cdot D_x\left(x\right)+x\cdot D_x\left(y^2\right)$
$=x^2\cdot\left[1y'\right]+y\cdot\left(2x\right)=x^2\cdot y'+2xy$	$=y^2\cdot\left[1\right]+x\cdot\left(2y\cdot y'\right)=y^2+2xy\cdot y'$

$$D_x\left(x^2\cdot y\right)+D_x\left(y^2\cdot x\right)=D_x\left(-2\right)$$

$$x^2\cdot y'+2xy+y^2+2xy\cdot y'=0$$

$x^2\cdot y'+2xy\cdot y'=-2xy-y^2$ Move all terms with no y' to the right

$y'\cdot\left(x^2+2xy\right)=-2xy-y^2$ Factor out y'

$$\frac{y'\cdot\left(x^2+2xy\right)}{\left(x^2+2xy\right)}=\frac{-2xy-y^2}{\left(x^2+2xy\right)}$$

$$y'=\frac{-2xy-y^2}{\left(x^2+2xy\right)}$$

6) Find y' for the implicit function $x^3y^3 - y = x$.

$$x^3y^3 - y = x$$

$$D_x\left(x^3y^3\right) - D_x\left(y\right) = D_x\left(x\right)$$

Note: $D_x\left(x^3 \cdot y^3\right)$ is derivative of a product:

$$D_x\left(x^3 \cdot y^3\right) = x^3 \cdot D_x\left(y^3\right) + y^3 \cdot D_x\left(x^3\right)$$

$$= x^3 \cdot \left[3y^2 \cdot y'\right] + y^3 \cdot \left(3x^2\right) = 3x^3y^2 \cdot y' + 3x^2y^3$$

$$D_x\left(x^3y^3\right) - D_x\left(y\right) = D_x\left(x\right)$$

$$3x^3y^2 \cdot y' + 3x^2y^3 - 1 \cdot y' = 1$$

$$3x^3y^2 \cdot y' - 1 \cdot y' = 1 - 3x^2y^3$$ Move all terms with no y' to the right

$$y' \cdot \left(3x^3y^2 - 1\right) = 1 - 3x^2y^3$$ Factor out y'

$$\frac{y' \cdot \left(3x^3y^2 - 1\right)}{\left(3x^3y^2 - 1\right)} = \frac{1 - 3x^2y^3}{\left(3x^3y^2 - 1\right)}$$

$$y' = \frac{1 - 3x^2y^3}{\left(3x^3y^2 - 1\right)}$$

7) Find y' for the implicit function $y^3 - x^2 = 4.$

Also, find equation of tangent line at the point (2, 2).

$$y^3 - x^2 = 4$$

$$D_x\left(y^3\right) - D_x\left(x^2\right) = D_x\left(4\right)$$

$$3y^2 \cdot y' - 2x = 0$$

$$3y^2 \cdot y' = 2x$$

$$\frac{3y^2 \cdot y'}{3y^2} = \frac{2x}{3y^2}$$

$$y' = \frac{2x}{3y^2}$$

slope of tangen line $= y'(2,2) = \frac{2x}{3y^2} = \frac{2(2)}{3(2)^2} = \frac{4}{12} = \frac{1}{3}$

equation of tangent line: $y - y_1 = m(x - x_1) \Leftrightarrow y - 2 = \frac{1}{3}(x-2)$

8) Find y' for the implicit function $(x+2)^2+(y-3)^2=37$.

Also, find equation of tangent line at the point (4, 4).

$(x+2)^2+(y-3)^2=37$

$x^2+4x+4+y^2-6y+9=37$

$D_x\left(x^2\right)+D_x\left(4x\right)+D_x\left(4\right)+D_x\left(y^2\right)-D_x\left(6y\right)+D_x\left(9\right)=D_x\left(37\right)$

$2x+4+0+2y\cdot y'-6\cdot y'+0=0$

$2x+4+2y\cdot y'-6\cdot y'=0$

$2y\cdot y'-6\cdot y'=-2x-4$ Move all terms with no y' to the right

$y'\cdot\left(2y-6\right)=-2x-4$ Factor out y'

$$\frac{y'\cdot\left(2y-6\right)}{\left(2y-6\right)}=\frac{-2x-4}{\left(2y-6\right)}$$

$$y'=\frac{-2x-4}{\left(2y-6\right)}$$

slope of tangen line = $y'(4,4)=\dfrac{-2x-4}{\left(2y-6\right)}=\dfrac{-2(4)-4}{\left(2(4)-6\right)}=\dfrac{-12}{2}=-6$

equation of tangent line: $y-y_1=m(x-x_1)\Leftrightarrow y-4=-6(x-4)$