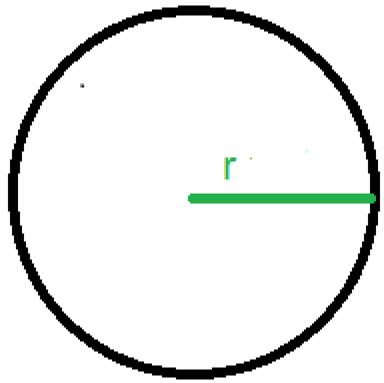
1)



Note: *Area* of circle $= \pi r^2$

Take implicit derivative with respect to time (t): $A' = \pi(2r)r'$

$A' = \frac{dA}{dt}$ and $r' = \frac{dr}{dt}$

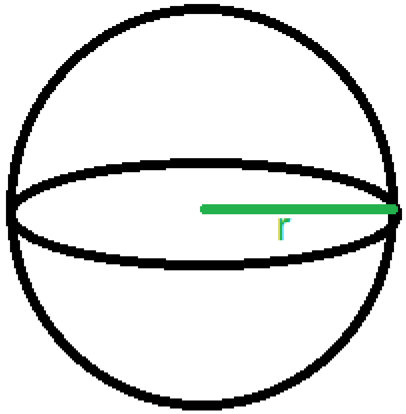
$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$ Note: $\frac{dA}{dt}$ = rate of change of area; $\frac{dr}{dt}$ = rate of change of radius

We are given that the radius is increasing at a rate of 7 in/min.

Hence, $\frac{dr}{dt} =$ 7 in/min.

Now we want to find $\frac{dA}{dt}$ when $r = 4$.

2)



Volume of sphere = $\frac{4}{3}\pi r^3$

Note: Volume of sphere = $\frac{4}{3}\pi r^3$

Take implicit derivative with respect to time (t): $V' = \frac{4}{3}(3\pi r^2)r' = 4\pi r^2 r' \quad \Rightarrow \quad \frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

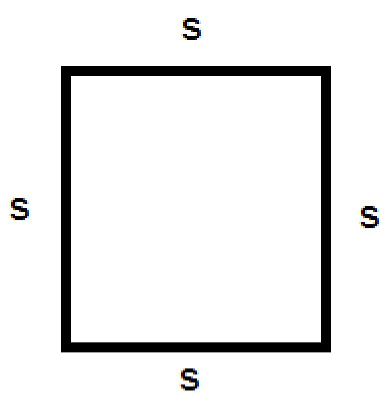
Note: $\frac{dV}{dt} =$ rate of change of volume; $\frac{dr}{dt} =$ rate of change of radius

We are given that the radius is increasing at a rate of 3 in/min.

Hence, $\frac{dr}{dt} =$ 3 in/min.

Now we want to find $\frac{dV}{dt}$ when $r = 4$.

3)



Area of square = s^2

Note: Area of square = s^2

Take implicit derivative with respect to time (t): $A' = 2s \cdot s' \quad \Rightarrow \quad \dfrac{dA}{dt} = 2s \cdot \dfrac{ds}{dt}$

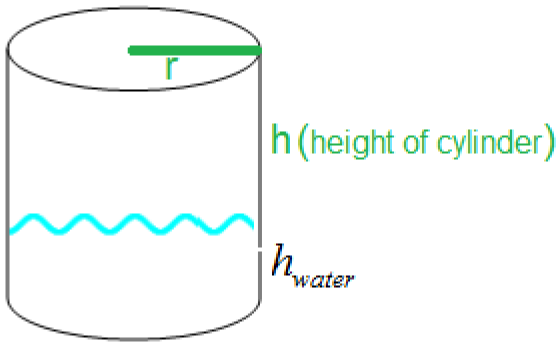
Note: $\dfrac{dA}{dt} =$ rate of change of area; $\dfrac{ds}{dt} =$ rate of change of s.

We are given that s is increasing at a rate of 2 in/min.

Hence, $\dfrac{ds}{dt} =$ 2 in/min.

Now we want to find $\dfrac{dA}{dt}$ when $s = 4$.

4)



Volume of cylinder = $\pi r^2 h$

Volume of water = $V_{\text{water}} = \pi r^2 h_{water}$

Note: Volume of cylinder = $\pi r^2 h$

Hence, Volume of water = $V_{\text{water}} = \pi r^2 h_{water}$ where h_{water} = height of water in tank

We are given that the radius is 10 feet.

$$V_{\text{water}} = \pi (10)^2 h_{water} \quad \Rightarrow \quad V_{\text{water}} = 100\pi h_{water}$$

Take implicit derivative with respect to time (t): $V'_{\text{water}} = 100\pi h'_{water} \quad \Rightarrow \quad \frac{dV_{\text{water}}}{dt} = 100\pi \cdot \frac{dh_{water}}{dt}$

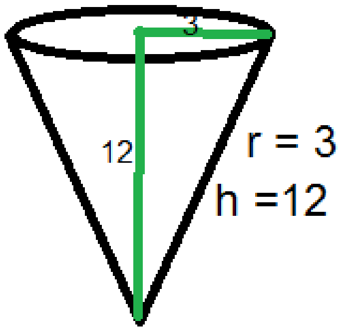
Note: $\frac{dV_{\text{water}}}{dt}$ = rate of change of volume of water; $\frac{dh_{water}}{dt}$ = rate of change of height of water.

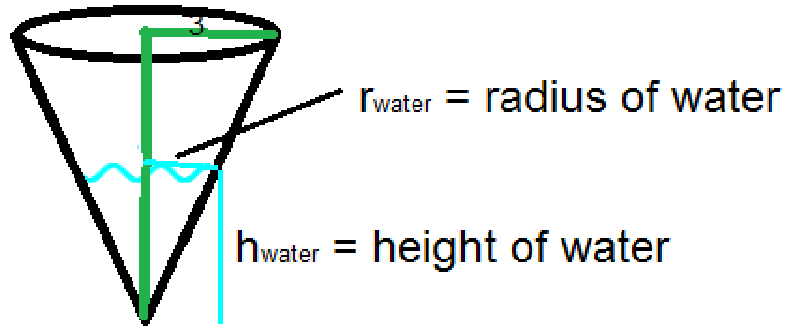
We are given that water is being drained at a rate of 2 ft^3/sec.

Hence, $\frac{dV_{\text{water}}}{dt} = 2\ \text{ft}^3/\text{sec}$.

Now we want to find $\frac{dh_{water}}{dt}$.

5)





Note: $\frac{r}{h} = \frac{3}{12} = \frac{1}{4}$

$\frac{r_{water}}{h_{water}} = \frac{1}{4}$

Note: the ratio $\frac{r}{h} = \frac{3}{12} = \frac{1}{4} \quad \Rightarrow \quad r = \frac{1}{4}h$

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{4}h\right)^2 h = \frac{1}{3}\pi\frac{1}{16}h^2 h = \frac{1}{48}\pi h^3$

Hence, Volume of water in the tank $= V_{water} = \frac{1}{48}\pi\left(h_{water}\right)^3$ where h_{water} = height of water in the tank

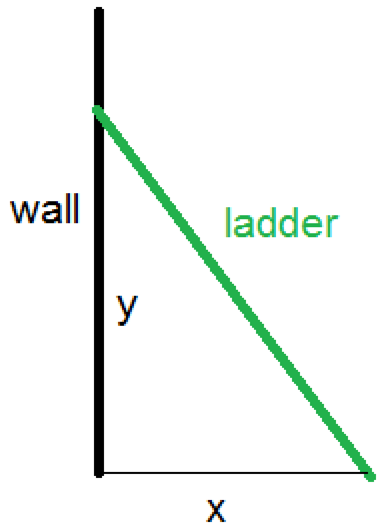
$$V_{water} = \frac{1}{48}\pi\left(h_{water}\right)^3$$

Take implicit derivative with respect to time (t): $V'_{water} = \frac{1}{48}\pi\left[3\left(h_{water}\right)\right]^2 h'_{water} \quad \Rightarrow \quad \frac{dV_{water}}{dt} = \frac{1}{48}\pi\left[3\left(h_{water}\right)\right]^2 \cdot \frac{dh_{water}}{dt}$

Note: $\frac{dV_{water}}{dt} =$ rate of change of volume of water = 1.5; $\frac{dh_{water}}{dt} =$ rate of change of height of water

We need to find $\frac{dh_{water}}{dt}$ when $h_{water} = 6$.

6)



x = distance of bottom of ladder from wall

y = distance from top of ladder to the ground.

Note: As x increases, y will decrease.

$Note: \ x^2 + y^2 = 12^2 \quad \Rightarrow \quad x^2 + y^2 = 144$

Take implicit derivative with respect to time (t): $\ 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

We are given that $\frac{dx}{dt} = 2$ ft/sec.

When $y = 7, \ x^2 + y^2 = 144 \Rightarrow \ x^2 + 7^2 = 144 \Rightarrow x^2 = 144 - 49 = 95 \Rightarrow x = \sqrt{95}$

Find $\frac{dy}{dt}$ when $\frac{dx}{dt} = 2$ ft/sec, $y = 7$, and $x = \sqrt{95}$.