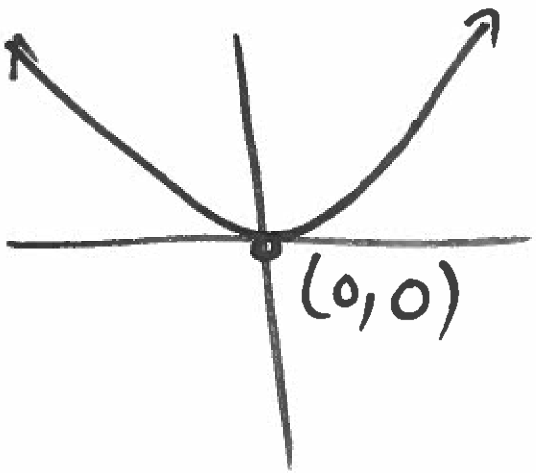
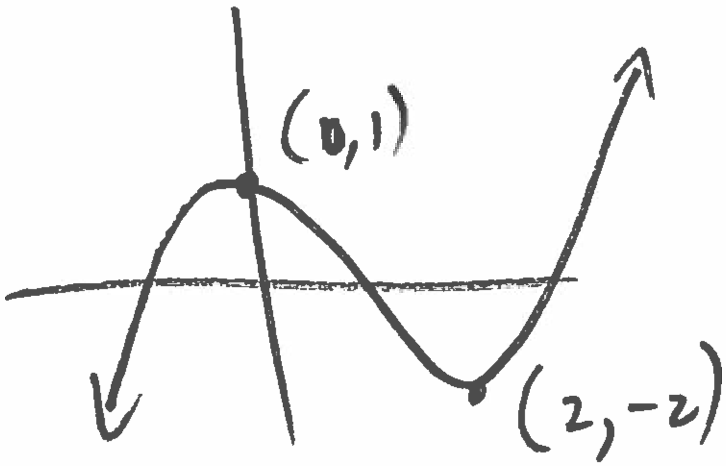
3.1 Extrema on an Interval



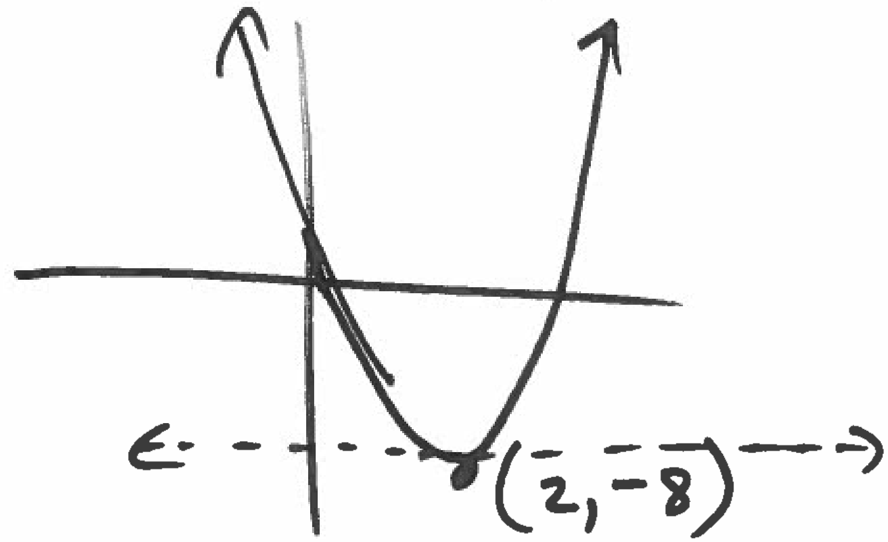
(0, 0) is an extremum point



(0, 1) ; (2, -2) are extremum points

$f(x) = 2x^2 - 8x$

Find extremum points



$f'(x) = 2 \cdot (2x) - 8$

$= 4x - 8$

Set $f'(x) = 0$

$4x - 8 = 0$

$4x = 8$

$x = 2$ Critical Number

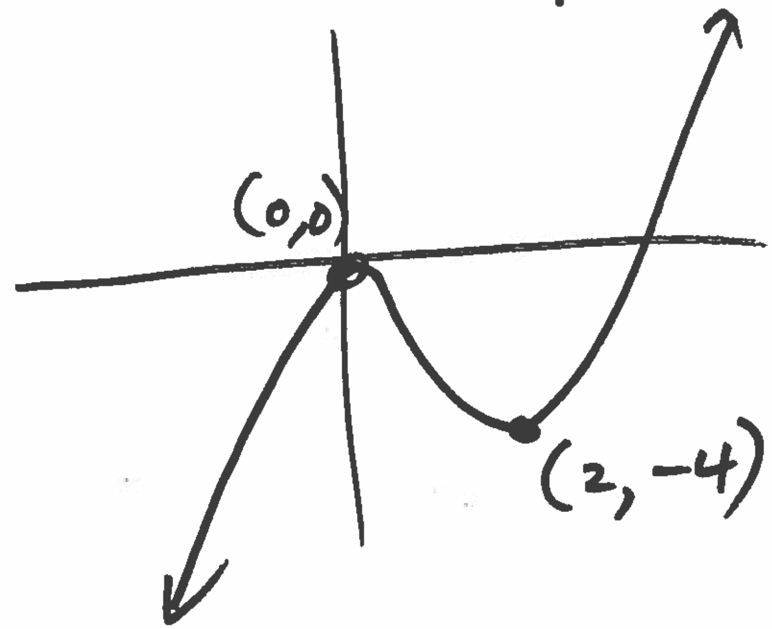
$Y = 2x^2 - 8x$

$Y = 2 \cdot (2)^2 - 8(2) = -8$

$(2, -8)$ is an extremum point.

$$f(x) = x^3 - 3 \cdot x^2$$

Find extremum points



$$f'(x) = 3x^2 - 3 \cdot (2x)$$

$$3x^2 - 6x$$

set $f'(x) = 0$

$$3x^2 - 6x = 0$$

$$(3x) \cdot (x - 2) = 0$$

$\Rightarrow 3x = 0$

$x = 0$

$Y = x^3 - 3x^2$

$Y = 0^3 - 3(0)^2 = 0$

$(0, 0)$

$x - 2 = 0$

$x = 2$

Critical Numbers

$Y = x^3 - 3x^2$

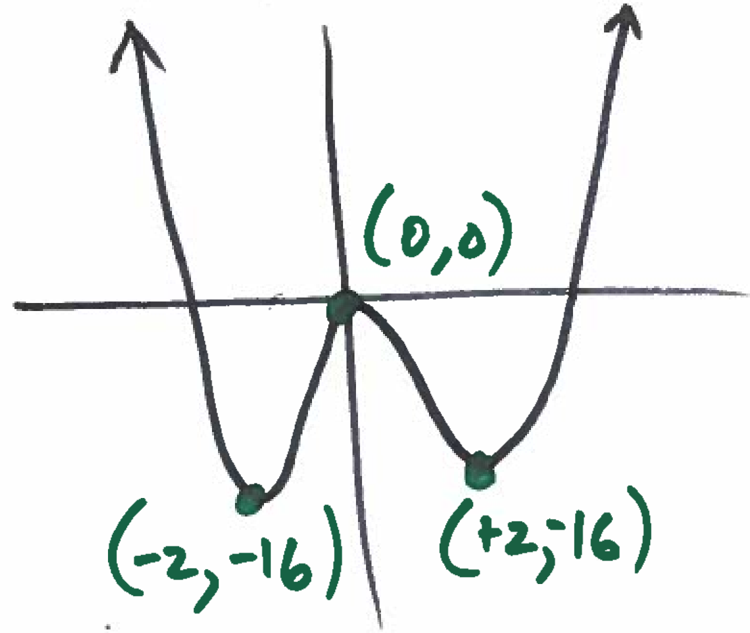
$Y = (2)^3 - 3 \cdot (2)^2 = -4$

$(2, -4)$

Extremum Points

$$f(x) = x^4 - 8 \cdot x^2$$

Find extremum Points



$$f'(x) = 4x^3 - 8 \cdot (2x)$$

$$4x^3 - 16x$$

set $f'(x) = 0$

$$4x^3 - 16x = 0$$

$$(4x) \cdot (x^2 - 4) = 0$$

$$\Rightarrow 4x = 0$$

$$x = 0$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$\sqrt{x^2} = \pm\sqrt{4}$$

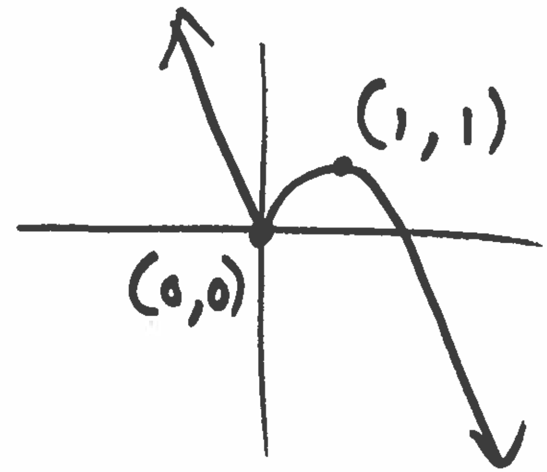
$$x = \pm 2$$

Critical Numbers : $x = 0, 2, -2$

$x = 0$	$x = 2$	$x = -2$
$y = 0$	$y = -16$	$y = -16$

Extremum Points: $(0,0)$ $(2,-16)$ $(-2,-16)$

$$f(x) = 3 \cdot x^{2/3} - 2x = 3x\hat{}(2/3) - 2x$$



$$f'(x) = 3 \cdot \left(\tfrac{2}{3} x^{-1/3}\right) - 2$$

$$f'(x) = \frac{2 \cdot}{x^{1/3}} - 2$$

set $f'(x) = 0$

$$\frac{2}{x^{1/3}} - \frac{2}{1} = 0$$

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$$

$$\frac{2 - 2x^{1/3}}{x^{1/3}} = 0$$

set $2 - 2x^{1/3} = 0$

$$2 = 2x^{1/3}$$

set $x^{1/3} = 0$

$$1 = x^{1/3}$$
$$x^{1/3} = 1$$
$$(x^{1/3})^3 = (1)^3$$
$$x = 1$$

$$(x^{1/3})^3 = (0)^3$$
$$x = 0$$

Critical Numbers: $x = 1, 0$

$x = 0$ $x = 1$

$y = 0$: $y = 1$

Extremum Points: $(0, 0)$; $(1, 1)$

$$f(x) = 3x^{2/3} - 2x$$

f(x) is not differentiable at $x = 0$ because the derivative is undefined when $x = 0$.

$$f'(x) = \frac{2 - x^{1/3}}{x^{1/3}}$$

Derivative Function

When $x = 0$, $f'(0) =$ undefined

Derivative is undefined when $x=0$

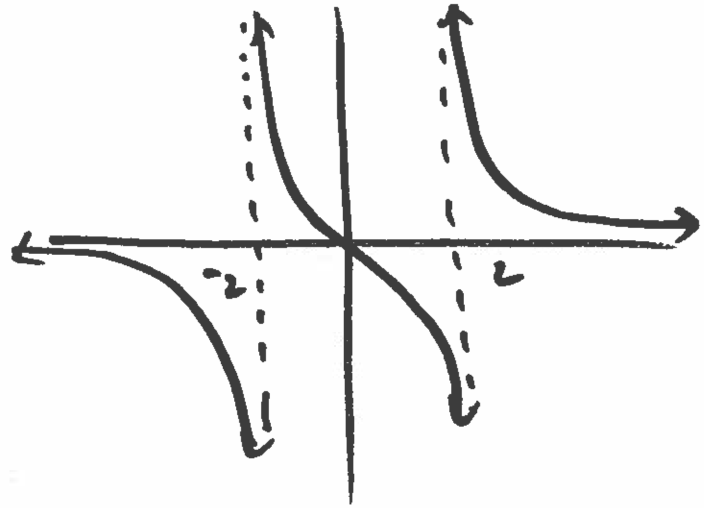
$$f(x) = \frac{2x}{x^2 - 4} = \frac{N}{D}$$

Find Extremum

$$= (2x)/(x^2-4)$$

$$f'(x) = \frac{D \cdot D_x(N) - N \cdot D_x(D)}{D^2}$$

$$f'(x) = \frac{(x^2-4)(2) - (2x)(2x)}{(x^2-4)^2}$$



set $(x^2-4)(2) - (2x)(2x) = 0$

$$2x^2 - 8 - 4x^2 = 0$$

$$-2x^2 - 8 = 0$$

$$-2x^2 = 8$$

$$x^2 = -4$$

$$\sqrt{x^2} = \pm\sqrt{-4}$$

$$x = \pm 2i$$

set $(x^2-4)^2 = 0$

$$\cancel{\sqrt{(x^2-4)^2}} = \pm\sqrt{0}$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$\sqrt{x^2} = \pm\sqrt{4}$$

$$x = \pm 2$$

Critical Numbers

$$h(x) = \sin^2 x + \cos x \qquad x \in (0, 2\pi)$$

$$h(x) = (\sin x)^2 + \cos x$$

$$(\text{base})^{\text{exponent}}$$

$$h'(x) = 2(\sin x)^1 \cdot D_x(\sin x) + -\sin x$$

$$h'(x) = 2 \sin x \cdot \cos x - \sin x$$

set $h'(x) = 0$

$$2 \cdot \sin x \cdot \cos x - \sin x = 0$$

$$(\sin x)(2\cos x - 1) = 0$$

set $\sin x = 0$

$$\cancel{x = 0}$$

$$x = \pi$$

set $2\cos x - 1 = 0$

$$2\cos x = 1$$

$$\cos x = 1/2$$

$$x = \pi/2 \;;\; 5\pi/3$$

Critical Numbers: $x = \pi$, $y = -1$ | $x = \pi/3$, $y = 1.25$ | $x = 5\pi/3$, $y = 1.25$