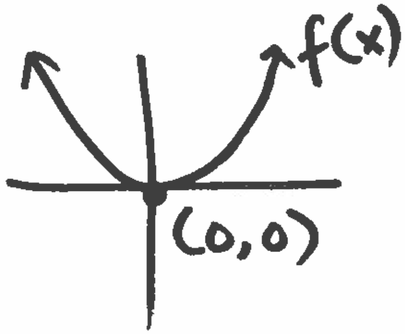
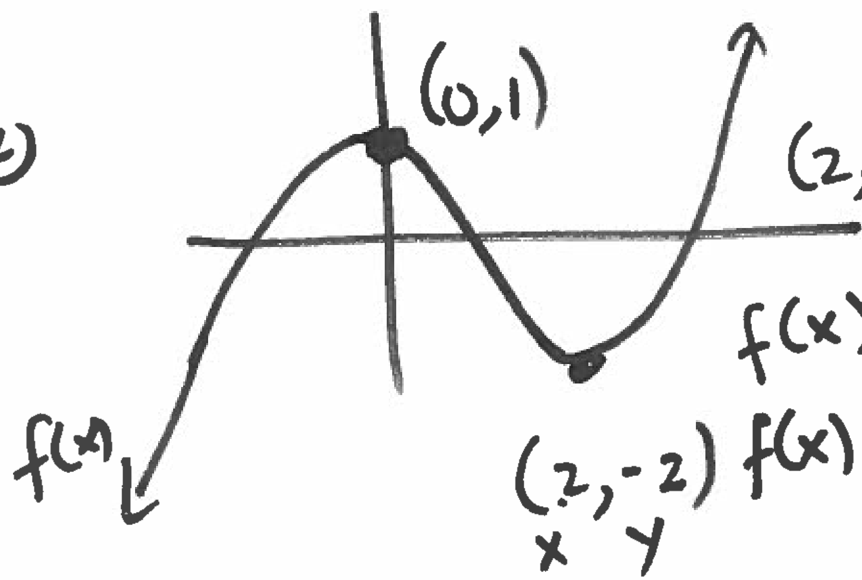
3.1 Extrema on an interval

① 

(0, 0) is an extremum point.

f(x) has a local minimum at $x = 0$.

Minimum function value is 0

② 

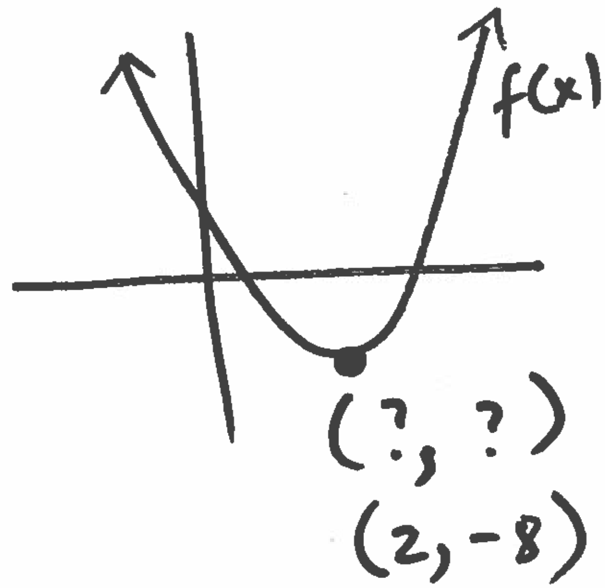
(0,1) is a local maximum point

(2,−2) is local minimum point

f(x) has local minimum at $x = 2$

f(x) has local minimum at $x =$

③ $f(x) = 2x^2 - 8x$. Find extremum point(s)



$f'(x) = 2(2x) - 8\cdot(1)$

$f'(x) = 4x - 8$

set $f'(x) = 0$

$4x - 8 = 0$

$4x = 8$

$x = 2$ ⟵ critical value

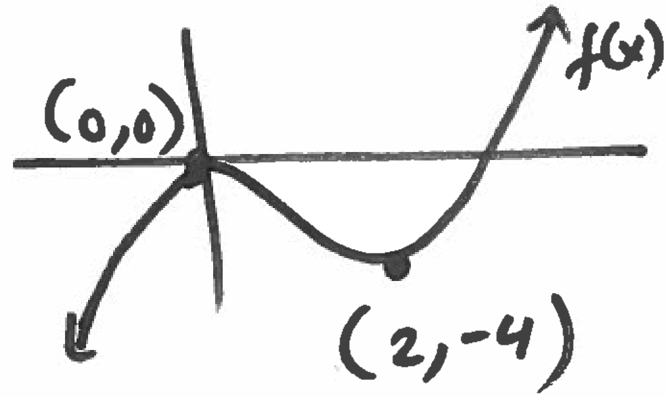
When $x = 2$, $y = 2x^2 - 8x$

$y = 2\cdot(2)^2 - 8(2) = -8$

$(2, -8)$ is an extremum point

(4) $f(x) = x^3 - 3x^2$

Find extremum points



$f'(x) = 3x^2 - 3\cdot(2x)$

$f'(x) = 3x^2 - 6x$

set $f'(x) = 0$

$3x^2 - 6x = 0$

$(3x)\cdot(x-2) = 0$

set $3x = 0$

$x = 0$

$y = x^3 - 3x^2$

$y = 0^3 - 3\cdot 0^2 = 0$

$(0, 0)$

set $x - 2 = 0$

$x = 2$

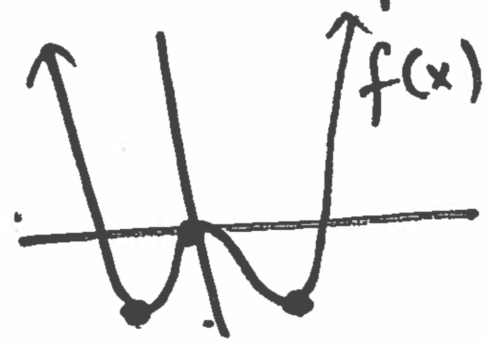
$y = x^3 - 3x^2$

$y = (2)^3 - 3\cdot(2)^2 = -4$

$(2, -4)$

⑤ $f(x) = x^4 - 8x^2$

Find extremum points



$f'(x) = 4x^3 - 8 \cdot 2x$

$f'(x) = 4x^3 - 16x$

set $f'(x) = 0$

$4x^3 - 16x = 0$

$(4x)(x^2 - 4) = 0$

set $4x = 0$

$x = 0$

set $x^2 - 4 = 0$

$x^2 = 4$

$\sqrt{x^2} = \pm\sqrt{4}$

$x = \pm 2$

Critical Values are $x = 0$, $x = 2$, $x = -2$

$x = 0$

$y = x^4 - 8x^2$

$y = 0^4 - 8(0)^2 = 0$

$(0, 0)$

$x = 2$

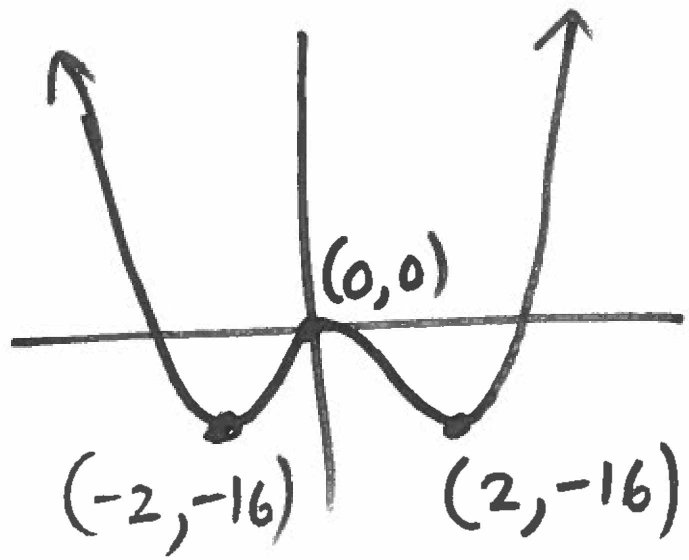
$y = (2)^4 - 8(2)^2 = -16$

$(2, -16)$

$x = -2$

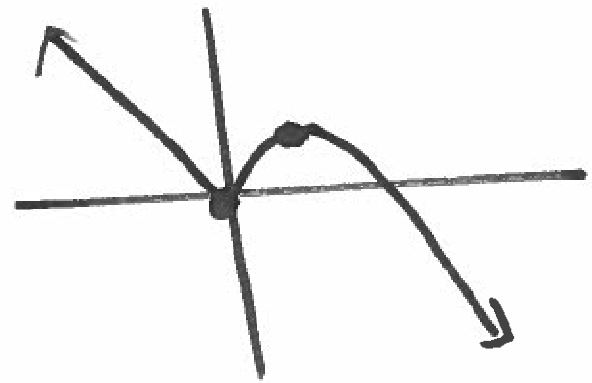
$y = (-2)^4 - 8\cdot(-2)^2 = -16$

$(-2, -16)$



⑥ $f(x) = 3x^{2/3} - 2x$

Find extremum points



$f'(x) = 3 \cdot \left(\frac{2}{3} x^{-\frac{1}{3}} \right) - 2 \cdot (1)$

$f'(x) = 2 \cdot \frac{1}{x^{1/3}} - 2$

$f'(x) = \frac{2}{x^{1/3}} - 2$

Set $f'(x) = 0$

$\frac{2}{x^{1/3}} - \frac{2}{1} = 0$

$\frac{2 \cdot 1 - 2 \cdot x^{1/3}}{x^{1/3} \cdot 1} = 0$

Recall:

$\frac{a}{b} \pm \frac{c}{d}$

$= \frac{ad \pm bc}{bd}$

$$f'(x) = \frac{2 - 2x^{1/3}}{x^{1/3}} = 0$$

set $2 - 2x^{1/3} = 0$

$2 = 2x^{1/3}$

$1 = x^{1/3}$

$x^{1/3} = 1$

$(x^{1/3})^3 = (1)^3$

$x^1 = 1$

$x = 1$

$y = 3x^{2/3} - 2x$

$y = 3\cdot(1)^{2/3} - 2(1) = 1$

$(1, 1)$

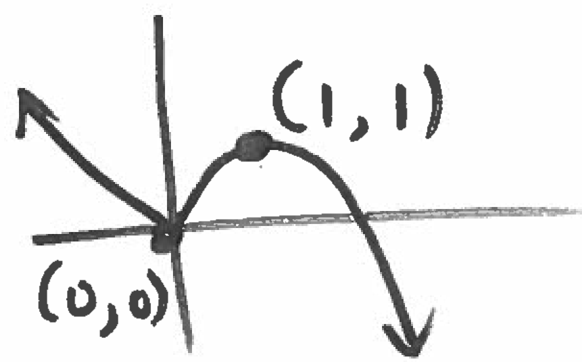
set $x^{1/3} = 0$

$(x^{1/3})^3 = 0^3$

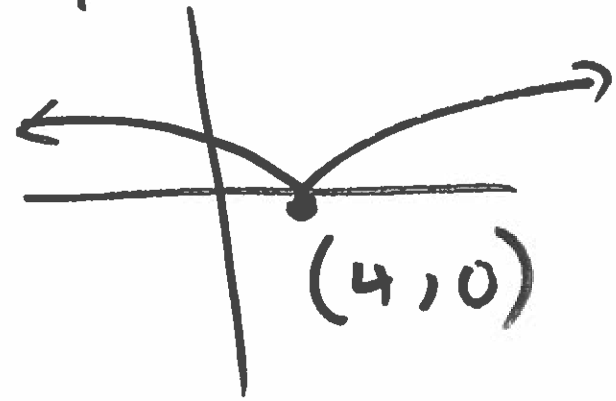
$x = 0$

$y = 3\cdot(0)^{2/3} - 2(0) = 0$

$(0, 0)$



⑦ $f(x) = (x-4)^{2/3}$ Find extremum



$$f'(x) = \frac{2}{3}\cdot(x-4)^{\frac{2}{3}-1}\cdot D_x(x-4)$$

$$f'(x) = \frac{2}{3}(x-4)^{-1/3}\cdot(1)$$

$$f'(x) = \frac{2}{3}\cdot\frac{1}{(x-4)^{1/3}} = \frac{2}{3(x-4)^{1/3}}$$

Set $2 = 0$

False

Set $3\cdot(x-4)^{1/3} = 0$

$$(x-4)^{1/3} = 0$$

$$\left((x-4)^{1/3}\right)^3 = 0^3 \quad (4, 0)$$

$$x-4 = 0$$

$$x = 4$$

$$y = (x-4)^{2/3} = (4-4)^{2/3} = 0$$

$f(x) = (x-4)^{2/3}$

$f'(x) = \dfrac{2}{3(x-4)^{1/3}}$

$f'(4) = \dfrac{2}{3(\cancel{x}4-4)^{1/3}} = \dfrac{2}{0} = \text{undefined}$

At $x=4$, $f'(x)$ is undefined.

We say the $f(x) = (x-4)^{2/3}$ is not differentiable at $x=4$ because $f'(4)$ is undefined.

$f'(3) = \dfrac{2}{3(3-4)^{1/3}} = \dfrac{2}{3(-1)^{1/3}} = \dfrac{2}{3\cdot(-1)} = -\dfrac{2}{3}$

At $x=3$, $f'(x)$ is defined

We say that $f(x) = (x-4)^{2/3}$ is differentiable at $x = 3$ because $f'(3)$ is defined (or equal to a real number).

$$f'(5) = \frac{2}{3(5-4)^{1/3}} = \frac{2}{3(1)^{1/3}} = \frac{2}{3}$$

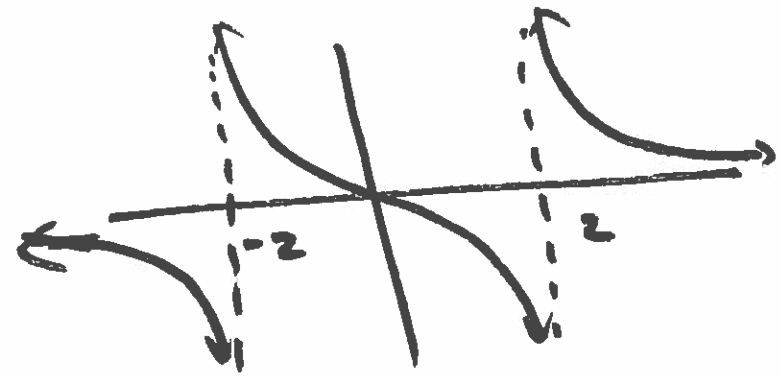
At $x=5$, $f'(x)$ is defined

We say that $f(x) = (x-4)^{2/3}$ is differentiable at $x = 5$ because $f'(5)$ is defined.

8)

$$f(x) = \frac{2x}{x^2-4}$$

Find extremum



$$f'(x) = \frac{L \cdot D_x(H) - H \cdot D_x(L)}{L^2}$$

$$f'(x) = \frac{(x^2-4) \cdot D_x(2x) - 2x \cdot D_x(x^2-4)}{(x^2-4)^2}$$

$$f'(x) = \frac{(x^2-4)(2) - 2x \cdot (2x)}{(x^2-4)^2}$$

$$f'(x) = \frac{2x^2-8 - 4x^2}{(x^2-4)^2} = \frac{-8-2x^2}{(x^2-4)^2}$$

set $-8-2x^2=0$

$$-2x^2 = 8$$

$$x^2 = -4$$

$$\sqrt{x^2} = \pm\sqrt{-4}$$

$$x = \pm 2i$$

$\Rightarrow$ No extremum

so $f(x)$ has no extremums

set $(x^2-4)^2=0$

$$\sqrt{(x^2-4)^2} = \pm\sqrt{0}$$

$$x^2-4 = 0$$

$$x^2 = 4$$

$$\sqrt{x^2} = \pm\sqrt{4}$$

$$x = \pm 2$$

At $x=2$,

$$y = \frac{2(2)}{(2)^2-4} = \frac{4}{0} = \text{undefined}$$

(2, undef)

At $x=-2$,

$$y = \frac{2(-2)}{(-2)^2-4} = \frac{-4}{0} = \text{undefined}$$

(−2, undef)

g) $h(x) = \sin^2 x + \cos x \qquad x \in [0, 2\pi]$

$h(x) = (\sin x)^2 + \cos x$

$h'(x) = 2 \cdot (\sin x)^{2-1} \cdot D_x(\sin x) + -\sin x$

$h'(x) = 2 \sin x \cdot \cos x - \sin x$

$h'(x) = \sin x \cdot \left(2\cos x - \frac{1}{\cancel{\sin x}}\right)$

Set $\sin x = 0$

$x = 0$

$x = \pi$

$x = 2\pi$

set $2\cos x - 1 = 0$

$2\cos x = 1$

$\cos x = 1/2$

$x = \pi/3$

$x = 5\pi/3$

Critical Values:

$x = 0 \qquad y = \sin^2 x + \cos x = \sin^2 0 + \cos(0) = 1$

$x = \pi \qquad y = -1$

$x = 2\pi \qquad y = 1$

$x = \pi/3 \qquad y = 1.25$

$x = 5\pi/3 \qquad y = 1.25$

Extremum points: $(0, 1)$, $(\pi, -1)$, $(2\pi, 1)$

$(\pi/3, 1.25)$, $(5\pi/3, 1.25)$