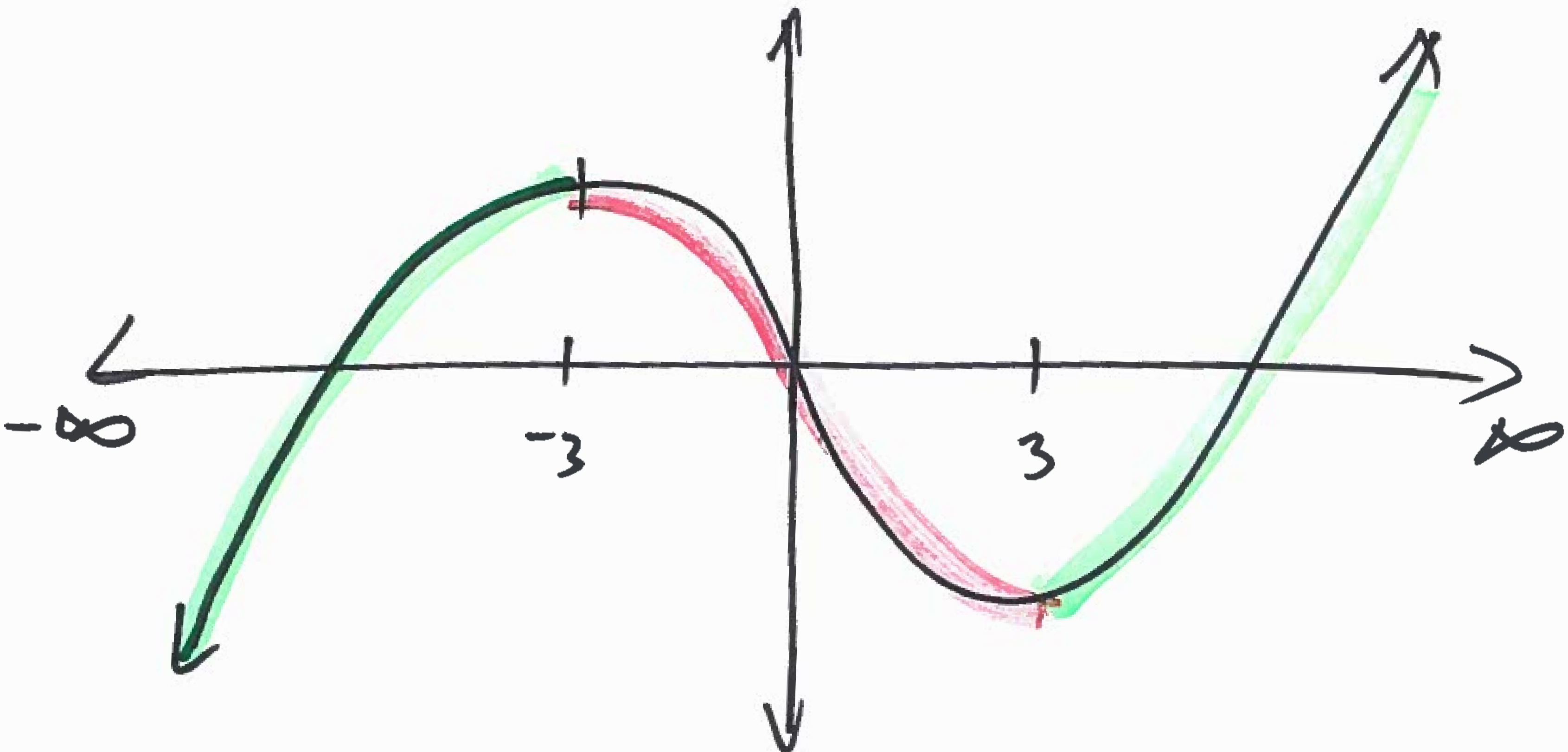
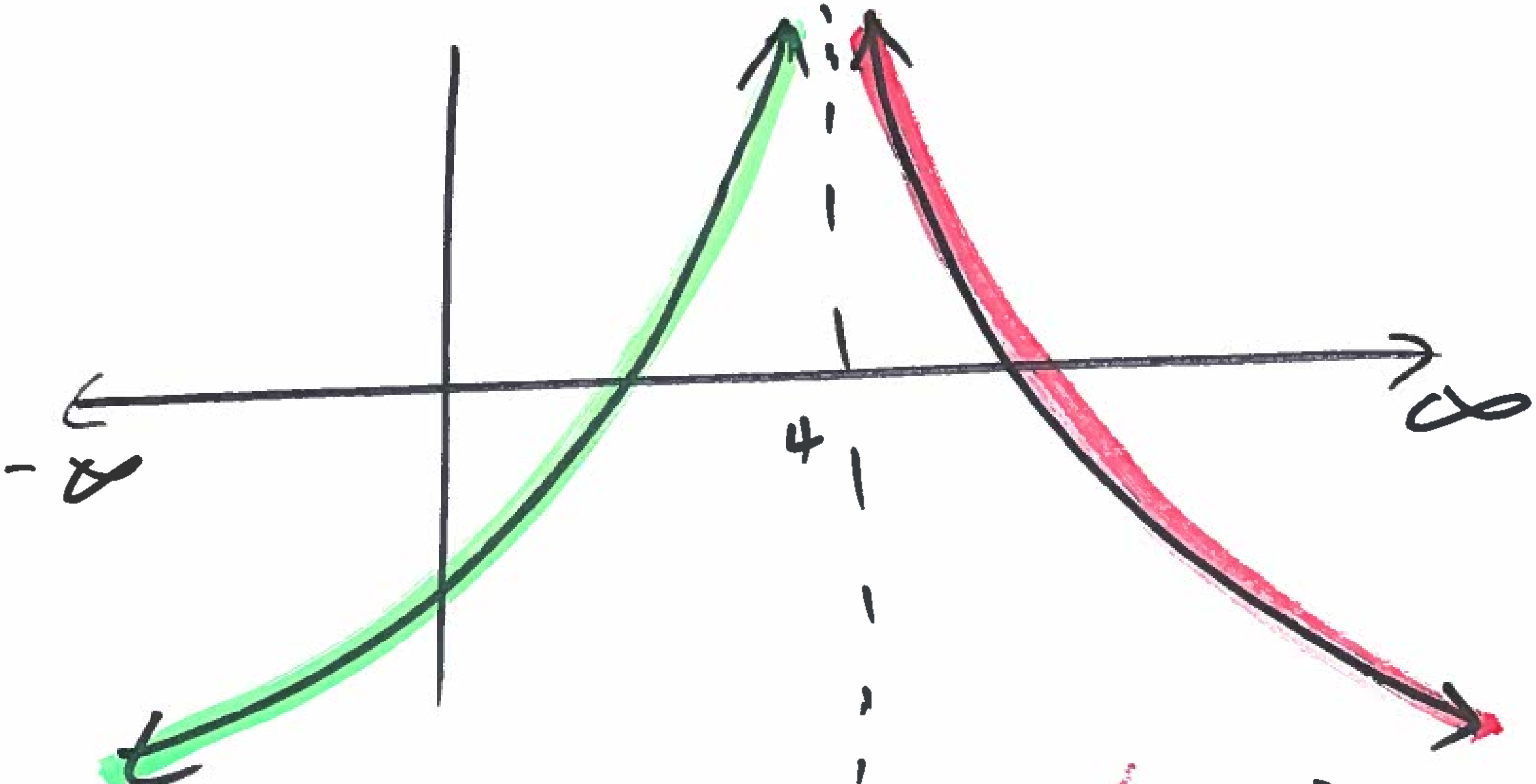
3.3 Derivative Application



$(-\infty, -3)$
Graph is increasing

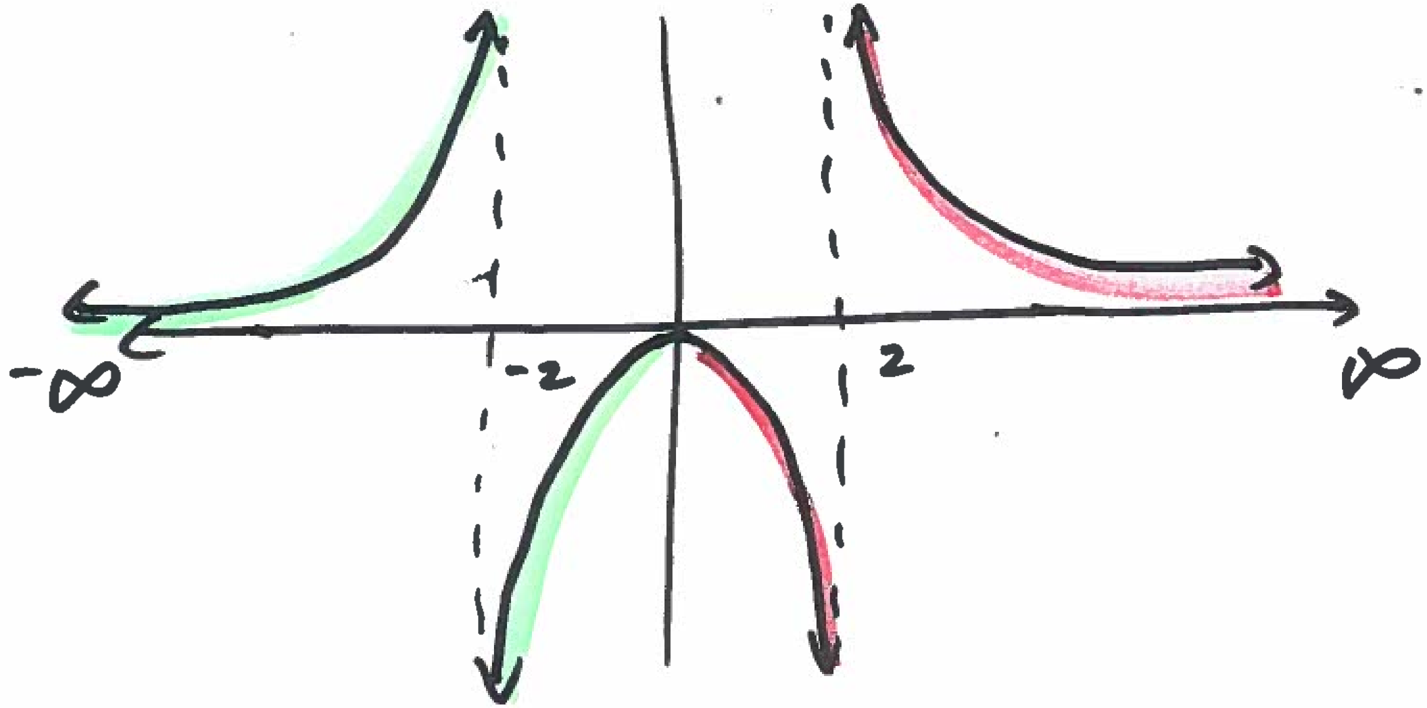
$(-3, 3)$
Graph is decreasing

$(3, \infty)$
Graph is increasing



$(-\infty, 4)$
Graph is increasing

$(4, \infty)$
Graph is decreasing.



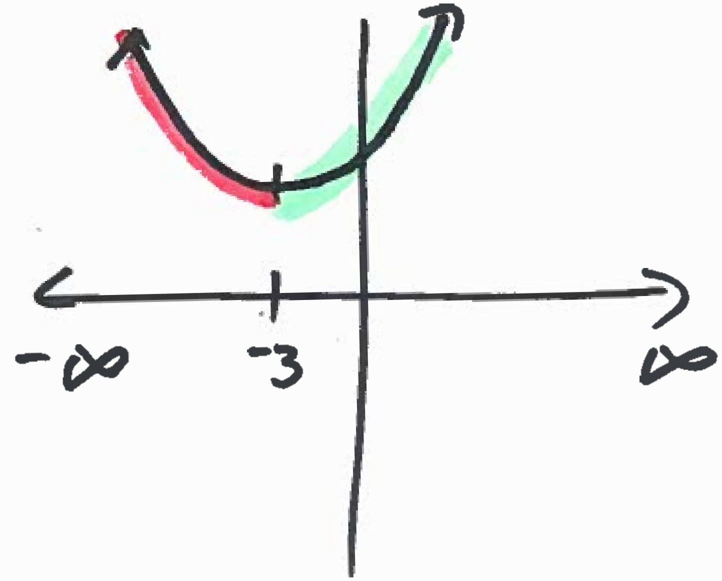
Graph is increasing on the intervals: $(-\infty, -2) \cup (-2, 0)$

Graph is decreasing on the intervals: $(0, 2) \cup (2, \infty)$

Example 1

$f(x) = x^2 + 6x + 10$

Find the intervals on which graph is increasing, decreasing, or constant.



$f'(x) = 2x + 6$

set $f'(x) = 0$

$2x + 6 = 0$

$2x = -6$

$x = -3$ Critical Number

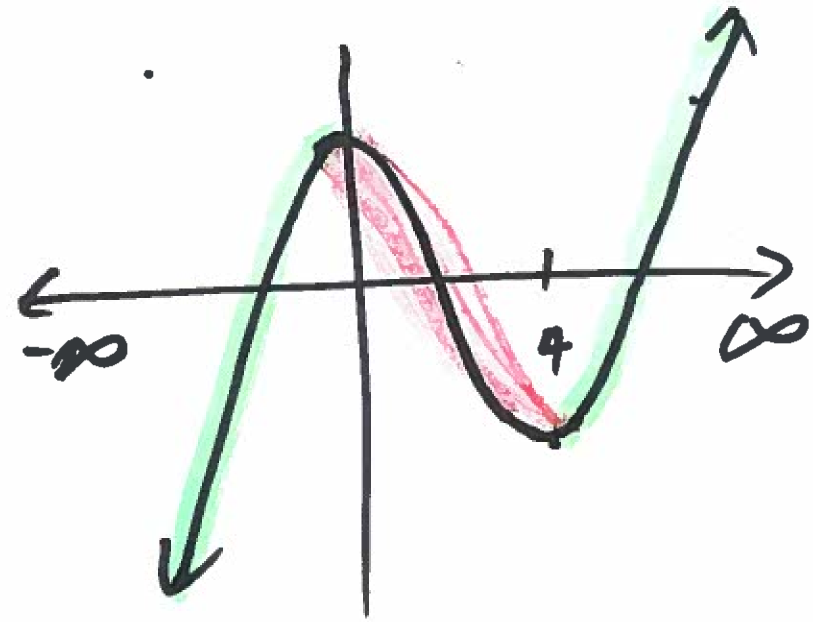
So, Graph decreases on $(-\infty, -3)$

Graph increases on $(-3, \infty)$

Example 2

$f(x) = x^3 - 6x^2 + 15$

Find the intervals on which graph is increasing, decreasing, or constant.



$f'(x) = 3x^2 - 12x$

set $f'(x) = 0$

$3x^2 - 12x = 0$

$(3x)(x-4) = 0$

$3x = 0$	$x - 4 = 0$
$x = 0$	$x = 4$

Critical Numbers $= 0; 4$

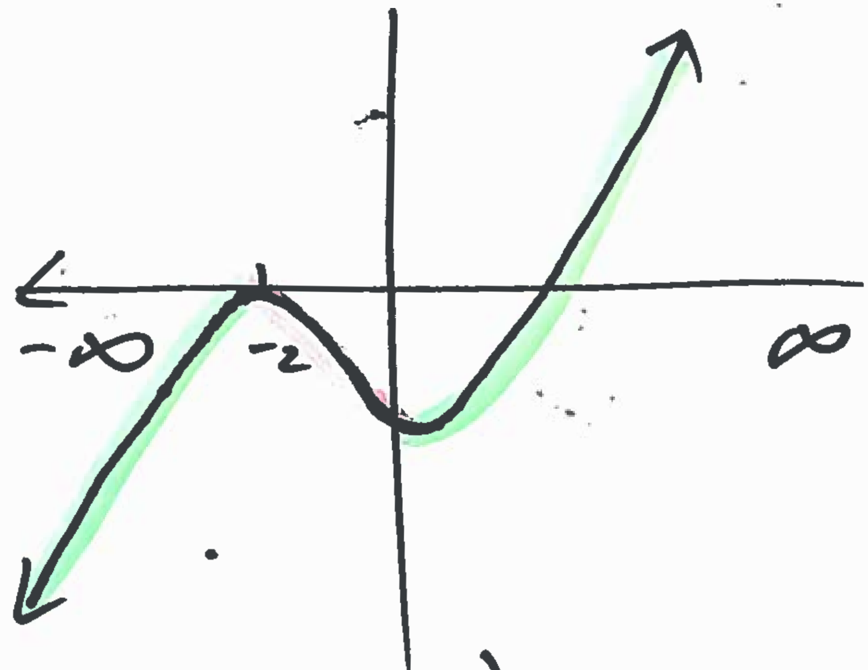
So: Graph increases on $(-\infty, 0) \cup (4, \infty)$

Graph decreases on $(0, 4)$

Example 3

$f(x) = (x+2)^2 \cdot (x-1)$

Find intervals on which graph is increasing, decreasing, or constant.



$f'(x) = (x+2)^2 \cdot D_x(x-1) + (x-1) \cdot D_x\left((x+2)^2\right)$

$f'(x) = (x+2)^2 \cdot (1) + (x-1) \cdot [\,2x+4\,]$

$f'(x) = x^2 + 4x + 4 + 2x^2 + 4x - 2x - 4$

$f'(x) = 3x^2 + 6x.$

Set $f'(x) = 0$

$3x^2 + 6x = 0$

**1

$(3x) \cdot (x+2) = 0$

$3x = 0$ | $x + 2 = 0$

$x = 0$ | $x = -2$

Critical Numbers $= 0; -2$

So: Graph increases on $(-\infty, -2) \cup (0, \infty)$

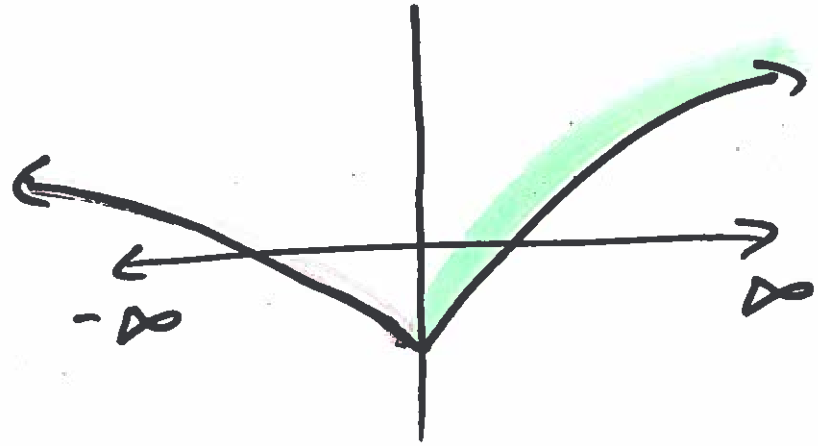
Graph decreases on $(-2, 0)$

**1

Example 4

$f(x) = x^{2/3} - 4 = x\wedge(2/3) - 4$

Find intervals on which graph is increasing and decreasing.



$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$

Set $f'(x) = 0$

$\frac{2}{3x^{1/3}} = 0$

set $2 \neq 0$

set $3x^{1/3} = 0$

$x^{1/3} = 0$

$(x^{1/3})^3 = (0)^3$

$x = 0$ Critical Number

**2

So: Graph decreases on $(-\infty, 0)$

Graph increases on $(0, \infty)$

**2

Example 5

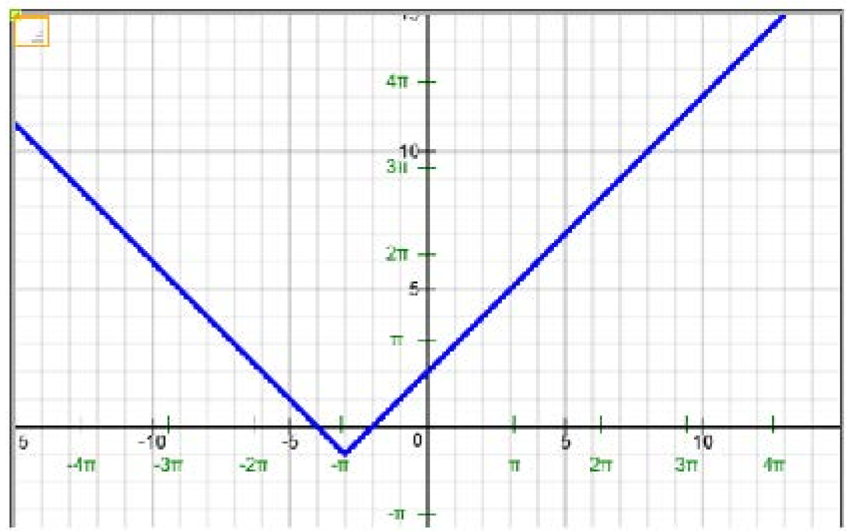
Let $f(x) = |x+3| - 1$

Find the intervals where $f(x)$ is increasing or decreasing.

For absolute value function, set inside of absolute value equal to zero to find critical value.

Set $x + 3 = 0$

$x = -3$ This is the critical value.



Hence,

Graph is decreasing on $(-\infty, -3)$.

Graph is increasing on $(-3, \infty)$.

Example 6:

Let $f\left(x\right)=\begin{cases}2x+1 & \text{if } x\le -1\\ x^2-2 & \text{if } x>-1\end{cases}$

Find the intervals where $f\left(x\right)$ is increasing or decreasing.

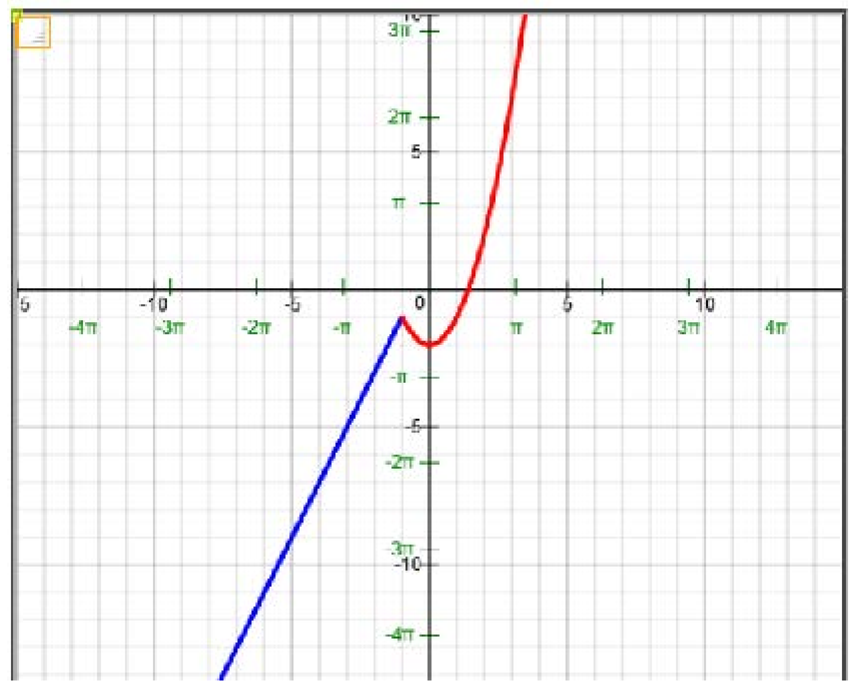
For piecewise function, look at graph to find critical value(s).

Alternatively, find derivative of $f\left(x\right)$ and set it equal to zero.

Hence (by looking at graph),

Graph is decreasing on $\left(-1,0\right)$.

Graph is increasing on $\left(-\infty,-1\right)$ and $\left(0,\infty\right)$.



Example 7:

Let $f\left(x\right) = x\sqrt{9-x^2} = x\left(9-x^2\right)^{1/2}$

Find the intervals where $f\left(x\right)$ is increasing or decreasing.

$$f'(x) = x \cdot D_x\left[\left(9-x^2\right)^{1/2}\right] + \left(9-x^2\right)^{1/2} \cdot D_x\left[x\right]$$

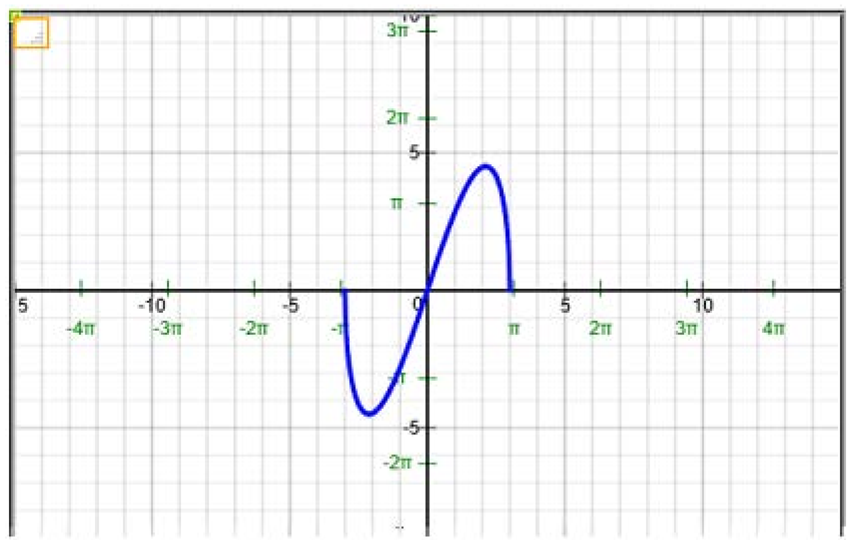
$$f'(x) = x \cdot \left[\frac{1}{2}\left(9-x^2\right)^{-1/2}\left(-2x\right)\right] + \left(9-x^2\right)^{1/2} \cdot \left[1\right]$$

Recall: $D_x\left[\left(base\right)^{\exp}\right] = \exp\left[\left(base\right)^{\exp-1}\right] \cdot D_x\left[\left(base\right)\right]$

$$f'(x) = -x^2 \cdot \left[\left(9-x^2\right)^{-1/2}\right] + \left(9-x^2\right)^{1/2}$$

$$f'(x) = -x^2 \cdot \left[\frac{1}{\left(9-x^2\right)^{1/2}}\right] + \left(9-x^2\right)^{1/2}$$

$$f'(x) = \frac{-x^2}{\left(9-x^2\right)^{1/2}} + \frac{\left(9-x^2\right)^{1/2}}{1}$$



Set $f'(x) = 0$

$$\frac{-x^2}{\left(9-x^2\right)^{1/2}} + \frac{\left(9-x^2\right)^{1/2}}{1} = 0$$

$$\frac{-x^2 + \left(9-x^2\right)^{1/2}\left(9-x^2\right)^{1/2}}{\left(9-x^2\right)^{1/2}} = 0$$

$$\frac{-x^2 + \left(9-x^2\right)^{1}}{\left(9-x^2\right)^{1/2}} = 0$$

Note: $\left(9-x^2\right)^{1/2}\left(9-x^2\right)^{1/2} = \left(9-x^2\right)^{1/2+1/2} = \left(9-x^2\right)^{1}$

To find critcal values:

Set $-x^2 + \left(9-x^2\right)^{1} = 0$ and $\left(9-x^2\right)^{1/2} = 0$

$9 - 2x^2 = 0$ $\quad$ $9 - x^2 = 0$

$2x^2 = 9$ $\quad$ $x^2 = 9$

$x = \pm\sqrt{9/2}$ $\quad$ $x = \pm 3$

From graph of $f(x) = x\sqrt{9-x^2}$, we can see that:

1) Graph is decreasing on $\left(-3, -\sqrt{9/2}\right)$ and $\left(\sqrt{9/2}, 3\right)$.

2) Graph is increasing on $(-\sqrt{9/2}, \sqrt{9/2})$

Example 8

Let $f(x) = \cos(2x) \qquad x \in [0, 2\pi]$

Find the intervals where $f(x)$ is increasing or decreasing.

$$f'(x) = -\sin(2x) \cdot D_x[2x] = -\sin(2x)(2) = -2\sin(2x)$$

Recall: $D_x\left[\cos(\text{expr})\right] = -\sin(\text{expr}) \cdot D_x[\text{expr}]$

Set $f'(x) = 0$

$-2\sin(2x) = 0$

$\sin(2x) = 0$

Recall: $\sin(0) = 0;\ \sin(\pi) = 0; \sin(2\pi) = 0; \sin(3\pi) = 0; \sin(4\pi) = 0; \sin(5\pi) = 0$

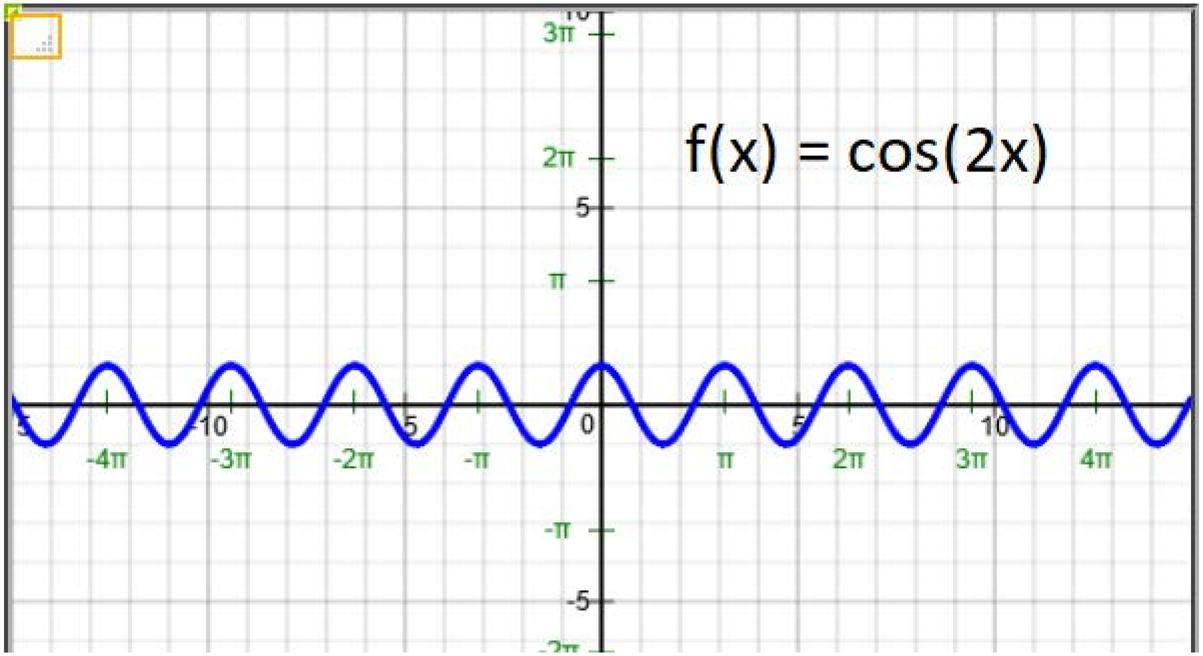
Hence, $2x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

By looking at graph of $f(x) = \cos(2x)$

Graph is decreasing on $(0, \pi/2)$ and $(\pi, 3\pi/2)$

Graph is increasing on $(\pi/2, \pi)$ and $(3\pi/2, 2\pi)$


f(x) = cos(2x)
3π
2π
5
π
-π
-5
-2π
-4π
-3π
-2π
-π
π
2π
3π
4π
-10
-5
0
5
10

Example 9

Let $f(x) = \sin(4x) - 2 \qquad x \in [0, 2\pi]$

Find the intervals where $f(x)$ is increasing or decreasing.

$$f'(x) = \cos(4x) \cdot D_x(4x) = \cos(4x) \cdot 4 = 4\cos(4x)$$

Set $f'(x) = 0$

$4\cos(4x) = 0$

$\cos(4x) = 0$

Recall: $\cos(\pi/2) = 0;\ \cos(3\pi/2) = 0;\ \cos(5\pi/2) = 0;\ \cos(7\pi/2) = 0;\ \cos(9\pi/2) = 0;\ \ldots$

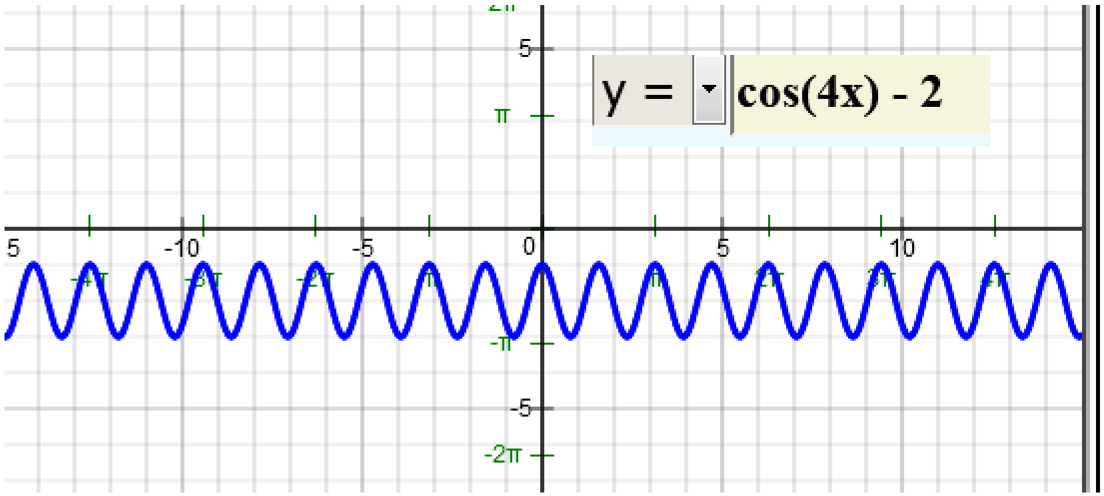
Hence, $4x = \pi/2;\ 3\pi/2;\ 5\pi/2;\ 7\pi/2;\ 9\pi/2;\ 11\pi/2;\ 13\pi/2;\ \ 15\pi/2$

$x = \pi/8;\ 3\pi/8;\ 5\pi/8;\ 7\pi/8;\ 9\pi/8;\ 11\pi/8;\ 13\pi/8;\ \ 15\pi/8$

By look at graph:

Graph increases on $(\pi/8, 3\pi/8) \cup (5\pi/8, 7\pi/8) \cup (9\pi/8, 11\pi/8) \cup (13\pi/8, 15\pi/8)$

Graph decreases on $(0, \pi/8) \cup (3\pi/8, 5\pi/8) \cup (7\pi/8, 9\pi/8) \cup (11\pi/8, 13\pi/8)$


y = cos(4x) - 2
5
π
-π
-5
-2π
0
-10
-5
5
10