3.5 Limits at Infinity

Review:

$$\frac{5}{\infty} = 0$$

- $$\frac{\text{Constant}}{\infty} = 0$$

$$\frac{\infty}{10} = \infty$$

$$\frac{\infty}{+\text{constant}} = +\infty$$

$$\frac{\infty}{-\text{constant}} = -\infty$$

$$\infty + \infty = \infty$$

$$\infty - \infty = \text{Indeterminate}$$

$$\infty \cdot \infty = \infty$$

$$\infty/\infty = \text{Indeterminate}$$

Review.

$5^{\infty} = \infty$

$\text{Constant}^{\infty} = \infty$

$5^{-\infty} = \frac{1}{5^{\infty}} = \frac{1}{\infty} = 0$

$\text{Constant}^{-\infty} = 0$

$\boxed{e^{\infty} = \infty}$

$e^{-\infty} = 0$

$\infty^{\text{positive power}} = \infty$

$\infty^{2} = \infty$

$\infty^{1/2} = \infty$

$\infty^{1/4} = \infty$

$\infty^{\text{Negative power}} = 0$

$\infty^{-2} = 1/\infty^{2} = \frac{1}{\infty} = 0$

Review

$$\sqrt{\infty} = \infty$$

$$\sqrt[n]{\infty} = \infty$$

$$\cancel{\sqrt[\text{odd}]{-\infty} = -\infty}$$

$$\cancel{\sqrt[\text{even}]{-\infty} =} \text{ imaginary}$$

$$\ln(\infty) = \infty$$

#1

$$\lim_{x \to \infty} \left(\frac{3 - 2x}{3x^3 - 1} \right) = \lim_{x \to \infty} \left(\frac{3/x^3 - 2x/x^3}{3x^3/x^3 - 1/x^3} \right)$$

$$= \lim_{x \to \infty} \left(\frac{3/x^3 - 2/x^2}{3 - 1/x^3} \right)$$

$$= \frac{3/\infty - 2/\infty}{3 - 1/\infty} = \frac{0 - 0}{3 - 0} = \frac{0}{3} = 0$$

#2

$$\lim_{x\to\infty} \frac{3-2x}{3x-1} = \lim_{x\to\infty}\left(\frac{3/x - 2x/x}{3x/x - 1/x}\right)$$

$$= \lim_{x\to\infty}\left(\frac{3/x - 2}{3 - 1/x}\right)$$

$$= \frac{3/\infty - 2}{3 - 1/\infty} = \frac{0-2}{3-0} = \frac{-2}{3}$$

#3

$$\lim_{x\to\infty}\left(\frac{5x^{3/2}}{4x^2+1}\right) = \lim_{x\to\infty}\left(\frac{5x^{3/2}/x^2}{4x^2/x^2 + 1/x^2}\right)$$

$$= \lim_{x\to\infty}\left(\frac{5\cdot x^{-1/2}}{4+1/x^2}\right)$$

$$= \lim_{x\to\infty}\left(\frac{5/x^{1/2}}{4+1/x^2}\right) = \frac{5/\infty^{1/2}}{4+1/\infty} = \frac{5/\infty}{4+1/\infty}$$

$$= \frac{0}{4+0} = 0$$

#4

$$\lim_{x\to\infty}\left(\frac{5x^{3/2}}{4\sqrt{x}+1}\right)=\lim_{x\to\infty}\left(\frac{5x^{3/2}}{4x^{1/2}+1}\right)$$

$$=\lim_{x\to\infty}\left(\frac{5x^{3/2}/x^{1/2}}{4x^{1/2}/x^{1/2}+1/x^{1/2}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{5\cdot x}{4+1/x^{1/2}}\right)=\frac{5\cdot\infty}{4+1/\infty^{1/2}}$$

$$=\frac{\infty}{4+0}=\frac{\infty}{4}=\infty$$

#5

$$\lim_{x\to\infty}\left(\frac{5x^3+1}{10x^3-3x^2+7}\right)$$

$$=\lim_{x\to\infty}\left(\frac{5x^3/x^3+1/x^3}{10x^3/x^3-3x^2/x^3+7/x^3}\right)$$

$$=\lim_{x\to\infty}\left(\frac{5+1/x^3}{10-3/x+7/x^3}\right)=\frac{5+1/\infty}{10-3/\infty+7/\infty}$$

$$=\frac{5+0}{10-0+0}=\frac{5}{10}=\frac{1}{2}$$

#6

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\sqrt{x^2} = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\sqrt{x^2} = \begin{cases} x & \text{if } x \to +\infty \\ -x & \text{if } x \to -\infty \end{cases}$$

#7

$$\lim_{x \to -\infty} \frac{x}{\sqrt{x^2}+1}$$

$$= \lim_{x \to -\infty} \frac{x/\sqrt{x^2}}{\sqrt{x^2+1}\,/\sqrt{x^2}}$$

$$= \lim_{x \to -\infty} \frac{x/(-x)}{\sqrt{\dfrac{x^2+1}{x^2}}}$$

$$= \lim_{x \to -\infty} \left(\frac{-1}{\sqrt{1+1/x^2}} \right) = \frac{-1}{\sqrt{1+1/\infty}} = \frac{-1}{1} = -1$$

Note

Since $x \to -\infty$,

$\sqrt{x^2} = -x$

$\sqrt{A}/\sqrt{B} = \sqrt{A/B}$

$\sqrt{A}/B = \sqrt{A}/B$

Review :) :

$$x^2 = \sqrt{x^4}$$

$$x^3 = \sqrt{x^6}$$

$$x^4 = \sqrt{x^8}$$

$$x^5 = \sqrt{x^{10}}$$

$$\lim_{x \to -\infty} \frac{\sqrt{x^4 - 1}}{x^3 - 1}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{x^4 - 1} \,/\, x^3}{x^3/x^3 - 1/x^3} = \lim_{x \to -\infty} \frac{\sqrt{x^4 - 1} \,/\, \sqrt{x^6}}{1 - 1/x^3}$$

$$= \lim_{x \to -\infty} \left(\frac{\sqrt{x^4/x^6 - 1/x^6}}{1 - 1/x^3} \right)$$

$$= \lim_{x \to -\infty} \frac{\sqrt{1/x^2 - 1/x^6}}{1 - 1/x^3} = \frac{\sqrt{0 - 0}}{1 - 0} = \frac{0}{1} = 0$$