Calculus I Section 3.7 Notes

1) A rectangle has a perimeter of 100 feet.

Find the length and width of a rectangle with a maximum area.

Let A = area

Let x = length of rectangle

Let y = width of rectangle

Note: $A = x \cdot y$; $P = 2x + 2y = 100$

a) Solve for y: $2x + 2y = 100 \Rightarrow y =$ _50 - x____

b) Write A as function of x only: $A = x \cdot y =$ _(x)(50-x) = 50x - x^2____

c) Find $A' = \dfrac{dA}{dx} =$ _50 - 2x______

d) Set $A' = 0$ and solve for x. $x =$ ____?_____

e) Area is a maximum when x = ____?_____ and y = ____?_____

2) A rectangle has an area of 80 square feet.

Find the length and width of a rectangle with a minimum perimeter.

Let P = perimeter

Let x = length of rectangle Let y = width of rectangle

Note: $A = x \cdot y = 80$; $P = 2x + 2y$

a) Solve for y: $x \cdot y = 80 \Rightarrow y = \dfrac{80}{x}$

b) Write perimeter P as a function of x only:

$$P = 2x + 2y = 2x + 2\left(\frac{80}{x}\right) = 2x + \frac{160}{x} = 20x + 160x^{-1}$$

c) Find $P' = \dfrac{dP}{dx} =$ ____?________

d) Set $P' = 0$ and solve for x. $x =$ ____?________

e) Perimeter is a minimum when x = ____?________ and y = ____?________

3) A container in the shape of a right circular cylinder with top and bottom has surface area 100 inch^2.

What height (h) and base radius (r) will maximize the volume of the cylinder ?

$S =$ Surface Area = Area of Top + Area of Bottom + Lateral Area

$S = \pi r^2 + \pi r^2 + 2\pi rh$

Note: $S = \pi r^2 + \pi r^2 + 2\pi rh = 100$

a) Solve $\pi r^2 + \pi r^2 + 2\pi rh = 100$ for h:

$$\pi r^2 + \pi r^2 + 2\pi rh = 100$$

$$2\pi r^2 + 2\pi rh = 100$$

$$2\pi rh = 100 - 2\pi r^2$$

$$h = \frac{100 - 2\pi r^2}{2\pi r}$$

b) Write volume $= \pi r^2 h$ as a function of r only:

$$V = \pi r^2 h = \pi r^2 \left(\frac{100 - 2\pi r^2}{2\pi r} \right) = r\left(\frac{100 - 2\pi r^2}{2} \right)$$

$$= r\left(50 - \pi r^2\right) = 50r - \pi r^3$$

c) Find $S' = \dfrac{dS}{dr} =$ ____?____

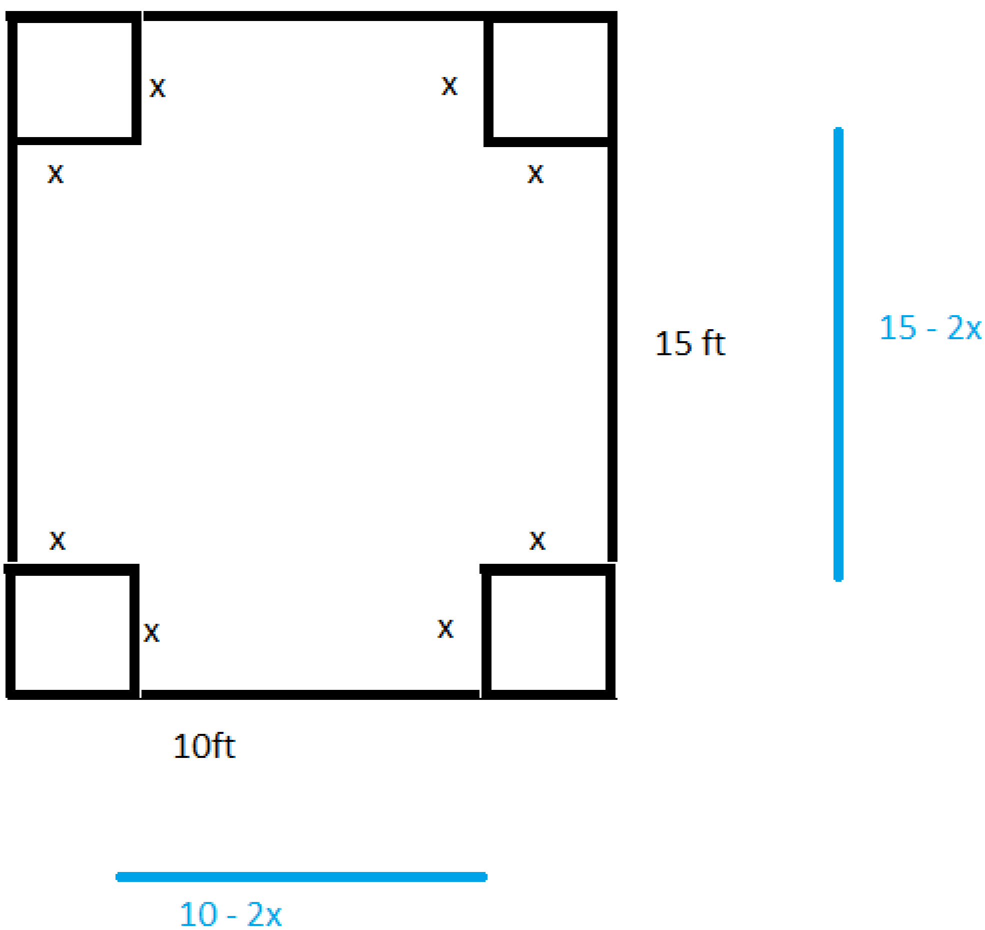
d) Set $S' = 0$ and solve for r. $r =$ ____?____

e) Cylinder is a maximum when $r =$ ____?____ and $h =$ ____?____

4) A sheet of cardboard 10 ft. by 15 ft. will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. Each side of the cut-out square has side length of x. Find x so that the box has the largest volume.

a) $x =$ _____?______

b) $V =$ Volume of box = _____?______



$$V = \text{Volume} = (length)(width)(height) = (10-2x)(15-2x)(x)$$

$$V = 150x - 50x^2 + 4x^3$$

$$\frac{dV}{dx} = 150 - 100x + 12x^2$$