Calculus I Section 3.9 Notes

Let Δx be some small number (like 0.001).

Let $\Delta y = f(x+\Delta x) - f(x)$

Let $dx =$ Differential of x

Let $dy =$ Differential of y

<u>Definition of Differentials</u>:

Let $y = f(x)$ be a function that is differentiable on an open interval $(a,\ b)$ such that $x \in (a,\ b)$. The differentials dx and dy are defined as follows: $dy = f'(x)\,dx.$

Example 1: Comparing Δy and dy

Let $f(x) = x^2$.

Suppose x changes from 1 to 1.01. What is the change in y? Or What is Δy?

Note: When x changes from $x = 1$ to $x = 1.01$, $\Delta x = 1.01 - 1 = 0.01$.

Hence, $f(1) = (1)^2 = 1$ and $f(1.01) = (1.01)^2 = 1.0201$

$\Delta y = f(1.01) - f(1) = 1.0201 - 1 = 0.0201$

$\Delta y = f(x + \Delta x) - f(x) = f(1.01) - f(1) = 1.0201 - 1 = 0.0201$

<u>Δy and dy:</u>

When $x = 1$ and $\Delta x = 0.01$, $\quad f(x + \Delta x) = f(1 + 0.01) = f(1.01) = (1.01)^2 = 1.0201$

$\Delta y = f(x + \Delta x) - f(x) = f(1.01) - f(1) = 1.0201 - 1 = 0.0201$

Also, $f'(x) = 2x$ and $f'(1) = 2(1) = 2$.

Now let $dx = \Delta x = 0.01$, then $dy = f'(x)dx = f'(1) \cdot \Delta x = (2)(0.01) = 0.02$

Comparing Δy and dy: $|0.0201 - 0.02| = 0.0001$

Note: Difference between Δy and dy is small; so $\Delta y \approx dy$ or $\Delta y \approx f'(x)dx$.

Example 2: Comparing Δy and dy

Let $f(x) = x^3 + 1.$

Suppose x changes from $x = 1$ to $x = 1.01$. What is the change in y? Or What is Δy?

Note: When x changes from $x = 1$ to $x = 1.01$, $\Delta x = 1.01 - 1 = 0.01.$

Hence, $f(1) = (1)^3 + 1 = 2$ and $f(1.01) = (1.01)^3 + 1 = 2.030301$

$\Delta y = f(1.01) - f(1) = 2.030301 - 2 = 0.030301$

$\Delta y = f(x + \Delta x) - f(x) = f(1.01) - f(1) = 2.030301 - 2 = 0.030301$

<u>Δy and dy:</u>

When $x = 1$ and $\Delta x = 0.01$, $f(x + \Delta x) = f(1 + 0.01) = f(1.01) = (1.01)^3 + 1 = 2.030301$

Hence, $\Delta y = f(x + \Delta x) - f(x) = f(1.01) - f(1) = 2.030301 - 2 = 0.030301$

Also, $f'(x) = 3x^2$ and $f'(1) = 3$

Now let $dx = \Delta x = 0.01$, then $dy = f'(x)dx = f'(1) \cdot \Delta x = (3)(0.01) = 0.03$

Comparing Δy and dy: $|0.030301 - 0.03| = 0.0003$

Note: Difference between Δy and dy is small; so $\Delta y \approx dy$ or $\Delta y \approx f'(x)dx$.

Example 3: Express dy in terms of $f'(x)$ and dx.

Let $f(x) = \sin x - 0.5.$

Then:

$f'(x) = \cos x$

$dy = f'(x)dx$

$dy = \cos x \cdot dx$

Example 4: Express *dy* in terms of $f'(x)$ and *dx*.

Let $f(x) = x^3 + 4$

Then:

$$\frac{dy}{dx} = f'(x) = 3x^2$$

$$dy = f'(x)dx$$

$$dy = 3x^2 \cdot dx$$

Example 5: Express *dy* in terms of $f'(x)$ and *dx*.

Let $f(x) = x^3 + x - 4$

Then:

a) $\frac{dy}{dx} = f'(x) = 3x + 1$

b) $dy = f'(x)dx = (3x+1)dx$

Example 6: Express *dy* in terms of $f'(x)$ and *dx*.

Let $f(x) = \sqrt{x-1} - x + 2 = (x-1)^{1/2} - x + 2$

Then:

a) $\frac{dy}{dx} = f'(x) = \frac{1}{2}(x-1)^{-1/2} - 1$

b) $dy = f'(x)dx = \left[\frac{1}{2}(x-1)^{-1/2} - 1\right]dx$