Calculus I Section 4.1 Notes

Let $f(x) = 5$

$F(x) =$ Antiderivative of $f(x) = 5x$ Note: $F'(x) = 5$

$F(x) =$ Antiderivative of $f(x) = 5x + 1$ Note: $F'(x) = 5$

$F(x) =$ Antiderivative of $f(x) = 5x + 2$ Note: $F'(x) = 5$

$F(x) =$ Antiderivative of $f(x) = 5x + 3$ Note: $F'(x) = 5$

In General: $F(x) =$ Antiderivative of $f(x) = 5x + C$

Let $f(x) = x$

$F(x) = \text{Antiderivative of } f(x) = \dfrac{x^2}{2}$ Note: $F'(x) = \dfrac{1}{2}(2x) = x$

$F(x) = \text{Antiderivative of } f(x) = \dfrac{x^2}{2} + 1$ Note: $F'(x) = \dfrac{1}{2}(2x) = x$

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$F(x) = \text{Antiderivative of } f(x) = \dfrac{x^2}{2} + 3$ Note: $F'(x) = \dfrac{1}{2}(2x) = x$

In General: $F(x) = \text{Antiderivative of } f(x) = \dfrac{x^2}{2} + C$

Let $f(x) = x^2$

$F(x) = \text{Antiderivative of } f(x) = \dfrac{x^3}{3}$ Note: $F'(x) = \dfrac{1}{3}\left(3x^2\right) = x^2$

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In General: $F(x) = \text{Antiderivative of } f(x) = \dfrac{x^3}{3} + C$

Let $f(x) = x^3$

$F(x) = \text{Antiderivative of } f(x) = \frac{x^4}{4}$ Note: $F'(x) = \frac{1}{4}\left(4x^3\right) = x^3$

$F(x) = \text{Antiderivative of } f(x) = \frac{x^4}{4} + 1$ Note: $F'(x) = \frac{1}{4}\left(4x^3\right) = x^3$

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In General: $F(x) = \text{Antiderivative of } f(x) = \frac{x^4}{4} + C$

Let $f(x) = x^n$

$F(x) = \text{Antiderivative of } f(x) = \dfrac{x^{n+1}}{n+1}$ Note: $F'(x) = \dfrac{1}{n+1}\left((n+1)x^n\right) = x^n$

$F(x) = \text{Antiderivative of } f(x) = \dfrac{x^{n+1}}{n+1} + 1$ Note: $F'(x) = \dfrac{1}{n+1}\left((n+1)x^n\right) = x^n$

$F(x) = \text{Antiderivative of } f(x) = \dfrac{x^{n+1}}{n+1} + 2$ Note: $F'(x) = \dfrac{1}{n+1}\left((n+1)x^n\right) = x^n$

$F(x) = \text{Antiderivative of } f(x) = \dfrac{x^{n+1}}{n+1} + 3$ Note: $F'(x) = \dfrac{1}{n+1}\left((n+1)x^n\right) = x^n$

In General: $F(x) = \text{Antiderivative of } f(x) = \dfrac{x^{n+1}}{n+1} + C = \dfrac{1}{n+1}x^{n+1} + C$

Let $f(x) = x + 2$

Find the antiderivative of $f(x)$.

$$F(x) = \int (x+2)\,dx = \frac{1}{2}x^2 + 2x + C$$

Also, $F'(x) = \frac{1}{2}(2x) + 2 = x + 2 = f(x)$

Let $f(x) = x^4 + 4$

Find the antiderivative of $f(x)$.

$$F(x) = \int \left(x^4 + 4\right) dx = \frac{x^5}{5} + 4x + C = \frac{1}{5}x^5 + 4x + C$$

Also, $F'(x) = \frac{1}{5}\left(5x^4\right) + 4 = x^4 + 4 = f(x)$

Let $f(x) = x^{3/4} + x + 9$

Find the antiderivative of $f(x)$.

$$F(x) = \int \left(x^{3/4} + x + 9\right) dx = \frac{x^{3/4+1}}{7/4} + \frac{x^2}{2} + 9x + C$$

$$= \frac{4}{7} x^{7/4} + \frac{1}{2} x^2 + 9x + C$$

Also, $F'(x) = \frac{4}{7}\left(\frac{7}{4} x^{3/4}\right) + \frac{1}{2}\left(2x\right) + 9 = x^{3/4} + x + 9 = f(x)$

Let $f(x) = \sqrt[3]{x^5}$ Hint: $\sqrt[3]{x^5} = x^{5/3}$

Find the antiderivative of $f(x)$.

$$F(x) = \int \left(\sqrt[3]{x^5}\right) dx = \int x^{5/3} dx = \frac{x^{5/3+1}}{8/3} + C$$

$$= \frac{3}{8} x^{8/3} + C$$

Also, $F'(x) = \frac{3}{8}\left(\frac{8}{3} x^{5/3}\right) = x^{5/3} = f(x)$

Let $f(x) = \dfrac{1}{x^3}$

Find $F(x) = \int \dfrac{1}{x^3}\,dx = ?$ Hint: $\dfrac{1}{x^3} = x^{-3}$

$$F(x) = \int \left(\frac{1}{x^3} \right) dx = \int x^{-3} dx = \frac{x^{-3+1}}{-2} + C = -\frac{1}{2} x^{-2} + C$$

Also, $F'(x) = -\dfrac{1}{2}\left(-2x^{-3}\right) = x^{-3} = f(x)$

Let $f(x) = \dfrac{x+4}{\sqrt[3]{x}}$

Find $F(x) = \displaystyle\int \frac{x+4}{\sqrt[3]{x}}\,dx.$

$$Note: \frac{x+4}{\sqrt[3]{x}} = \frac{x+4}{x^{1/3}} = \frac{x}{x^{1/3}} + \frac{4}{x^{1/3}} = x^{2/3} + 4x^{-1/3}$$

$$F(x) = \int \frac{x+4}{\sqrt[3]{x}}\,dx = \frac{x^{2/3+1}}{5/3} + 4\left[\frac{x^{-1/3+1}}{2/3}\right] + \mathrm{C}$$

$$= \frac{3}{5}x^{5/3} + 4\cdot\frac{3}{2}\cdot x^{2/3} + C$$

$$= \frac{1}{3}x^{5/3} + 6x^{2/3} + C$$

Let $f(x) = (2x+1)(4x-5)$

$$Note: f(x) = (2x+1)(4x-5) = 8x^2 - 6x - 5$$

$$F(x) = \int (2x+1)(4x-5)\,dx = \int \left(8x^2 - 6x - 5\right) dx$$

$$= 8\left(\frac{x^3}{3}\right) - 6\left(\frac{x^2}{2}\right) - 5x + C$$

$$= \frac{8}{3}x^3 - 3x^2 - 5x + C$$

Antiderivative for Trigonometric Functions

$$\int (\cos x)dx = \sin x + C$$

$$\int (\sin x)dx = -\cos x + C$$

$$\int (\tan x)dx = -\ln\left|\cos x\right| + C$$

$$\int (\sec x)dx = \ln\left|\sec x + \tan x\right| + C$$

$$\int (\csc x)dx = -\ln\left|\csc x + \cot x\right| + C$$

$$\int (\cot x)dx = \ln\left|\sin x\right| + C$$

Let $f(x) = 5\sin x + 3\cos x$

$$F(x) = \int (5\sin x + 3\cos x)dx = 5\left(-\cos x\right) + 3\left(\sin x\right) + C$$

$$= -5\cos x + 3\sin x + C$$

Let $f(x) = \sec x - \cos x$

$$F(x) = \int (\sec x - \cos x)dx = \ln\left|\sec x + \tan x\right| - \sin x + C$$