Calculus I Section 4.2 Notes

Summation Notation Review:

$$\sum_{i=1}^{3}(4i+2) = 6 + 10 + 14 = 30$$

$$\sum_{k=0}^{6}\frac{3}{k^2+4} = \frac{3}{0^2+4}+\frac{3}{1^2+4}+\frac{3}{2^2+4}+\frac{3}{3^2+4}+\frac{3}{4^2+4}$$

$$+\frac{3}{5^2+4}+\frac{3}{6^2+4} = 2.2842175$$

$$\sum_{i=1}^{6} b = b+b+b+b+b+b = 6b$$

$$\sum_{i=1}^{4} 8 = 8 + 8 + 8 + 8 = 32$$

$$\sum_{i=1}^{5} 3i = 3 + 6 + 9 + 12 + 15 = 45$$

$$\sum_{i=1}^{5} (i - 4)^2 = 9 + 4 + 1 + 0 + 1 = 15$$

$$\sum_{i=1}^{10} \frac{i-7}{10^2} = -0.15$$

$$\sum_{i=1}^{100} \frac{i-7}{100^2} = 0.4350$$

$$\sum_{i=1}^{1000} \frac{i-7}{1000^2} = 0.4935$$

Example 1: Finding Area Under Curve

$f(x) = 2x + 5$

Let n = number of subintervals = 4

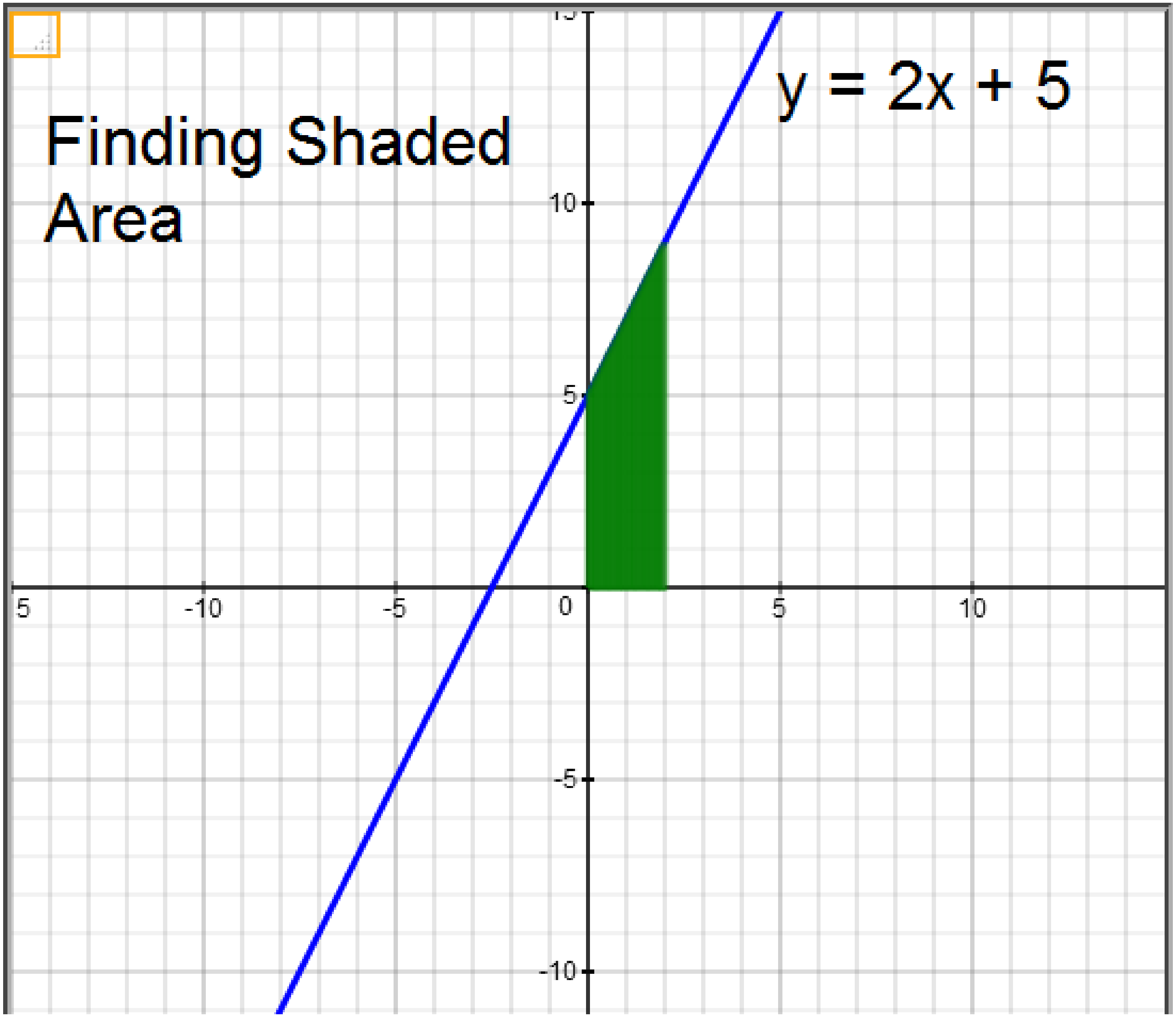
Let $a = 0$ and $b = 2$

a) $\Delta x = \dfrac{b - a}{n} = \dfrac{2}{4} = 0.5$

Find approximate area under graph with 4 rectangles:

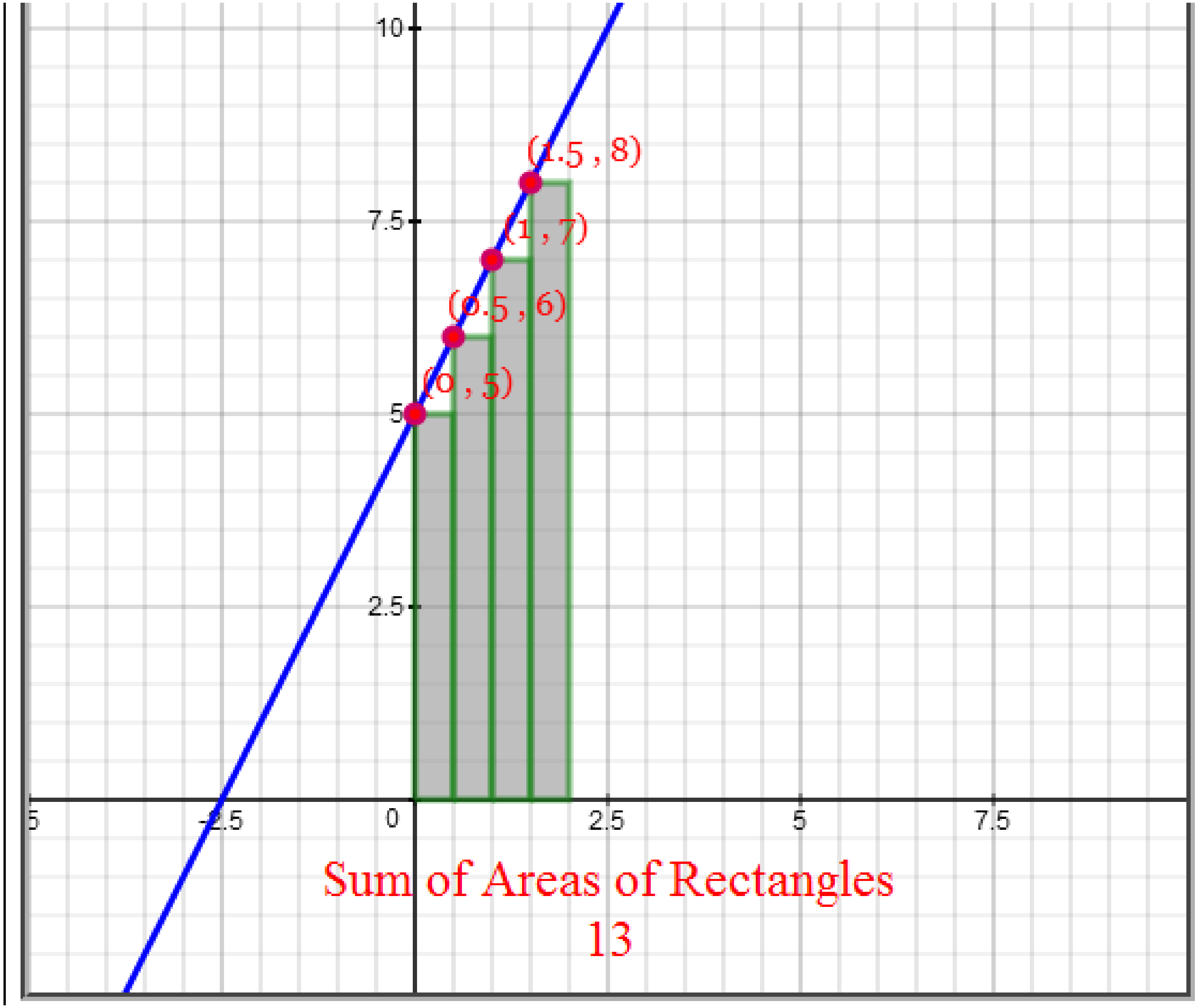
b) Using left endpoints: area ≈ 13

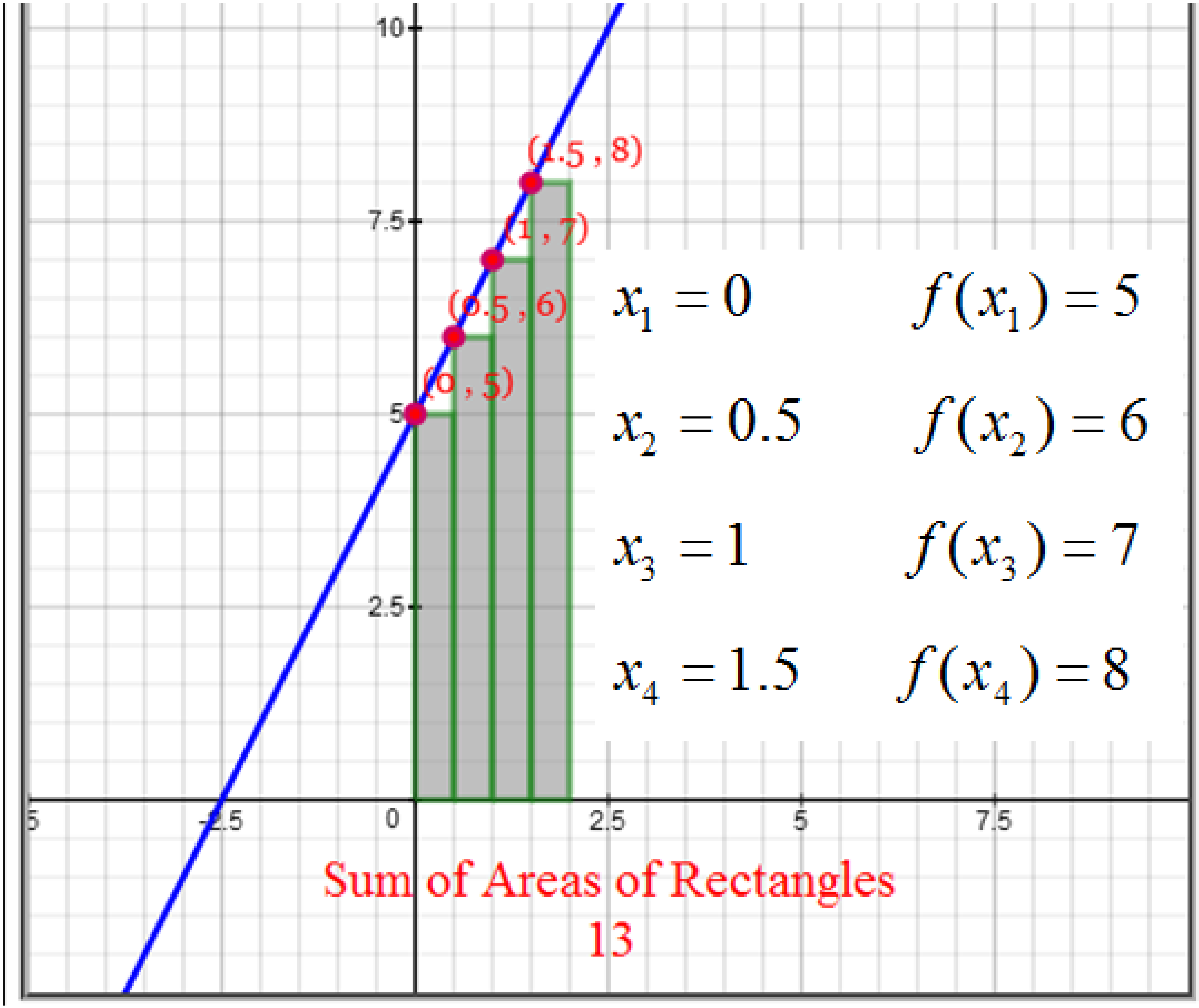
c) Using right endpoints: area ≈ 15



Finding Shaded Area
y = 2x + 5
10
5
0
-5
-10
-10
-5
5
10

Using Left-Endpoint Method:




(1.5 , 8)
(1 , 7)
(0.5 , 6)
(0 , 5)
$x_1 = 0$ $f(x_1) = 5$
$x_2 = 0.5$ $f(x_2) = 6$
$x_3 = 1$ $f(x_3) = 7$
$x_4 = 1.5$ $f(x_4) = 8$
Sum of Areas of Rectangles
13

Rectangle 1: width = 0.5 height = 5 Area $=(0.5)(5)=2.5$

$\Delta x_1 = 0.5$ $f(x_1) = 5$ Area $=(0.5)(5)=2.5$

Rectangle 2: width = 0.5 height = 6 Area $=(0.5)(6)=3$

$\Delta x_2 = 0.5$ $f(x_2) = 6$ Area $=(0.5)(6)=3$

Rectangle 3: width = 0.5 height = 7 Area $=(0.5)(7)=3.5$

$\Delta x_3 = 0.5$ $f(x_3) = 7$ Area $=(0.5)(7)=3.5$

Rectangle 4: width = 0.5 height = 8 Area $=(0.5)(8)=4$

$\Delta x_4 = 0.5$ $f(x_4) = 8$ Area $=(0.5)(8)=4$

Approximate Area Under Curve:

Shaded Area $\approx$ 2.5 + 3 + 3.5 + 4 = 13

Shaded Area $\approx f(x_1)\cdot\Delta x_1 + f(x_2)\cdot\Delta x_2 + f(x_3)\cdot\Delta x_3 + f(x_4)\cdot\Delta x_4$

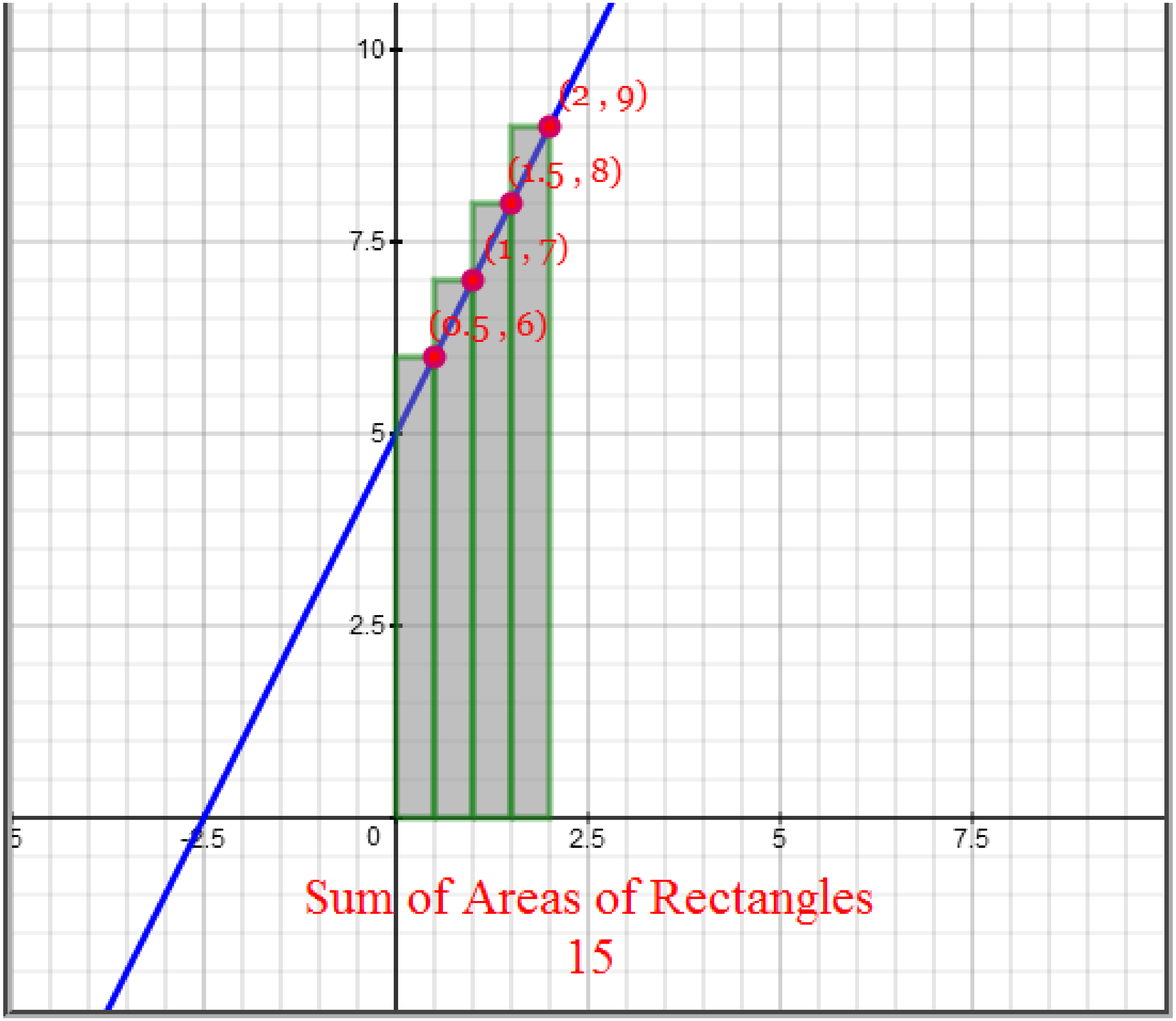
Shaded Area $\approx \sum_{i=1}^{4} f(x_i)\cdot\Delta x_i$

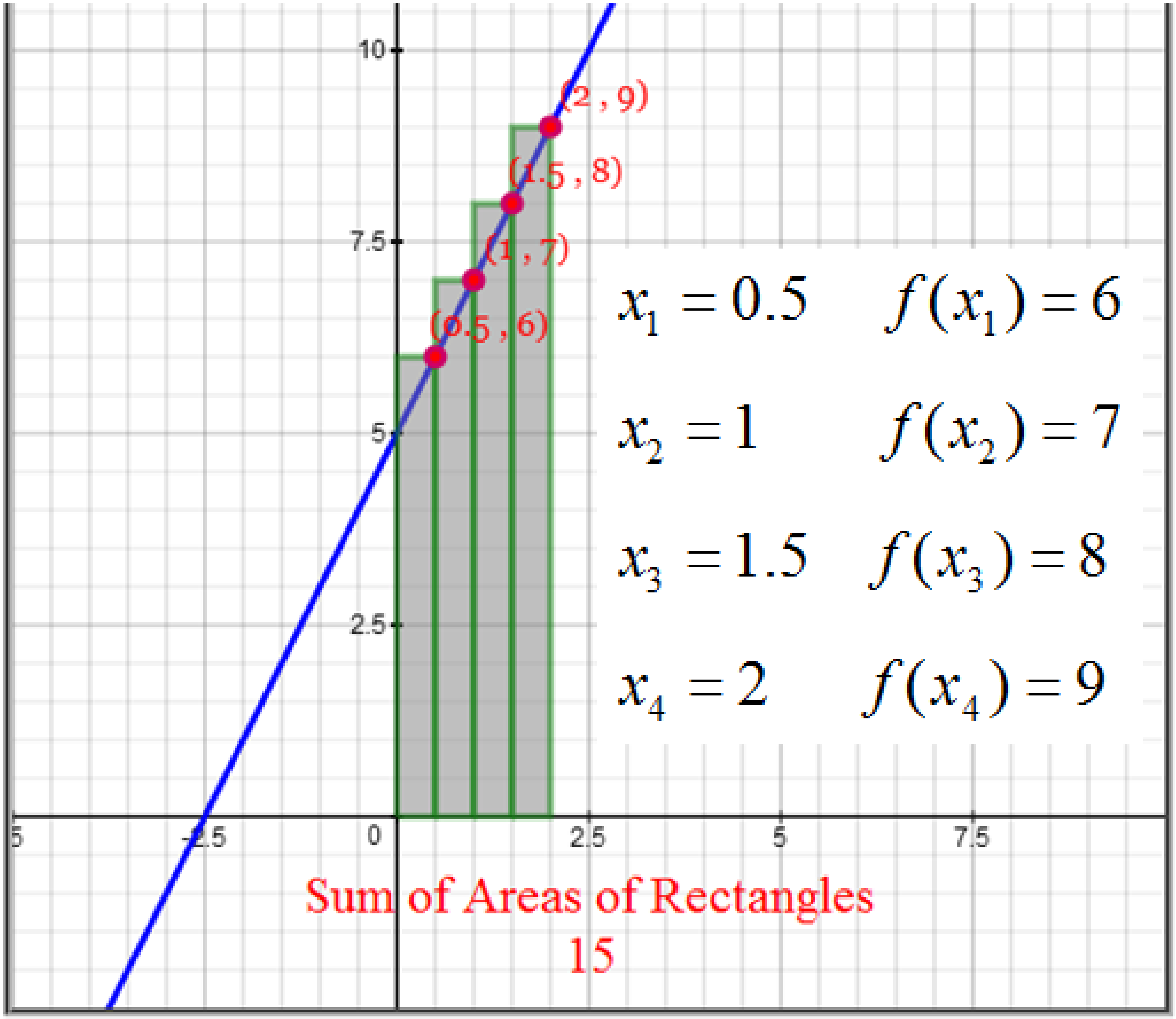
Exact Area Under Curve:

Shaded Area $= \sum_{i=1}^{\infty} f(x_i)\cdot\Delta x_i = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\cdot\Delta x_i$

$$= \int_0^2 f(x)dx = \int_0^2 (2x+5)dx$$

Using Right-Endpoint Method:




(2 , 9)
(1.5 , 8)
(1 , 7)
(0.5 , 6)
10
7.5
5
2.5
-2.5
0
2.5
5
7.5
$x_1 = 0.5$ $f(x_1) = 6$
$x_2 = 1$ $f(x_2) = 7$
$x_3 = 1.5$ $f(x_3) = 8$
$x_4 = 2$ $f(x_4) = 9$
Sum of Areas of Rectangles
15

Rectangle 1: width = 0.5   height = 6   Area $= (0.5)(6) = 3$

$\Delta x_1 = 0.5 \qquad f(x_1) = 6 \qquad \text{Area} = (0.5)(6) = 3$

Rectangle 2: width = 0.5   height = 7   Area $= (0.5)(7) = 3.5$

$\Delta x_2 = 0.5 \qquad f(x_2) = 7 \qquad \text{Area} = (0.5)(7) = 3.5$

Rectangle 3: width = 0.5   height = 8   Area $= (0.5)(8) = 4$

$\Delta x_3 = 0.5 \qquad f(x_3) = 8 \qquad \text{Area} = (0.5)(8) = 4$

Rectangle 4: width = 0.5   height = 9   Area $= (0.5)(5) = 4.5$

$\Delta x_4 = 0.5 \qquad f(x_4) = 9 \qquad \text{Area} = (0.5)(5) = 4.5$

Approximate Area Under Curve:

Shaded Area ≈ 3 + 3.5 + 4 + 4.5 = 15

Shaded Area $\approx f(x_1)\cdot\Delta x_1 + f(x_2)\cdot\Delta x_2 + f(x_3)\cdot\Delta x_3 + f(x_4)\cdot\Delta x_4$

Shaded Area $\approx \sum_{i=1}^{4} f(x_i)\cdot\Delta x_i$

Exact Area Under Curve:

$$\text{Shaded Area} = \sum_{i=1}^{\infty} f(x_i)\cdot\Delta x_i = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\cdot\Delta x_i$$

$$= \int_0^2 f(x)dx = \int_0^2 (2x+5)dx$$

Example 2: Finding Area Under Curve

$f(x) = -4x + 5$ (height of rectangle)

Let n = number of subintervals = 6

Let $a = 2$ and $b = 5$

a) $\Delta x = \dfrac{b - a}{n} = \dfrac{3}{6} = 0.5$ (width of rectangle)

Find approximate area under graph with 6 rectangles:

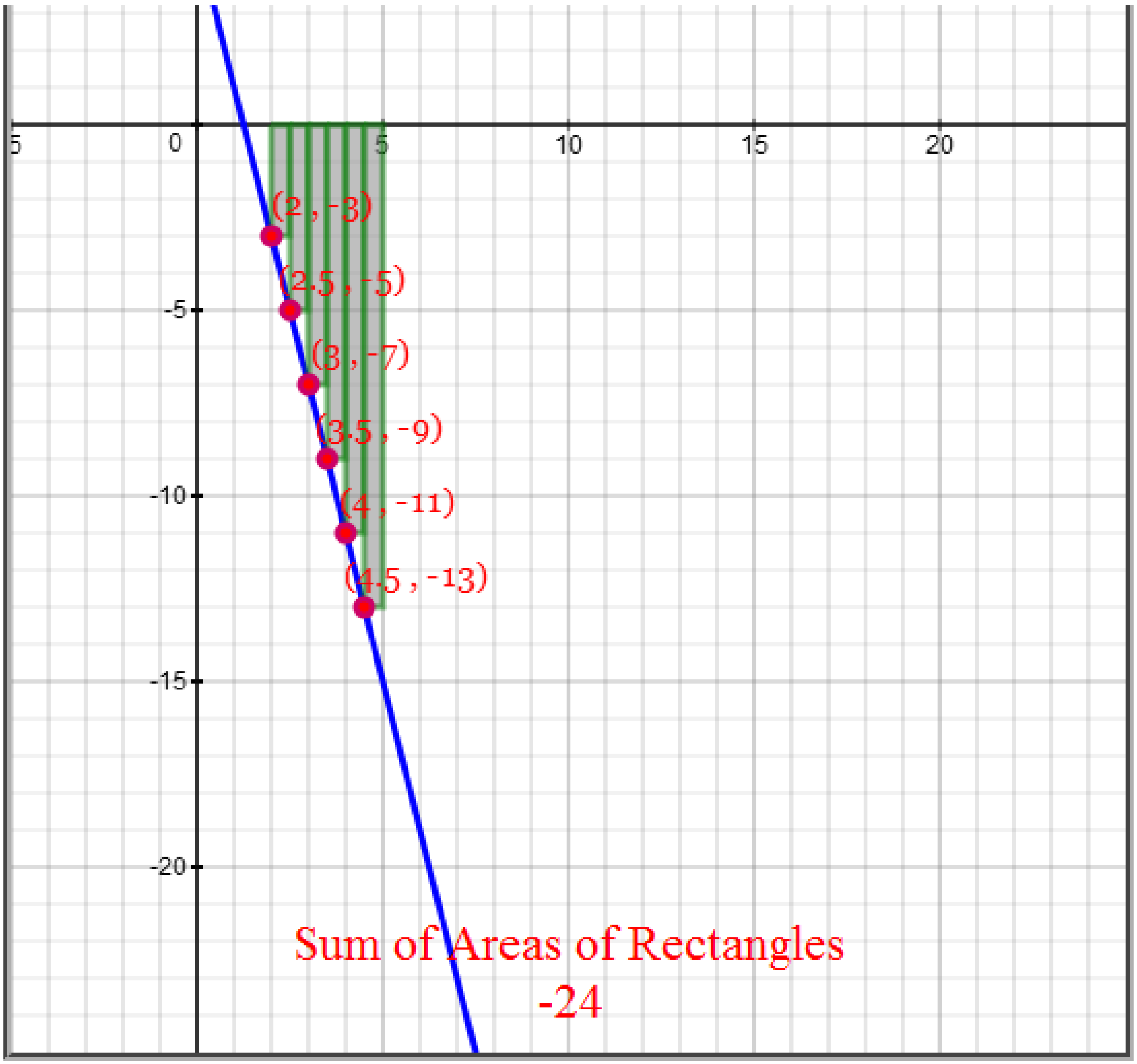
b) Using left endpoints: area ≈ -24

c) Using right endpoints: area ≈ -30

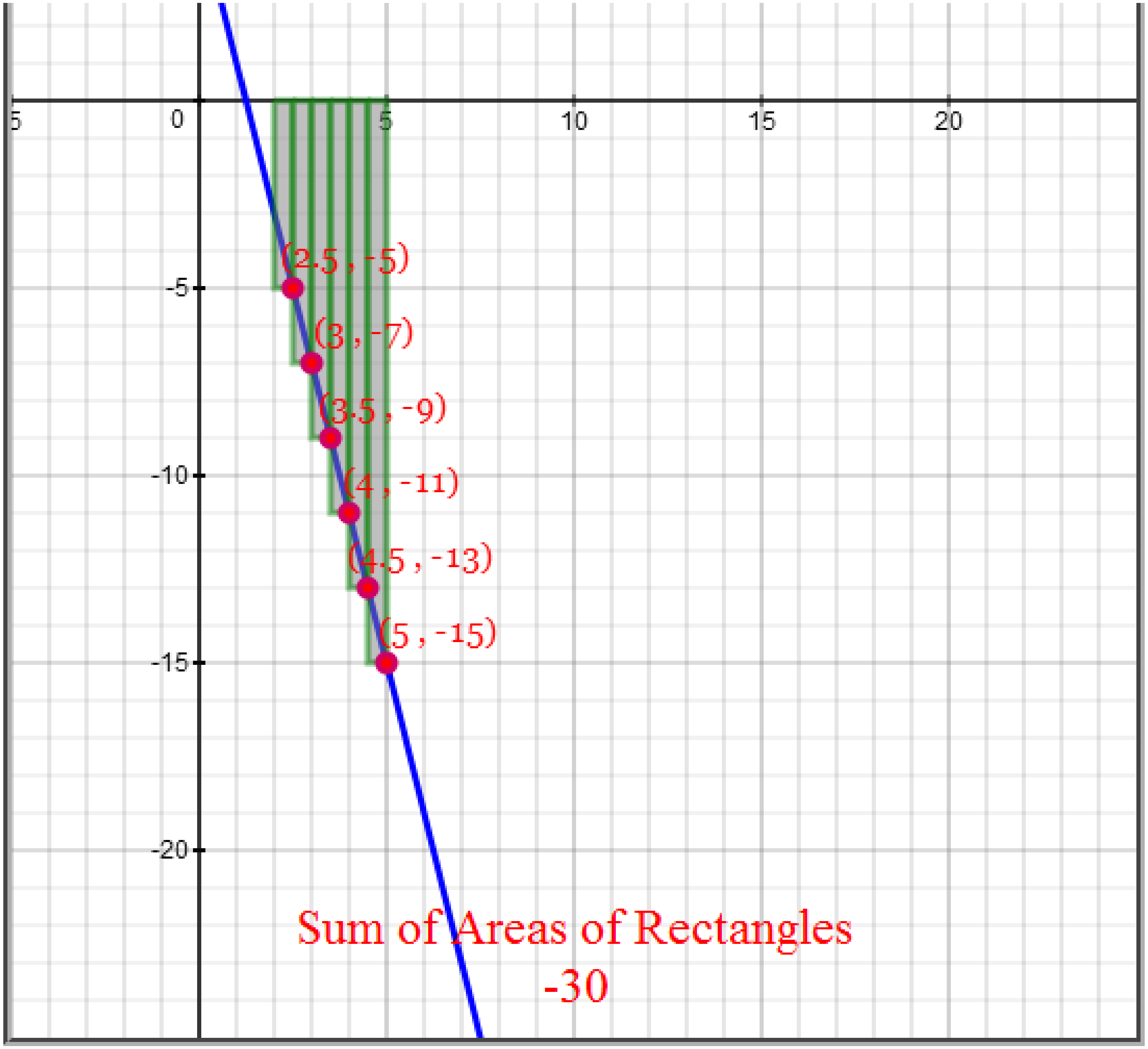
d) Exact Area = $\displaystyle\sum_{i=1}^{\infty} f(x_i)\Delta x_i = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$

$$= \int_2^5 f(x)\,dx = \int_2^5 (-4x + 5)\,dx$$

Using Left-End Point:



Using Right-End Point:



Example 3: Finding Area Under Curve

$f(x) = x^2$ (height of rectangle)

Let n = number of subintervals = 4

Let $a = 0$ and $b = 2$

a) $\Delta x = \dfrac{b - a}{n} = \dfrac{2}{4} = 0.5$ (width of rectangle)

Find approximate area under graph with 4 rectangles:

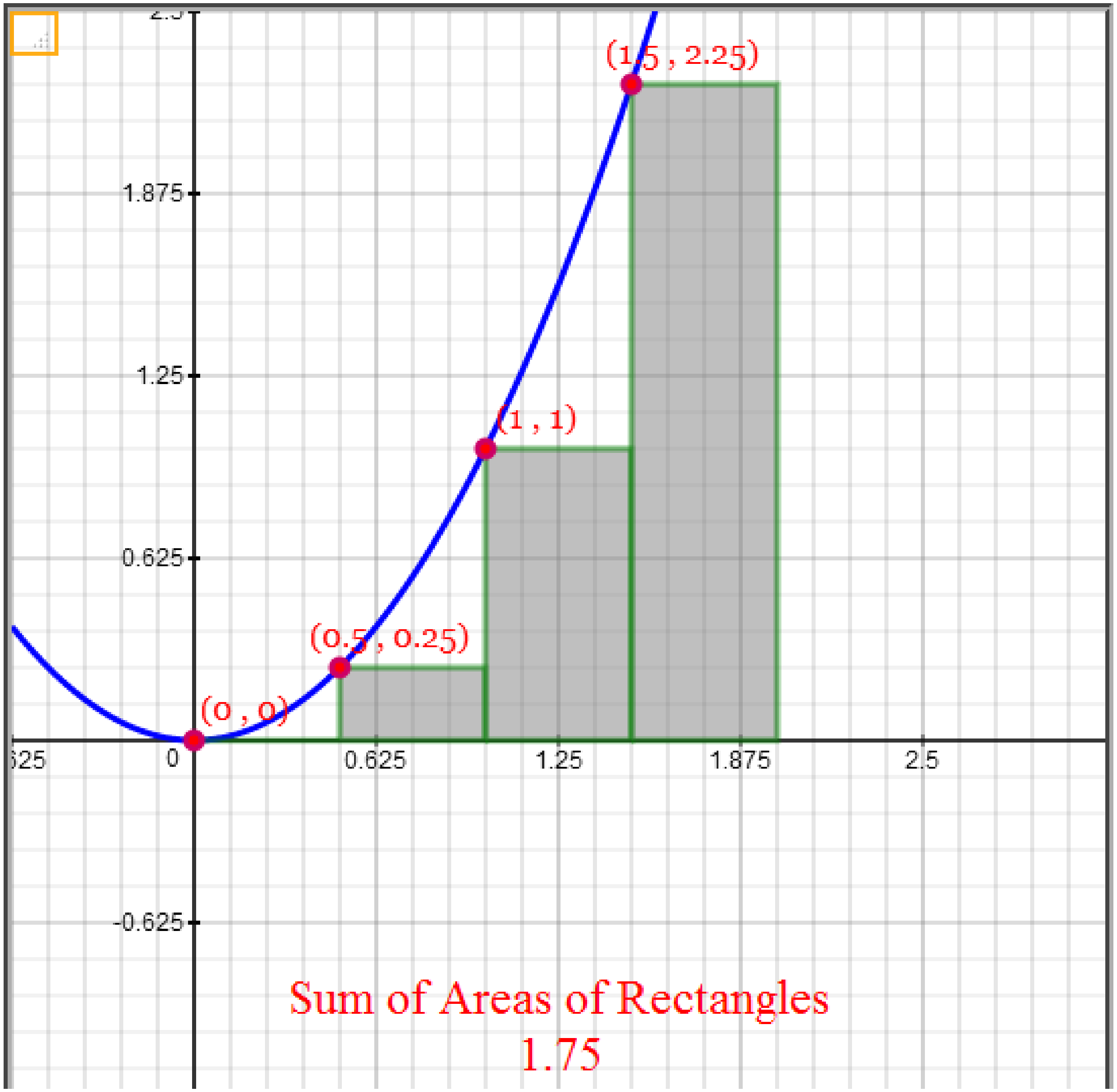
b) Using left endpoints: area ≈ 1.75

c) Using right endpoints: area ≈ 3.75

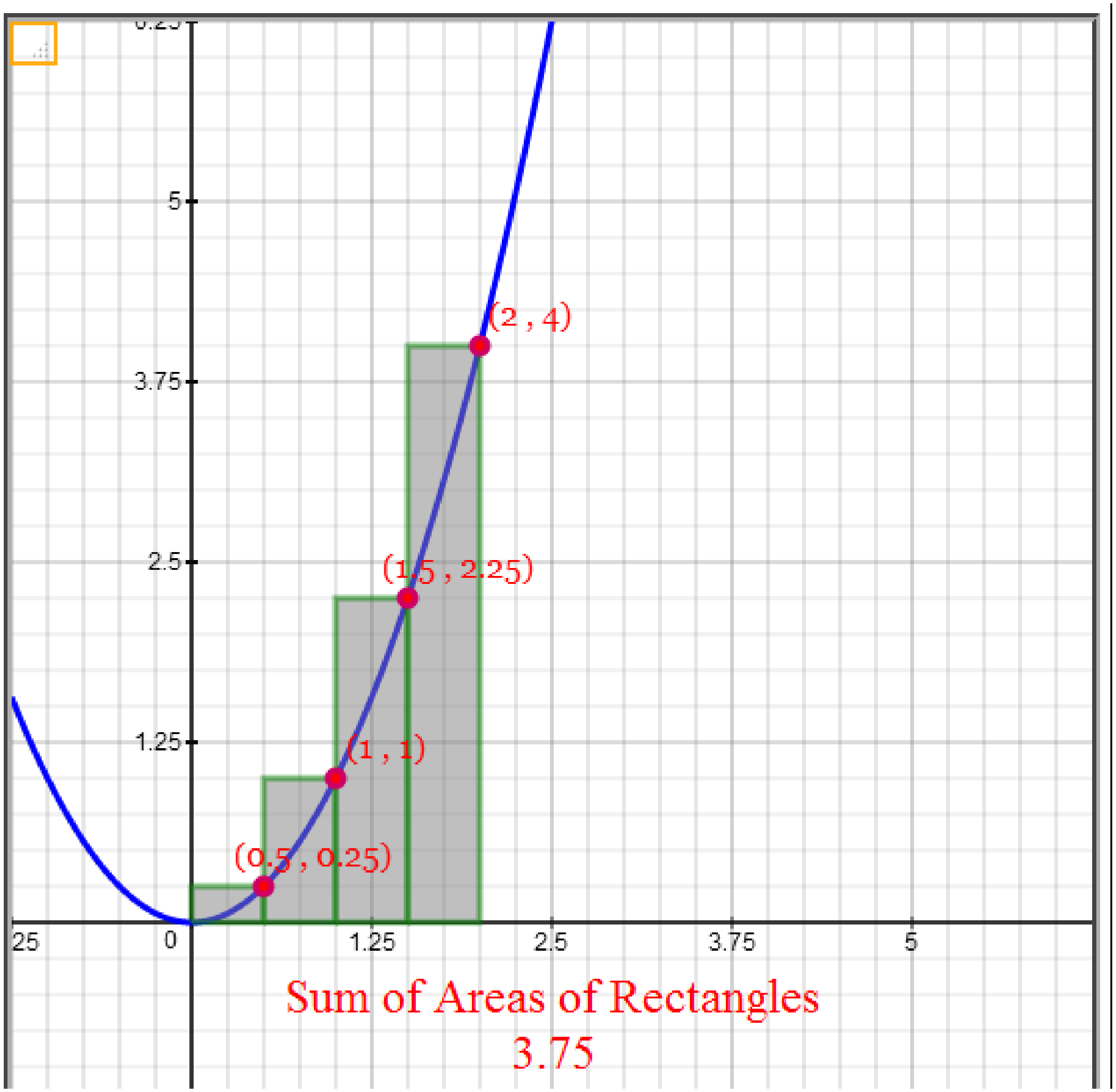
d) Exact Area = $\sum_{i=1}^{\infty} f(x_i)\Delta x_i = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$

$$= \int_0^2 f(x)dx = \int_0^2 \left(x^2\right)dx$$

Using Left-End Points



Using Right-End Points



Example 4: Finding Area Under Curve

$f(x) = \sin x$ (height of rectangle)

Let n = number of subintervals = 6

Let $a = 0$ and $b = \pi$

a) $\Delta x = \dfrac{b - a}{n} = \dfrac{\pi - 0}{6} = \dfrac{\pi}{6}$ (width of rectangle)

Find approximate area under graph with 4 rectangles:

b) Using left endpoints: area ≈ 1.954

c) Using right endpoints: area ≈ 1.954

$$f(x) = \sin x$$

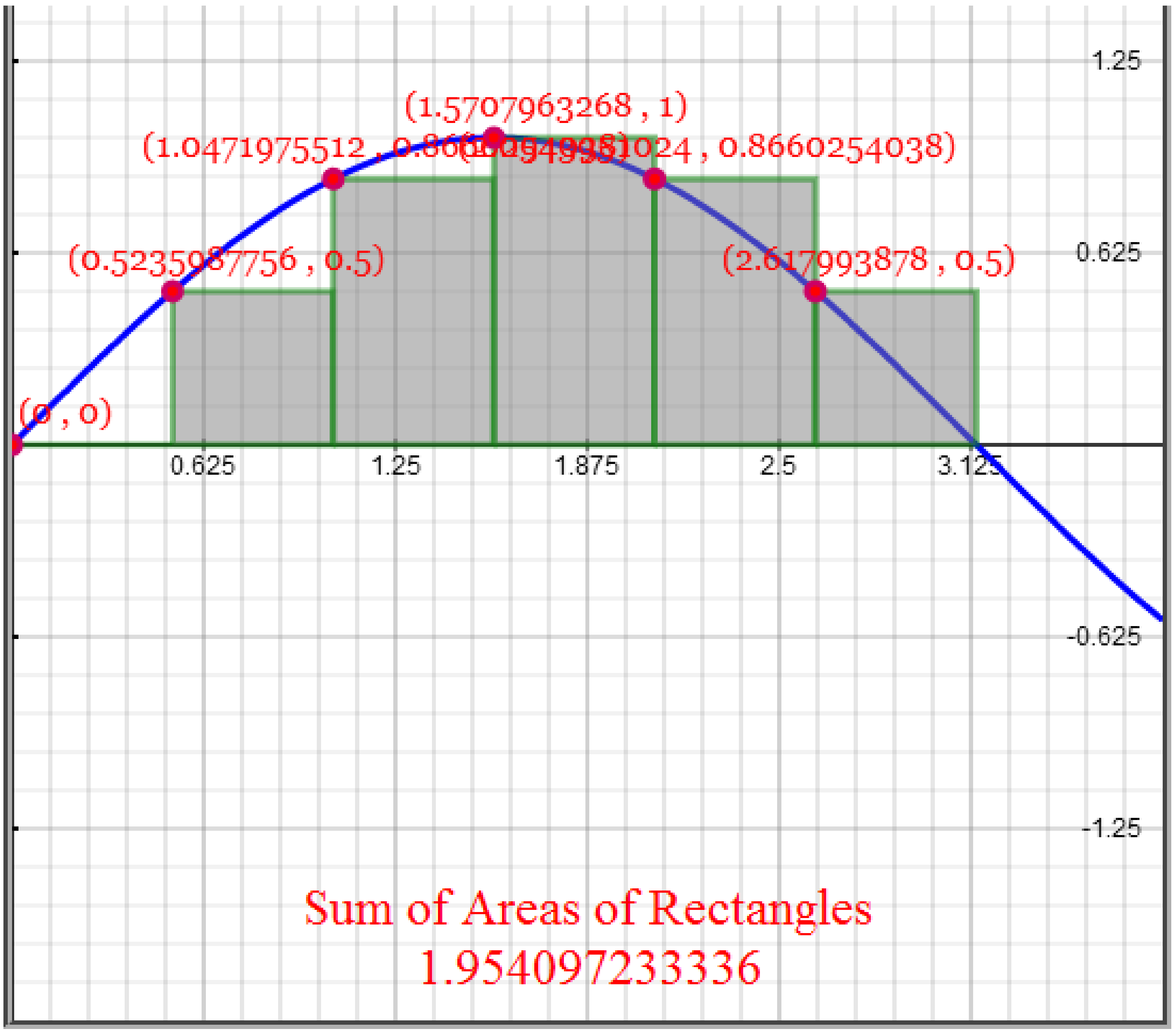
Let $a = 0$ and $b = \pi$

d) Exact Area = $\sum_{i=1}^{\infty} f(x_i)\Delta x_i = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$

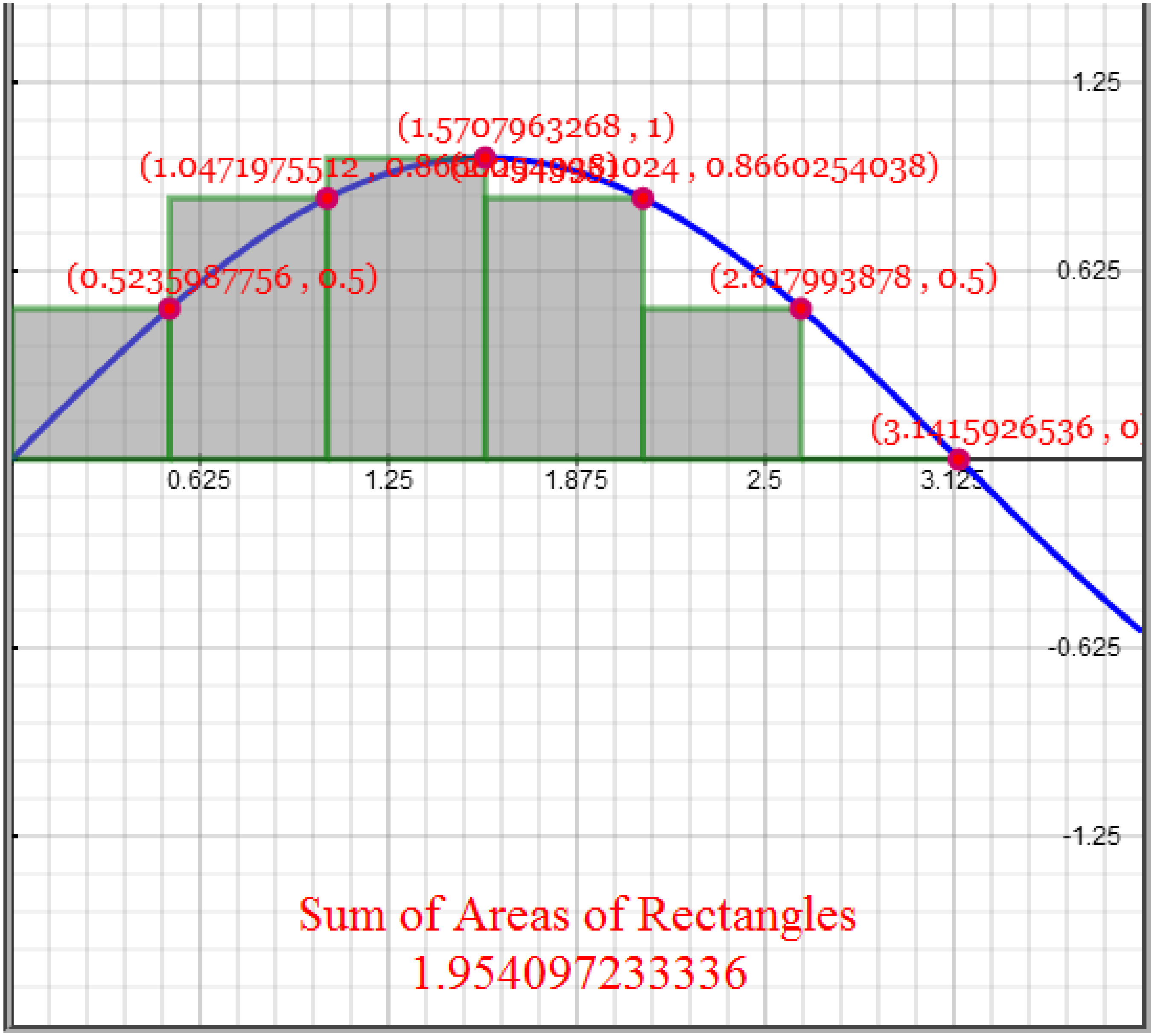
$$= \int_0^{\pi} f(x)dx = \int_0^{\pi} \sin x dx = \left[-\cos x\right]_0^{\pi}$$

$$= \left[-\cos \pi\right] - \left[-\cos 0\right] = -(-1) - (-1) = 2$$

Using Left-End Points



Using Right-End Points



Example 5: Finding Area Under Curve

$$f(x) = \frac{1}{x-4} \qquad \text{(height of rectangle)}$$

Let n = number of subintervals = 4

Let $a = 5$ and $b = 6$

a) $\Delta x = \dfrac{b - a}{n} = \dfrac{6 - 5}{4} = 0.25$ (width of rectangle)

Find approximate area under graph with 4 rectangles:

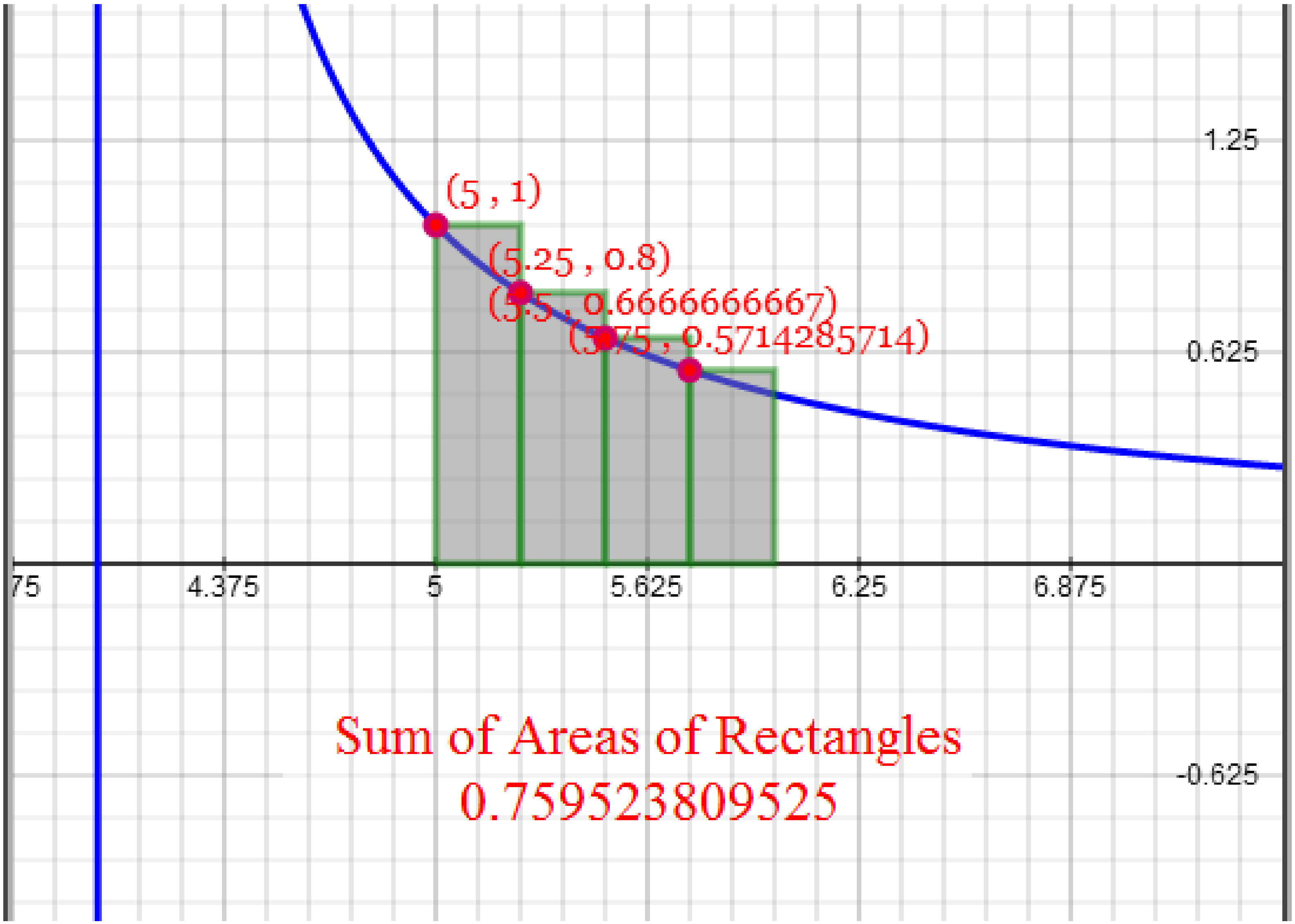
b) Using left endpoints: area ≈ 0.7595

c) Using right endpoints: area ≈ 0.6345

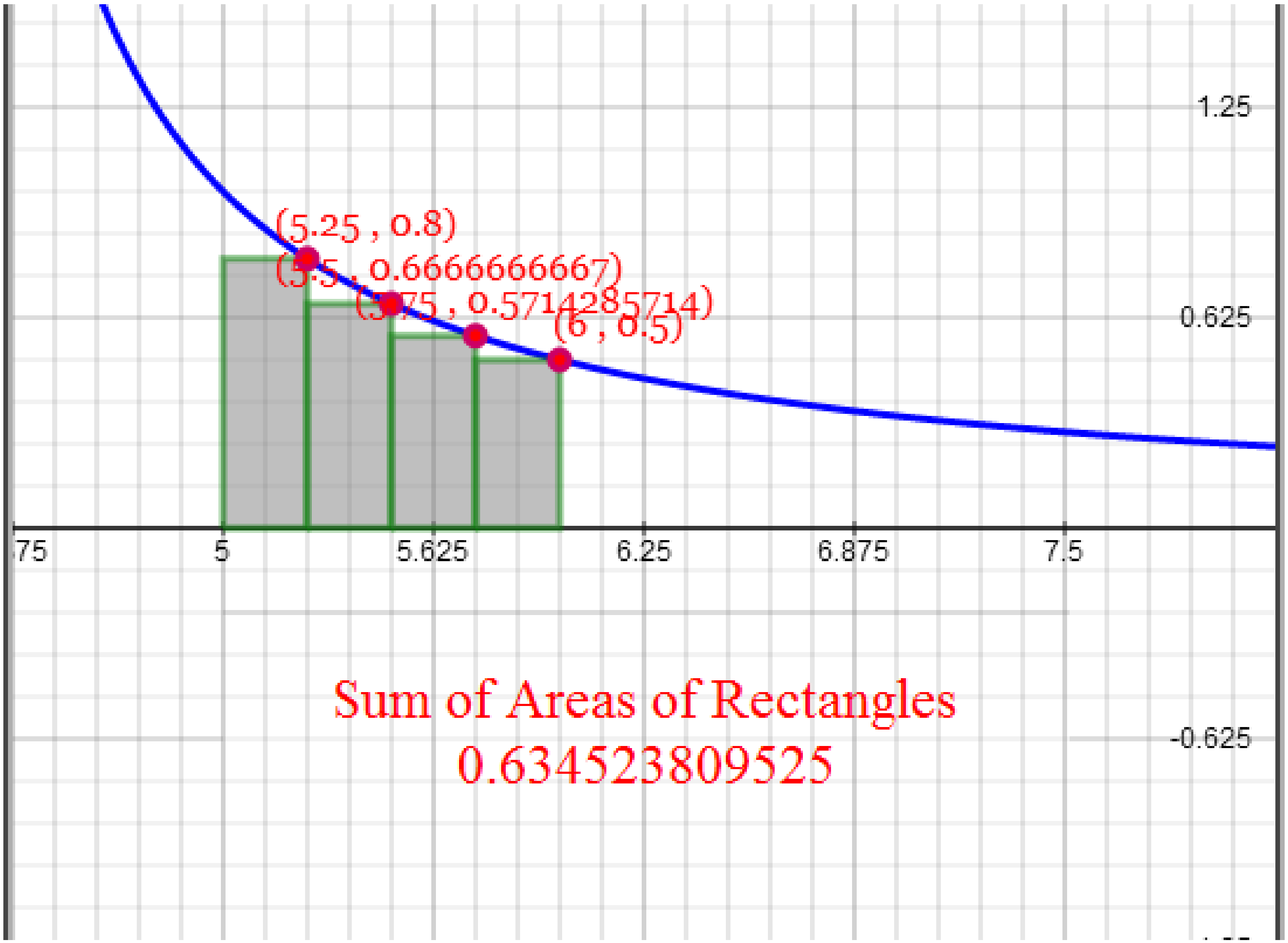
d) Exact Area = $\sum_{i=1}^{\infty} f(x_i)\Delta x_i = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$

$$= \int_5^6 f(x)dx = \int_5^6 \left(\frac{1}{x-4}\right)dx$$

Using Left-End Points



Using Right-End Points



Example 6: Finding Area Under Curve

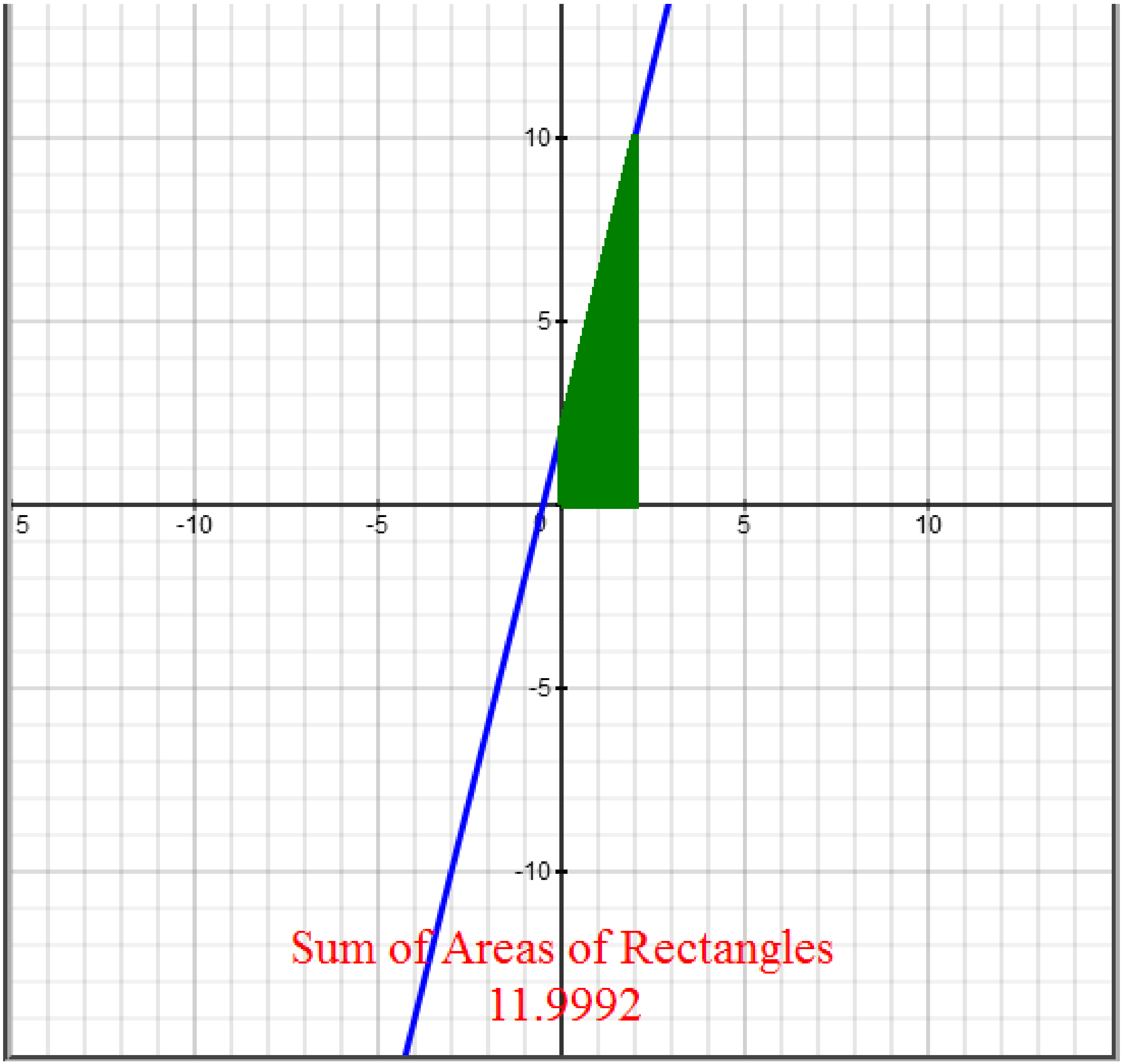
$f(x) = 4x + 2$

Let n = number of subintervals = 10000

Let $a = 0$ and $b = 2$

a) $\Delta x = \dfrac{b - a}{n} = \dfrac{2 - 0}{10000} = 0.0002$

b) Find area between graph and x-axis?


10
5
-5
-10
5
-10
-5
0
5
10
Sum of Areas of Rectangles
11.9992

Example 7: Finding Area Under Curve

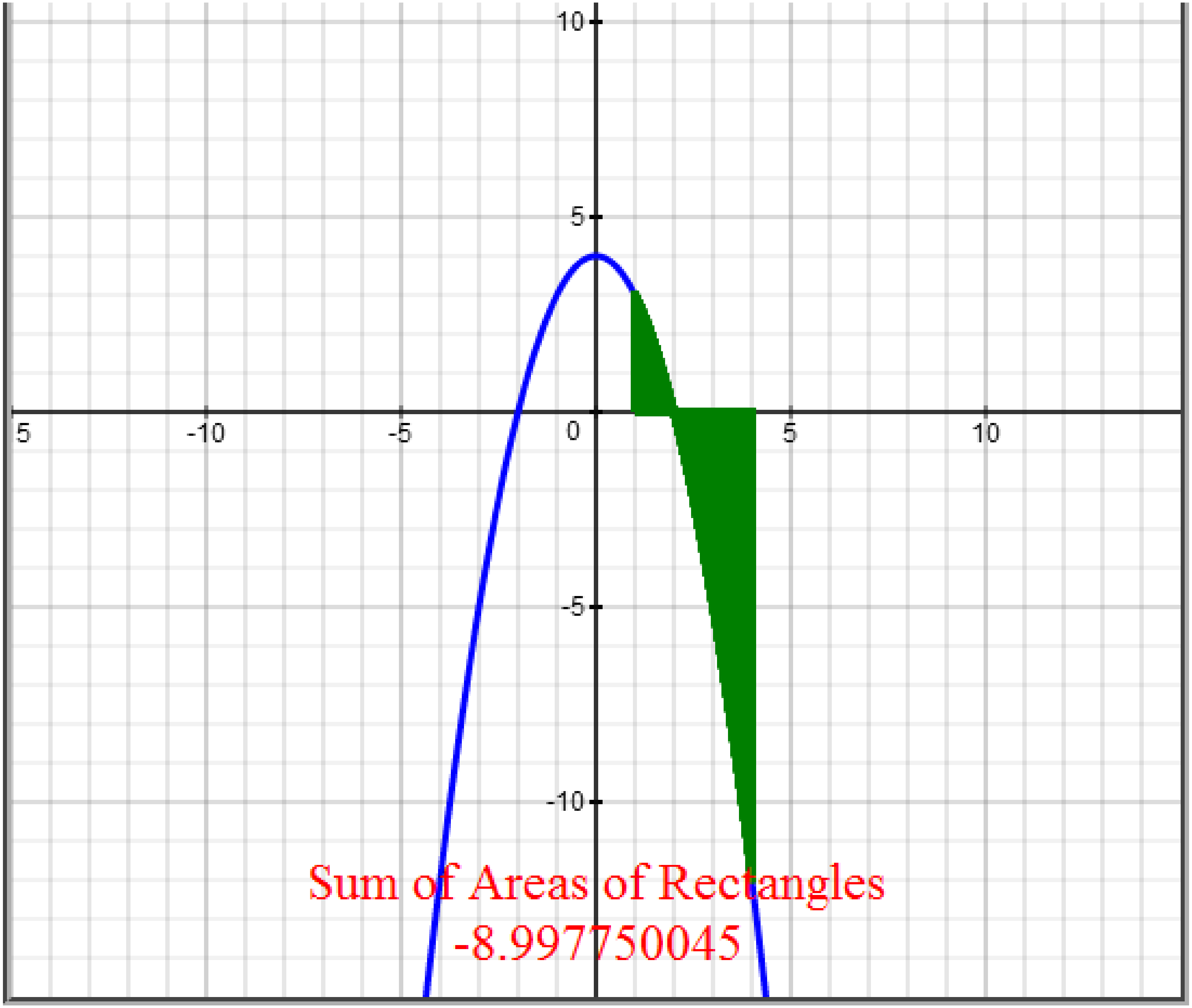
$$f(x) = 4 - x^2$$

Let n = number of subintervals = 10000

Let $a = 1$ and $b = 4$

a) $\Delta x = \dfrac{b - a}{n} = \dfrac{4-1}{10000} = 0.0003$

b) Find area between graph and x-axis?


10
5
-5
-10
5
-10
-5
0
5
10
Sum of Areas of Rectangles
-8.997750045

Example 8: Finding Area Under Curve

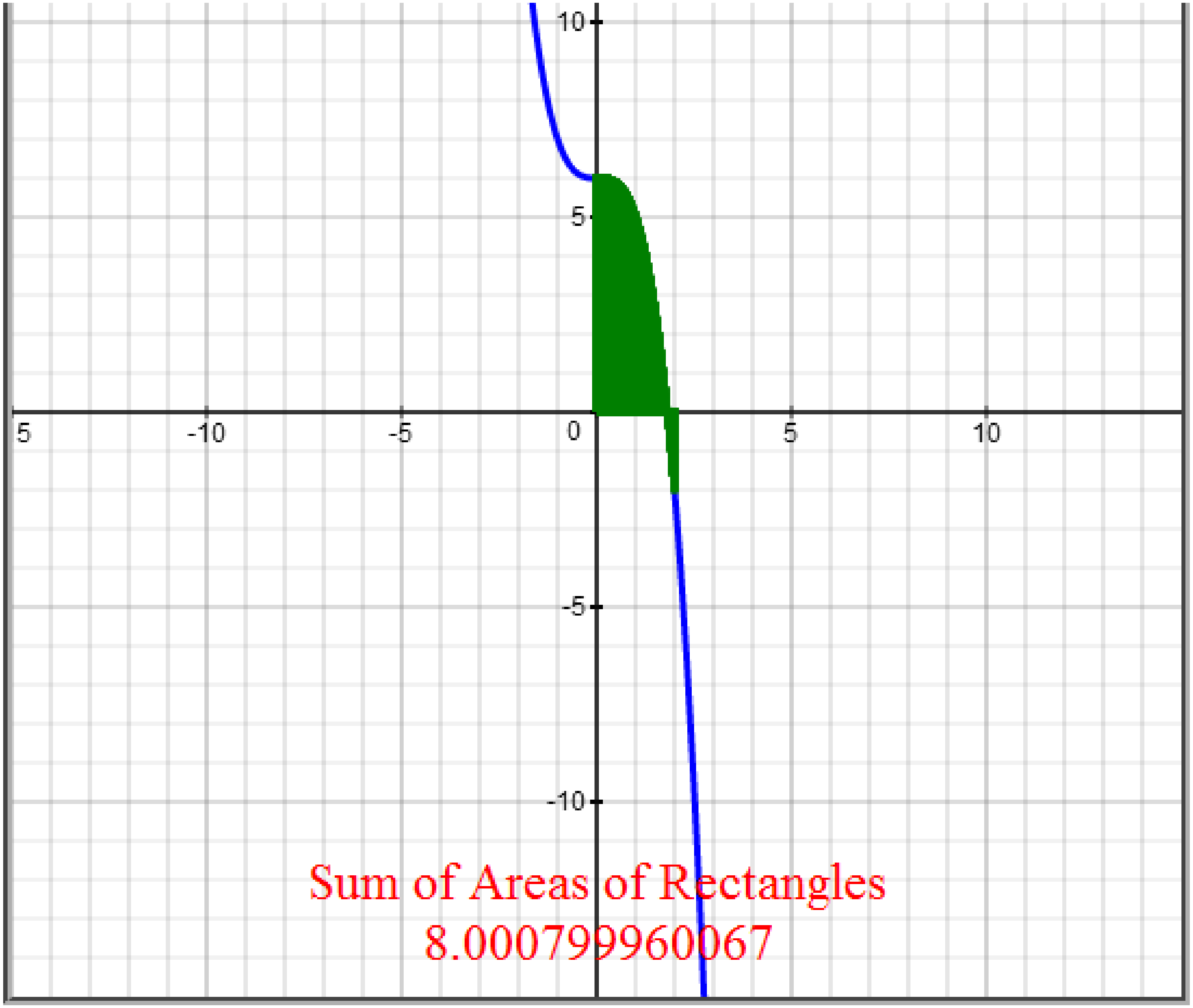
$$f(x) = 6 - x^3$$

Let n = number of subintervals = 10000

Let $a = 0$ and $b = 2$

a) $\Delta x = \dfrac{b - a}{n} == \dfrac{2-0}{10000} = 0.0002$

b) Find area between graph and x-axis?


10
5
-5
-10
5
-10
-5
0
5
10
Sum of Areas of Rectangles
8.000799960067

Example 9: Finding Area Under Curve

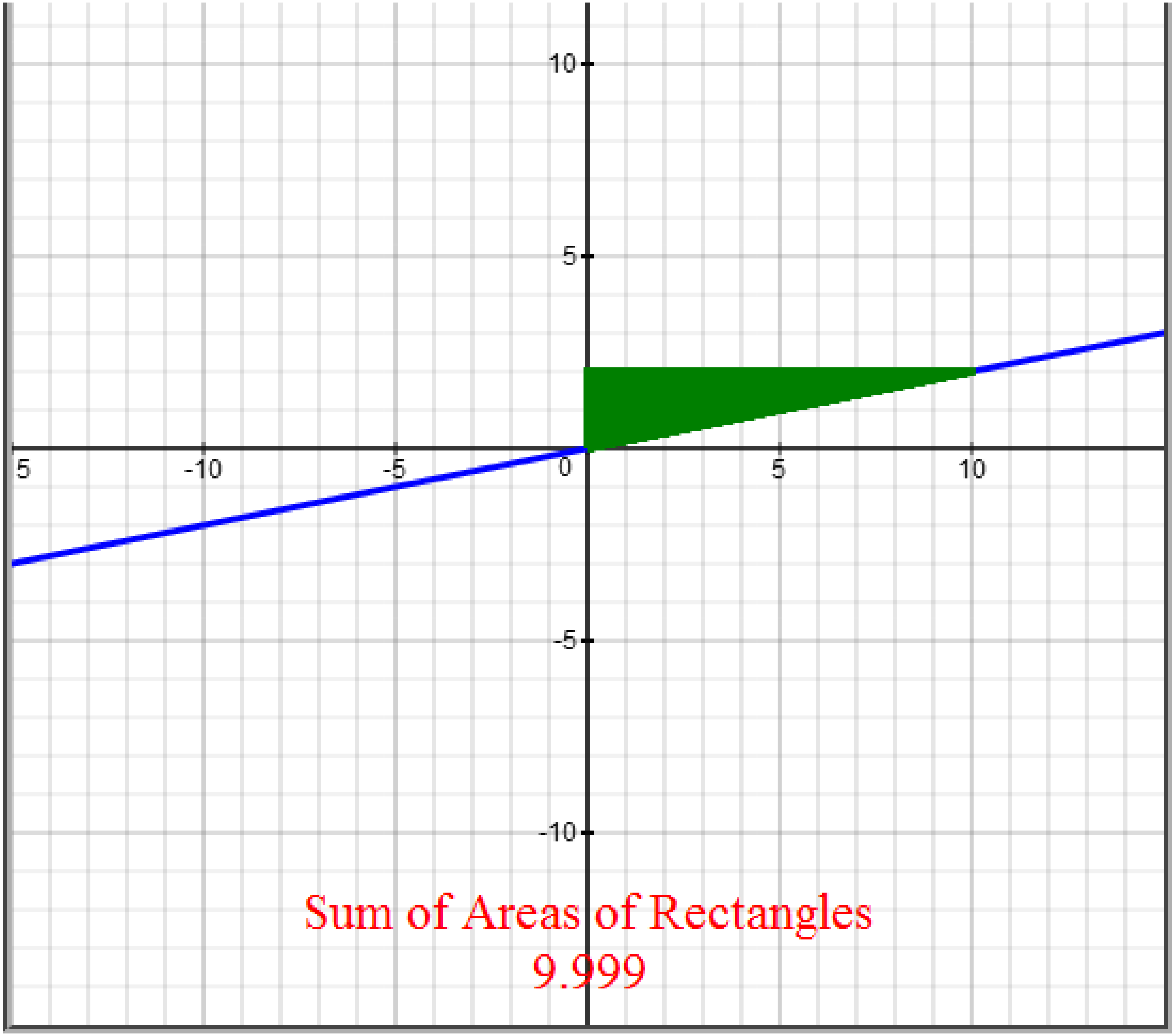
$g(y) = 5y \qquad\qquad 0 \le y \le 2$

Let n = number of subintervals = 10000

Let c = 0 and d = 2

a) $\Delta y = \dfrac{d - c}{n} = \dfrac{2-0}{10000} = 0.0002$

b) Find area between graph and y-axis?


10
5
-5
-10
5
-10
-5
0
5
10
Sum of Areas of Rectangles
9.999

Example 10: Finding Area Under Curve

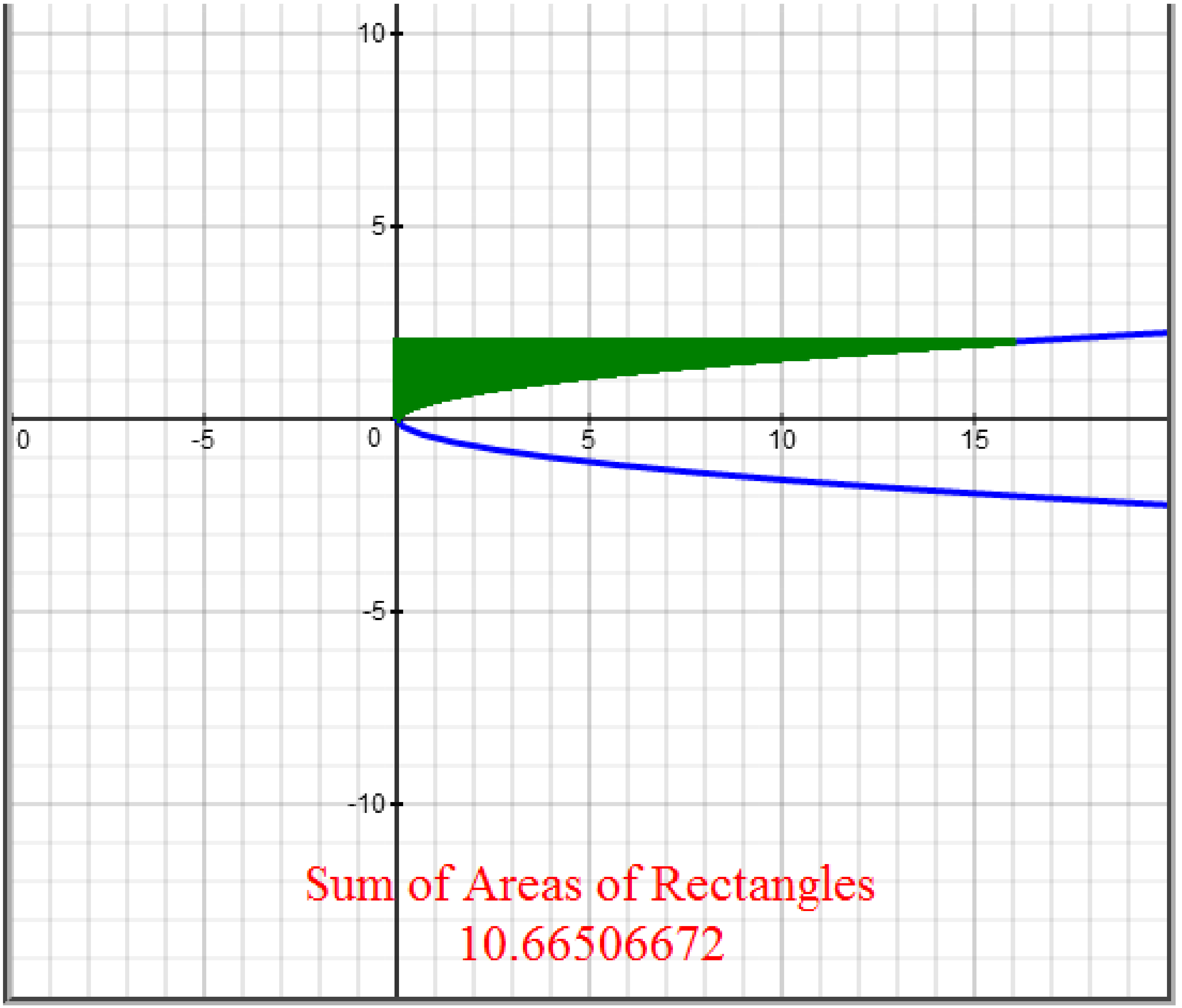
$$g(y) = 4y^2 \qquad 0 \le y \le 2$$

Let n = number of subintervals = 10000

Let $c = 0$ and $d = 2$

a) $$\Delta y = \frac{d - c}{n} = \frac{2-0}{10000} = 0.0002$$

b) Find area between graph and y-axis?


10
5
-5
-10
0
-5
0
5
10
15
Sum of Areas of Rectangles
10.66506672