Calculus I Section 4.4 Notes

Find $\int_0^2 5x\,dx$.

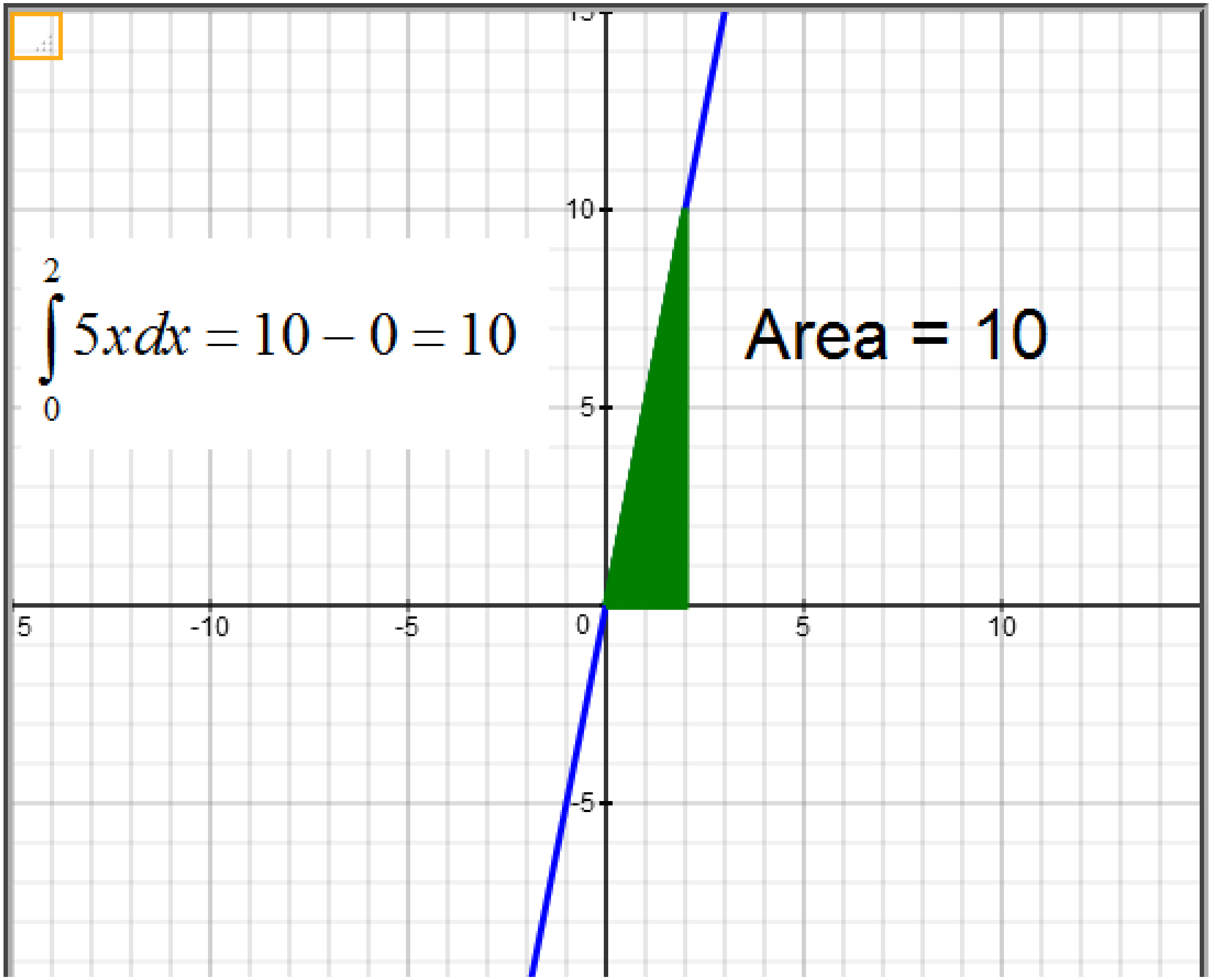
Let $f(x) = 5x$ $\quad a = 0 \quad b = 2$

Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b) - F(a)$

a) Antiderivative $F(x) = \int 5x\,dx = 5\left(\frac{x^2}{2}\right) = \frac{5x^2}{2}$

b) $F(0) = \frac{5x^2}{2} = \frac{5(0)^2}{2} = 0$ c) $F(2) = \frac{5x^2}{2} = \frac{5(2)^2}{2} = 10$

d) $\int_0^2 5x\,dx = 10 - 0 = 10$



$\int_0^2 5x\,dx = 10 - 0 = 10$
Area = 10
10
5
-5
-10
0

Find $\int_{-1}^{1}\left(x^2-4\right)dx.$

Let $f(x)=x^2-4$ $\quad a=-1 \quad b=1$

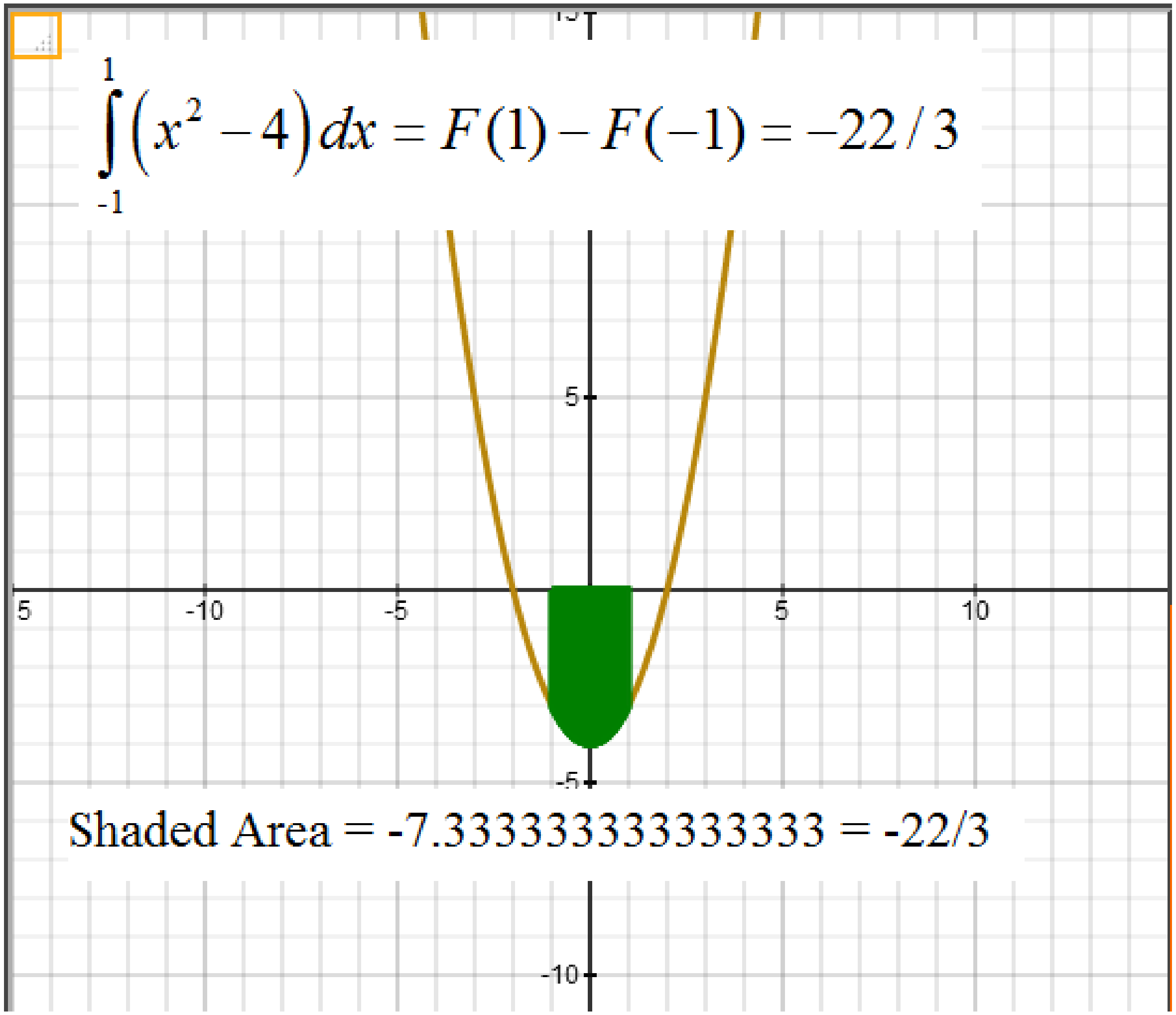
Fundamental Theorem of Calculus: $\int_{a}^{b} f(x)dx = F(b)-F(a)$

a) Antiderivative $F(x)= \int\left(x^2-4\right)dx=\frac{x^3}{3}-4x$

b) $F(1)=\frac{x^3}{3}-4x=\frac{\left(1\right)^3}{3}-4\left(1\right)$ = -11/3

c) $F(-1)=\frac{x^3}{3}-4x=\frac{\left(-1\right)^3}{3}-4\left(-1\right)$ = 11/3

d) $\int_{-1}^{1}\left(x^2-4\right)dx=F(1)-F(-1)=$ -11/3 - 11/3 $=-22/3$


$\int_{-1}^{1} (x^2 - 4)\,dx = F(1) - F(-1) = -22/3$
5
-10
-5
5
10
5
-5
-10
Shaded Area = -7.33333333333333 = -22/3

Find $\int_0^1 (3t-1)^2\, dt$. Let $f(t) = (3t-1)^2$ $a = 0$ $b = 1$

Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b) - F(a)$

a) Antiderivative $F(t) = \int (3t-1)^2\, dt = \int \left(9t^2 - 6t + 1\right) dt$

$$= 9\left(\frac{t^3}{3}\right) - 6\left(\frac{t^2}{2}\right) + t = 3t^3 - 3t^2 + t$$

b) $F(0) = 3t^3 - 3t^2 + t = 3(0)^3 - 3(0)^2 + (0) = 0$

c) $F(1) = 3t^3 - 3t^2 + t = 3(1)^3 - 3(1)^2 + (1) = 1$

d) $\int_0^1 (3t-1)^2\, dt = F(1) - F(0) = 1 - 0 = 1$

Find $\int_1^2 \left(\frac{5}{x^4} \text{ - } 8 \right) dx.$ Let $f(x) = \frac{5}{x^4} \text{ - } 8$ $a = 1$ $b = 2$

Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b) - F(a)$

a) Antiderivative $F(x) = \int \left(\frac{5}{x^4} \text{ - } 8 \right) dx = \int \left(5x^{-4} \text{ - } 8 \right) dx$

$$= \frac{5x^{-3}}{-3} - 8x = -\frac{5}{3x^3} - 8x$$

$b)$ $F(1) = -\frac{5}{3x^3} - 8x = -\frac{5}{3(1)^3} - 8(1) = \text{-19/3}$

c) $F(2) = -\frac{5}{3x^3} - 8x = -\frac{5}{3(2)^3} - 8(2) = \text{-389/24}$

d) $\int_1^2 \left(\frac{5}{x^4} \text{ - } 8 \right) dx = F(2) - F(1) = \text{-389/24 - -19/3} = \text{-79/8}$

Find $\int_1^4 \left(\frac{7x-5}{\sqrt{x}} \right) dx.$ Let $f(x) = \frac{7x-5}{\sqrt{x}}$ $a = 1$ $b = 4$

Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b) - F(a)$

a) Antiderivative $F(x) = \int \left(\frac{7x-5}{\sqrt{x}} \right) dx = ?$

Note: $\frac{7x-5}{\sqrt{x}} = \frac{7x}{x^{1/2}} - \frac{5}{x^{1/2}} = 7x^{1/2} - 5x^{-1/2}$

$$\int \left(\frac{7x-5}{\sqrt{x}} \right) dx = \int \left(7x^{1/2} - 5x^{-1/2} \right) dx$$

$$= 7\frac{x^{3/2}}{3/2} - 5\frac{x^{1/2}}{1/2} = 7 \cdot \frac{2}{3} \cdot x^{3/2} - 5 \cdot \frac{2}{1} \cdot x^{1/2} = \frac{14}{3}x^{3/2} - 10x^{1/2}$$

Let $f(x) = \dfrac{7x-5}{\sqrt{x}}$ $\qquad a = 1 \qquad b = 4$

b) $F(1) = \dfrac{14}{3}x^{3/2} - 10x^{1/2} = \dfrac{14}{3}(1)^{3/2} - 10(1)^{1/2}$ = -16/3

c) $F(4) = \dfrac{14}{3}x^{3/2} - 10x^{1/2} = \dfrac{14}{3}(4)^{3/2} - 10(4)^{1/2}$ = 52/3

d) $\displaystyle\int_1^4 \left(\frac{7x-5}{\sqrt{x}}\right)dx = F(4) - F(1) =$ 52/3 - -16/3 = 68/3

Find $\int_0^{\pi}\left(2+\sin x\right)dx.$ Let $f(x)=2+\sin x$ $a=0$ $b=\pi$

Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b)-F(a)$

a) Antiderivative $F(x)=\int\left(2+\sin x\right)dx$

$$=2x+\left(-\cos x\right)=2x-\cos x$$

b) $F(0)=2x-\cos x=2\left(0\right)-\cos\left(0\right)=0\ -1=-1$

c) $F(\pi)=2x-\cos x=2\left(\pi\right)-\cos\left(\pi\right)=2\pi\ -\left(-1\right)=2\pi+1$

d) $\int_0^{\pi}\left(2+\sin x\right)dx=F(\pi)-F(0)=2\pi+1--1=2\pi+2$

Find $\int_0^{\pi/4} \tan x dx.$ Let $f(x) = \tan x$ $a = 0$ $b = \pi/4$

Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b) - F(a)$

a) Antiderivative $F(x) = \int \tan x dx = -\ln|\cos x|$

b) $F(0) = -\ln|\cos x| = -\ln|\cos 0| = -\ln|1| = 0$

c) $F(\pi/4) = -\ln|\cos x| = -\ln\left|\cos\left(\pi/4\right)\right| = 0.34657359027$

d) $\int_0^{\pi/4} \tan x dx = F(\pi/4) - F(0) = -\ln\left|\cos\left(\pi/4\right)\right|$

$= 0.34657359027$

Find the area bounded by the following graphs:

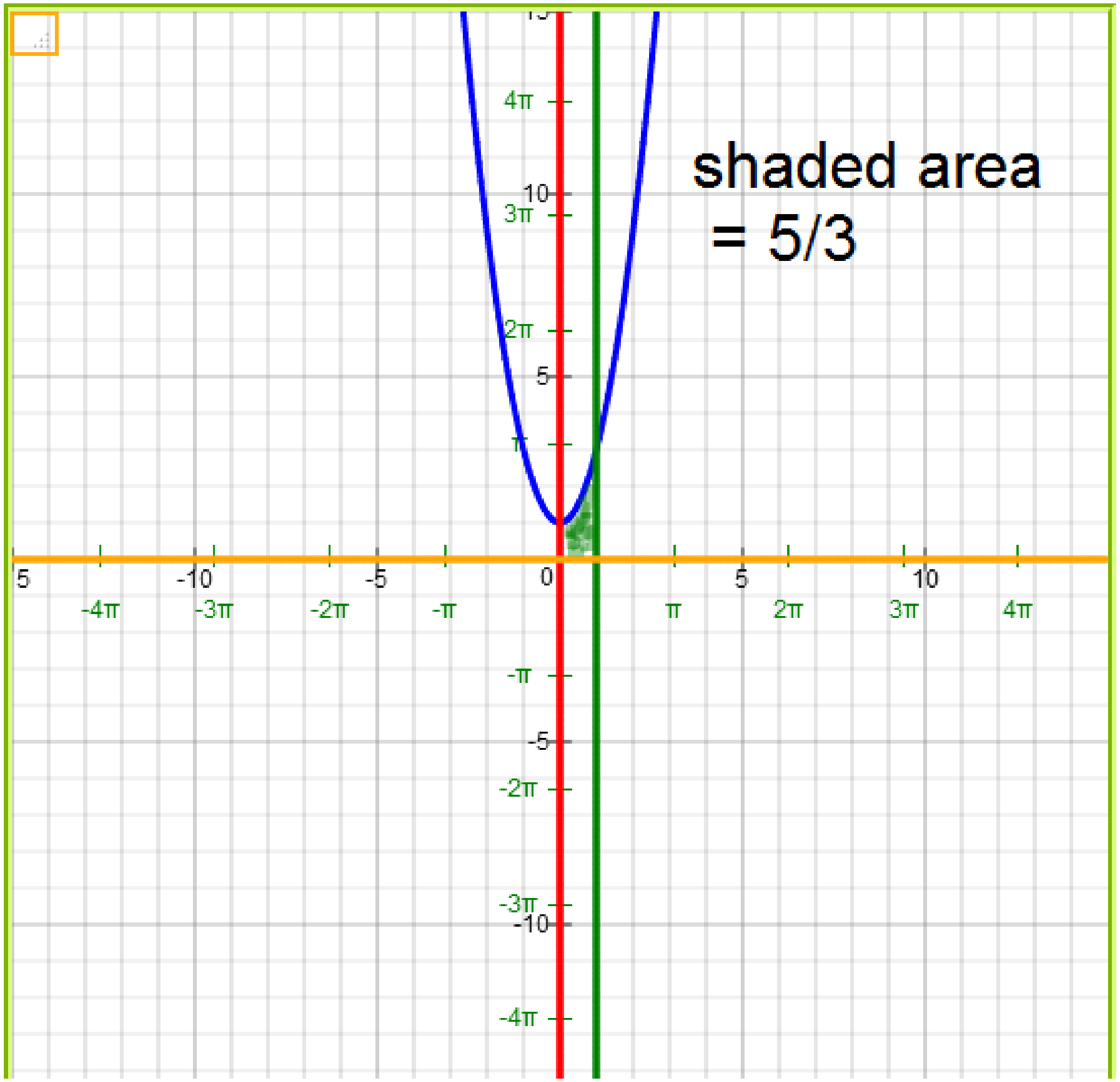
$$y = 2x^2 + 1; \qquad x = 0; \qquad x = 1; \qquad y = 0$$

$$\text{Area} = \int_0^1 \left(2x^2 + 1\right) dx$$

$$f(x) = 2x^2 + 1$$

$$F(x) = 2\left(\frac{x^3}{3}\right) + x = \frac{2x^3}{3} + x$$

$$\text{Area} = F(1) - F(0) = \left[\frac{2\left(1\right)^3}{3} + \left(1\right)\right] - \left[\frac{2\left(0\right)^3}{3} + \left(0\right)\right] = \frac{5}{3}$$


shaded area
= 5/3

Find the area bounded by the following graphs:

$$y = \sqrt[3]{x}; \qquad x = 0; \qquad x = 2; \qquad y = 0$$

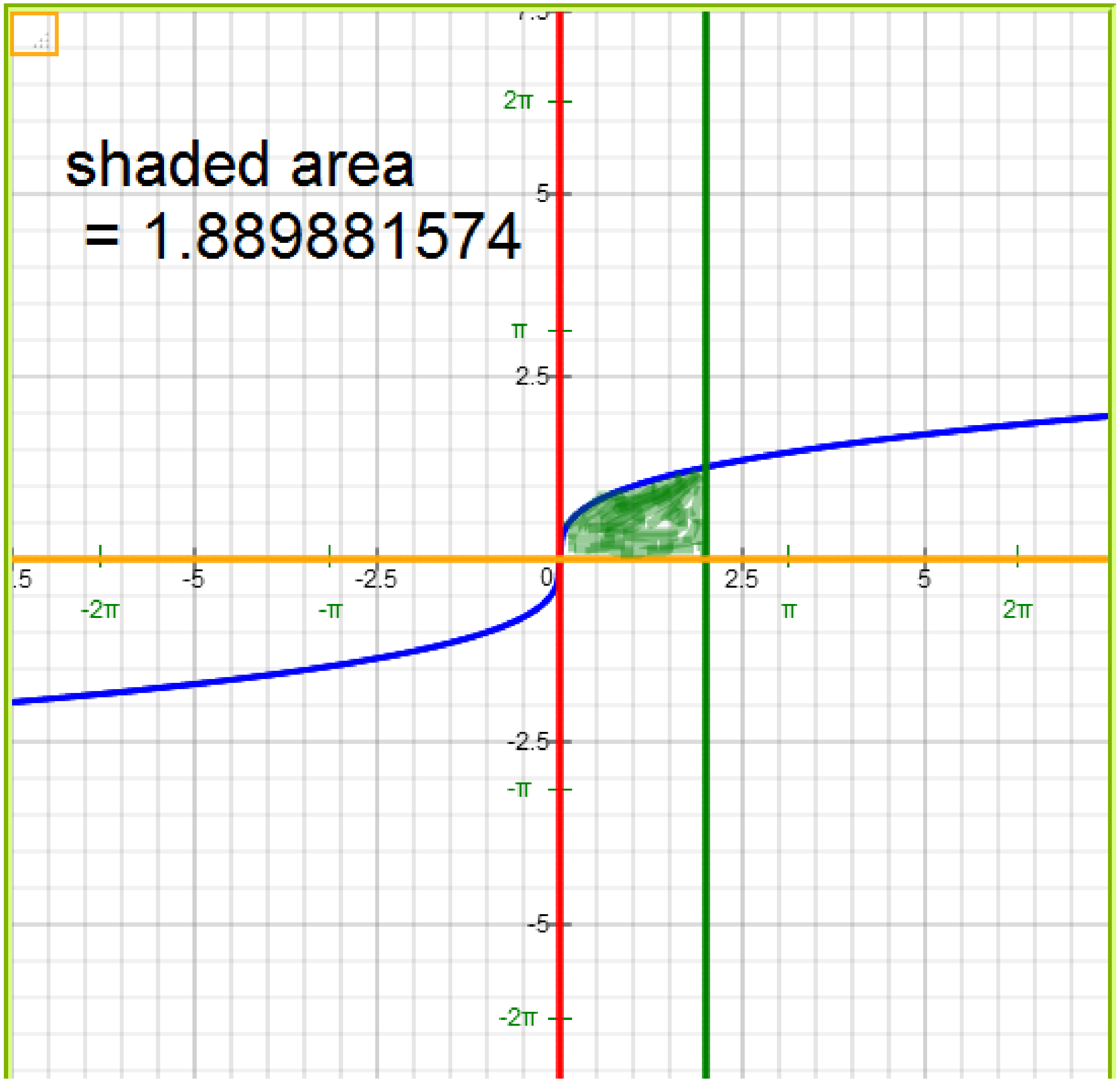
$$\text{Area} = \int_0^2 \left(\sqrt[3]{x}\right) dx = \int_0^2 \left(x^{1/3}\right) dx$$

$$f(x) = x^{1/3}$$

$$F(x) = \frac{x^{1/3+1}}{4/3} = \frac{3}{4} x^{4/3}$$

$$\text{Area} = F(2) - F(0) = \left[\frac{3}{4}(2)^{4/3}\right] - \left[\frac{3}{4}(0)^{4/3}\right]$$

$$= 1.889881574$$


shaded area
= 1.889881574
2π
5
π
2.5
-2.5
-π
-5
-2π
-5
-2.5
0
2.5
5
-2π
-π
π
2π

Second Fundamental Theorem of Calculus: $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = dx$ where a is a constant

Example:

Let $f(t) = t^2 + 1$

$$\int_a^x f(t)\,dt = \int_a^x \left(t^2+1\right)dt = \left[\frac{1}{3}t^3 + t\right]_a^x = \left(\frac{1}{3}x^3 + x\right) - \left(\frac{1}{3}(a)^3 + a\right)$$

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = \frac{d}{dx}\left[\left(\frac{1}{3}x^3 + x\right) - \left(\frac{1}{3}(a)^3 + a\right)\right] = \frac{1}{3}\left(3x^2\right) + 1 - 0 = x^2 + 1$$

Hence, $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = x^2 + 1 = f(x)$

Second Fundamental Theorem of Calculus: $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = dx$ where a is a constant

Example:

Let $f(t) = \sqrt{t^2+1}$

$$\int_a^x f(t)\,dt = \int_a^x \sqrt{t^2+1}\,dt = \sqrt{x^2+1} = f(x)$$

Second Fundamental Theorem of Calculus: $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = dx$ where a is a constant

Example:

Let $f(t) = t\sqrt{t^4+1}$

$$\int_a^x f(t)\,dt = \int_a^x t\sqrt{t^4+1}\,dt = x\sqrt{x^4+1} = f(x)$$