

$$\text{Find } \int_0^2 5x dx.$$

$$\text{Let } f(x) = 5x \qquad a = 0 \qquad b = 2$$

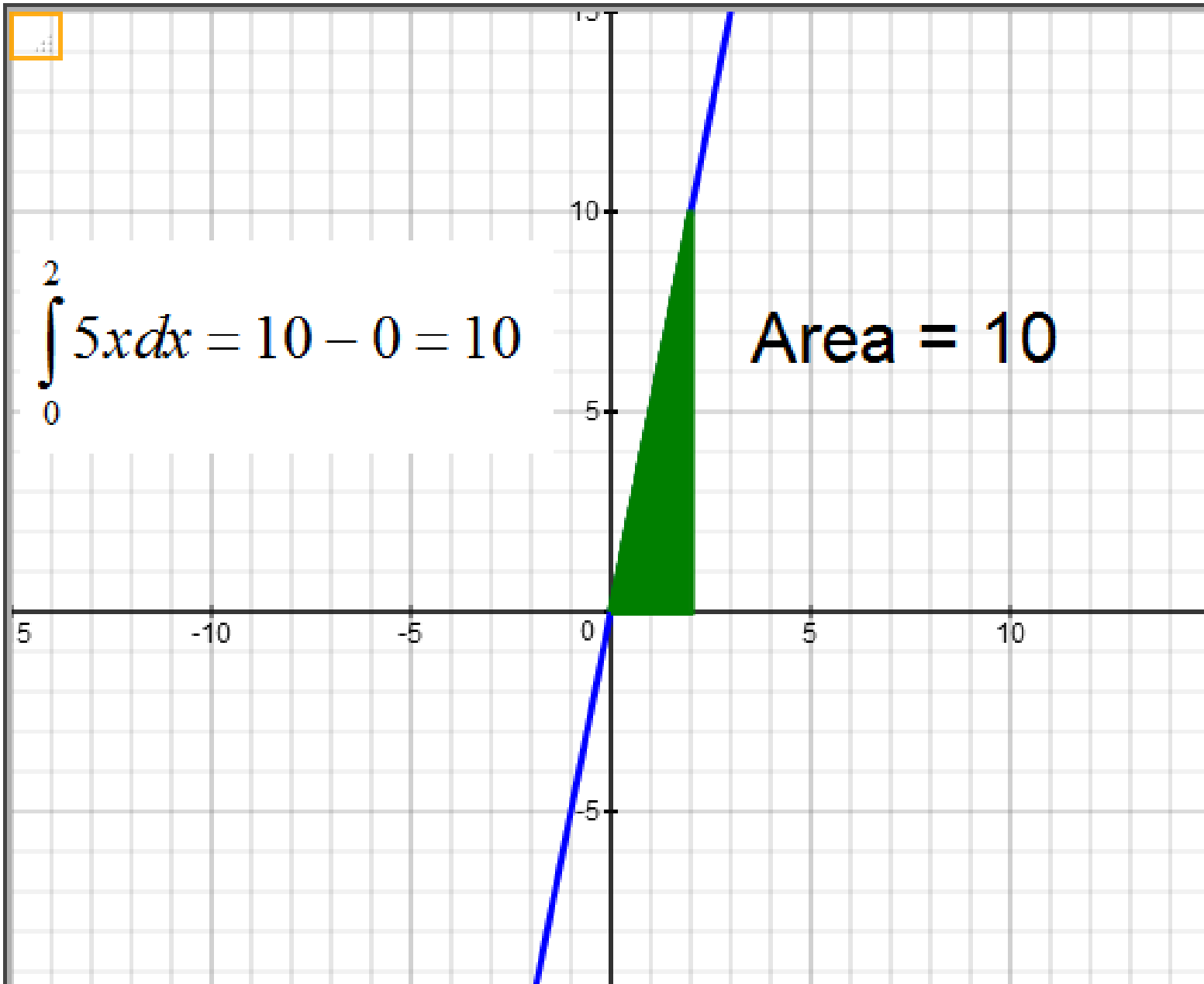
$$\text{Fundamental Theorem of Calculus: } \int_a^b f(x) dx = F(b) - F(a)$$

$$\text{a) Antiderivative } F(x) = \int 5x dx = 5 \left(\frac{x^2}{2} \right) = \frac{5x^2}{2}$$

$$\text{b) } F(0) = \frac{5x^2}{2} = \frac{5(0)^2}{2} = 0$$

$$\text{c) } F(2) = \frac{5x^2}{2} = \frac{5(2)^2}{2} = 10$$

$$\text{d) } \int_0^2 5x dx = 10 - 0 = 10$$



Find $\int_{-1}^1 (x^2 - 4) dx$.

Let $f(x) = x^2 - 4$ $a = -1$ $b = 1$

Fundamental Theorem of Calculus: $\int_a^b f(x) dx = F(b) - F(a)$

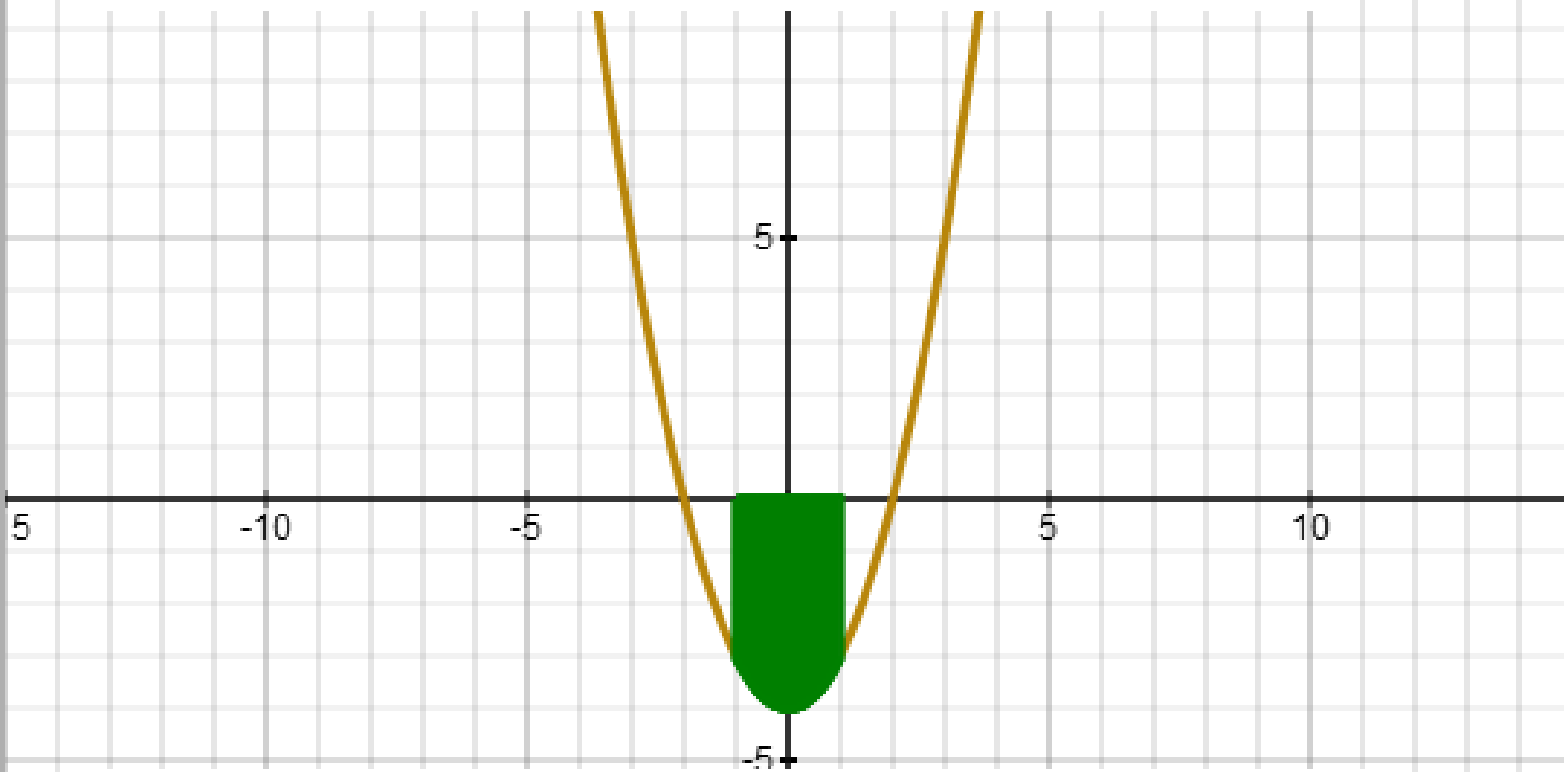
a) Antiderivative $F(x) = \int (x^2 - 4) dx = \frac{x^3}{3} - 4x$

b) $F(1) = \frac{x^3}{3} - 4x = \frac{(1)^3}{3} - 4(1) = -11/3$

c) $F(-1) = \frac{x^3}{3} - 4x = \frac{(-1)^3}{3} - 4(-1) = 11/3$

d) $\int_{-1}^1 (x^2 - 4) dx = F(1) - F(-1) = -11/3 - 11/3 = -22/3$

$$\int_{-1}^1 (x^2 - 4) dx = F(1) - F(-1) = -22/3$$



Shaded Area = $-7.33333333333333333333 = -22/3$

Find $\int_0^1 (3t - 1)^2 dt$. Let $f(t) = (3t - 1)^2$ $a = 0$ $b = 1$

Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b) - F(a)$

a) Antiderivative $F(t) = \int (3t - 1)^2 dt = \int (9t^2 - 6t + 1) dt$

$$= 9\left(\frac{t^3}{3}\right) - 6\left(\frac{t^2}{2}\right) + t = 3t^3 - 3t^2 + t$$

b) $F(0) = 3t^3 - 3t^2 + t = 3(0)^3 - 3(0)^2 + (0) = 0$

c) $F(1) = 3t^3 - 3t^2 + t = 3(1)^3 - 3(1)^2 + (1) = 1$

d) $\int_0^1 (3t - 1)^2 dt = F(1) - F(0) = 1 - 0 = 1$

Find $\int_1^2 \left(\frac{5}{x^4} - 8 \right) dx$. Let $f(x) = \frac{5}{x^4} - 8$ $a = 1$ $b = 2$

Fundamental Theorem of Calculus: $\int_a^b f(x) dx = F(b) - F(a)$

a) Antiderivative $F(x) = \int \left(\frac{5}{x^4} - 8 \right) dx = \int (5x^{-4} - 8) dx$

$$= \frac{5x^{-3}}{-3} - 8x = -\frac{5}{3x^3} - 8x$$

b) $F(1) = -\frac{5}{3x^3} - 8x = -\frac{5}{3(1)^3} - 8(1) = -19/3$

c) $F(2) = -\frac{5}{3x^3} - 8x = -\frac{5}{3(2)^3} - 8(2) = -389/24$

d) $\int_1^2 \left(\frac{5}{x^4} - 8 \right) dx = F(2) - F(1) = -389/24 - (-19/3) = -79/8$

Find $\int_1^4 \left(\frac{7x-5}{\sqrt{x}} \right) dx$. Let $f(x) = \frac{7x-5}{\sqrt{x}}$ $a = 1$ $b = 4$

Fundamental Theorem of Calculus: $\int_a^b f(x) dx = F(b) - F(a)$

a) Antiderivative $F(x) = \int \left(\frac{7x-5}{\sqrt{x}} \right) dx = ?$

Note: $\frac{7x-5}{\sqrt{x}} = \frac{7x}{x^{1/2}} - \frac{5}{x^{1/2}} = 7x^{1/2} - 5x^{-1/2}$

$$\int \left(\frac{7x-5}{\sqrt{x}} \right) dx = \int (7x^{1/2} - 5x^{-1/2}) dx$$

$$= 7 \frac{x^{3/2}}{3/2} - 5 \frac{x^{1/2}}{1/2} = 7 \cdot \frac{2}{3} \cdot x^{3/2} - 5 \cdot \frac{2}{1} \cdot x^{1/2} = \frac{14}{3} x^{3/2} - 10x^{1/2}$$

$$\text{Let } f(x) = \frac{7x-5}{\sqrt{x}} \quad a=1 \quad b=4$$

$$\text{b) } F(1) = \frac{14}{3}x^{3/2} - 10x^{1/2} = \frac{14}{3}(1)^{3/2} - 10(1)^{1/2} = -16/3$$

$$\text{c) } F(4) = \frac{14}{3}x^{3/2} - 10x^{1/2} = \frac{14}{3}(4)^{3/2} - 10(4)^{1/2} = 52/3$$

$$\text{d) } \int_1^4 \left(\frac{7x-5}{\sqrt{x}} \right) dx = F(4) - F(1) = 52/3 - -16/3 = 68/3$$

Find $\int_0^{\pi} (2 + \sin x) dx$. Let $f(x) = 2 + \sin x$ $a = 0$ $b = \pi$

Fundamental Theorem of Calculus: $\int_a^b f(x) dx = F(b) - F(a)$

a) Antiderivative $F(x) = \int (2 + \sin x) dx$
 $= 2x + (-\cos x) = 2x - \cos x$

b) $F(0) = 2x - \cos x = 2(0) - \cos(0) = 0 - 1 = -1$

c) $F(\pi) = 2x - \cos x = 2(\pi) - \cos(\pi) = 2\pi - (-1) = 2\pi + 1$

d) $\int_0^{\pi} (2 + \sin x) dx = F(\pi) - F(0) = 2\pi + 1 - (-1) = 2\pi + 2$

Find $\int_0^{\pi/4} \tan x dx$. Let $f(x) = \tan x$ $a = 0$ $b = \pi/4$

Fundamental Theorem of Calculus: $\int_a^b f(x) dx = F(b) - F(a)$

a) Antiderivative $F(x) = \int \tan x dx = -\ln|\cos x|$

b) $F(0) = -\ln|\cos x| = -\ln|\cos 0| = -\ln|1| = 0$

c) $F(\pi/4) = -\ln|\cos x| = -\ln|\cos(\pi/4)| = 0.34657359027$

d) $\int_0^{\pi/4} \tan x dx = F(\pi/4) - F(0) = -\ln|\cos(\pi/4)|$
 $= 0.34657359027$

Find the area bounded by the following graphs:

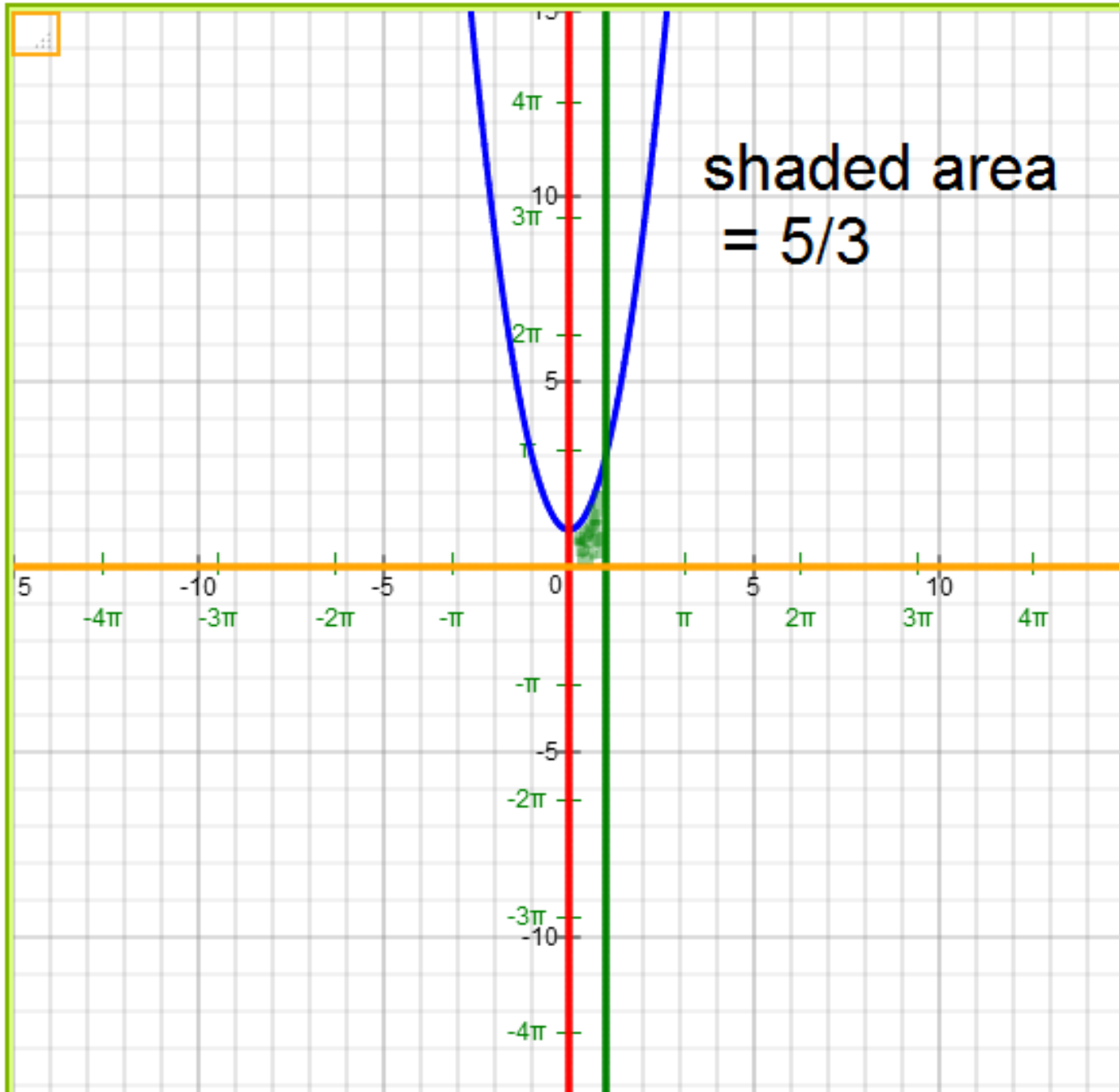
$$y = 2x^2 + 1; \quad x = 0; \quad x = 1; \quad y = 0$$

$$\text{Area} = \int_0^1 (2x^2 + 1) dx$$

$$f(x) = 2x^2 + 1$$

$$F(x) = 2\left(\frac{x^3}{3}\right) + x = \frac{2x^3}{3} + x$$

$$\text{Area} = F(1) - F(0) = \left[\frac{2(1)^3}{3} + (1) \right] - \left[\frac{2(0)^3}{3} + (0) \right] = \frac{5}{3}$$



Find the area bounded by the following graphs:

$$y = \sqrt[3]{x}; \quad x = 0; \quad x = 2; \quad y = 0$$

$$\text{Area} = \int_0^2 \left(\sqrt[3]{x} \right) dx = \int_0^2 \left(x^{1/3} \right) dx$$

$$f(x) = x^{1/3}$$

$$F(x) = \frac{x^{1/3+1}}{4/3} = \frac{3}{4} x^{4/3}$$

$$\begin{aligned} \text{Area} &= F(2) - F(0) = \left[\frac{3}{4} (2)^{4/3} \right] - \left[\frac{3}{4} (0)^{4/3} \right] \\ &= 1.889881574 \end{aligned}$$

