

Find the indefinite integral  $\int (4 + 5x)^4 (5) dx$

Let  $u = 4 + 5x$

a)  $\frac{du}{dx} = 5 \Rightarrow du = 5dx$

b)  $\int (4 + 5x)^4 (5) dx = \int (u)^4 du = \frac{u^5}{5} + C = \frac{(4 + 5x)^5}{5} + C$

*Note*:  $D_x \left[ \frac{(4 + 5x)^5}{5} + C \right] = (4 + 5x)^4 (5)$

Find the indefinite integral  $\int \sqrt{6 - 4x^2} (-8x) dx$

Let  $u = 6 - 4x^2$

$$\text{a) } \frac{du}{dx} = -8x \quad \Rightarrow \quad du = -8x dx$$

$$\begin{aligned} \text{b) } \int \sqrt{6 - 4x^2} (-8x) dx &= \int \sqrt{u} du = \int u^{1/2} du = \frac{u^{3/2}}{3/2} + C \\ &= \frac{2}{3} u^{3/2} + C = \frac{2}{3} (6 - 4x^2)^{3/2} + C \end{aligned}$$

$$\text{Note: } D_x \left[ \frac{2}{3} (6 - 4x^2)^{3/2} + C \right] = \sqrt{6 - 4x^2} (-8x)$$

Find the indefinite integral  $\int x(2x^2 + 4)^2 dx$

Note:  $\int x(2x^2 + 4)^2 dx = \int (2x^2 + 4)^2 \cdot x dx$

Let  $u = 2x^2 + 4$

a)  $\frac{du}{dx} = 4x$

b)  $du = 4x dx \Rightarrow \frac{1}{4} du = \frac{1}{4} \cdot 4x dx \Rightarrow \frac{1}{4} du = x dx$

c)  $\int x(2x^2 + 4)^2 dx = \int (2x^2 + 4)^2 \cdot x dx = \int (u)^2 \cdot \frac{1}{4} du$   
 $= \frac{1}{4} \int u^2 du = \frac{1}{4} \left( \frac{u^3}{3} \right) + C = \frac{1}{12} u^3 + C = \frac{1}{12} (2x^2 + 4)^3 + C$

Note:  $D_x \left[ \frac{1}{12} (2x^2 + 4)^3 + C \right] = x(2x^2 + 4)^2$

Find the indefinite integral  $\int 4x(5 - x^2)^{1/3} dx$

Note:  $\int 4x(5 - x^2)^{1/3} dx = 4 \int (5 - x^2)^{1/3} \cdot x dx$

Let  $u = 5 - x^2$

a)  $\frac{du}{dx} = -2x$

b)  $du = -2x dx \Rightarrow \frac{-1}{2} du = \frac{-1}{2} (-2x dx) \Rightarrow \frac{-1}{2} du = x dx$

c)  $\int 4x(5 - x^2)^{1/3} dx = 4 \int (5 - x^2)^{1/3} \cdot x dx = 4 \int (u)^{1/3} \cdot \frac{-1}{2} du$

$$= -2 \int (u)^{1/3} du = -2 \left( \frac{u^{4/3}}{4/3} \right) + C = -2 \cdot \frac{3}{4} \cdot u^{4/3} + C$$

$$= -\frac{3}{2} \cdot (5 - x^2)^{4/3} + C$$

Find the indefinite integral  $\int \frac{5x^2}{(3+x^3)^2} dx$

*Note:*  $\int \frac{5x^2}{(3+x^3)^2} dx = 5 \int \frac{1}{(3+x^3)^2} \cdot x^2 dx$

Let  $u = 3 + x^3$

a)  $\frac{du}{dx} = 3x^2$

b)  $du = 3x^2 dx \Rightarrow \frac{1}{3} du = \frac{1}{3} (3x^2 dx) \Rightarrow \frac{1}{3} du = x^2 dx$

c)  $\int \frac{5x^2}{(3+x^3)^2} dx = 5 \int \frac{1}{(3+x^3)^2} \cdot x^2 dx = 5 \int \frac{1}{(u)^2} \cdot \frac{1}{3} du$   
 $= \frac{5}{3} \int u^{-2} du = \frac{5}{3} \left( \frac{u^{-1}}{-1} \right) + C = -\frac{5}{3} (3+x^3)^{-1} + C$

Find the indefinite integral  $\int \left(5 + \frac{1}{x}\right)^2 \left(\frac{1}{x^2}\right) dx$

$$\text{Let } u = 5 + \frac{1}{x} = 5 + x^{-1}$$

$$\text{a) } \frac{du}{dx} = -1x^{-2} = \frac{-1}{x^2}$$

$$\text{b) } du = \frac{-1}{x^2} dx \Rightarrow (-1) du = (-1) \frac{-1}{x^2} dx \Rightarrow (-1) du = \frac{1}{x^2} dx$$

$$\text{c) } \int \left(5 + \frac{1}{x}\right)^2 \left(\frac{1}{x^2}\right) dx = \int (u)^2 (-1) du = (-1) \int (u)^2 du$$

$$= (-1) \frac{u^3}{3} + C = \frac{-1}{3} \left(5 + \frac{1}{x}\right)^3 + C$$

Find the indefinite integral  $\int \pi \sin \pi x dx = \int (\sin \pi x) \cdot \pi \cdot dx$

Let  $u = \pi x$

a)  $\frac{du}{dx} = \pi$

b)  $du = \pi dx$

c)  $\int \pi \sin \pi x dx = \int (\sin \pi x) \cdot \pi \cdot dx = \int (\sin u) \cdot du$   
 $= -\cos u + C = -\cos(\pi x) + C$

Find the indefinite integral  $\int \tan x \cdot \sec^2 x \cdot dx$

Let  $u = \tan x$

a)  $\frac{du}{dx} = \sec^2 x$

b)  $du = \sec^2 x du$

c)  $\int \tan x \cdot \sec^2 x \cdot dx = \int u \cdot du = \frac{u^2}{2} + C = \frac{(\tan x)^2}{2} + C$