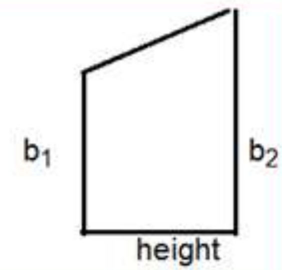
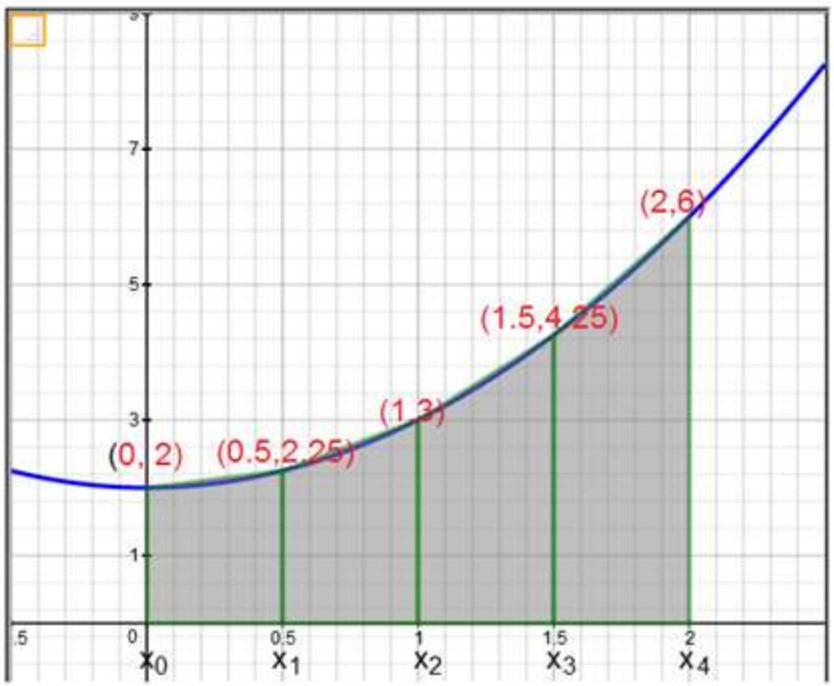
Find the definite integral $\int_0^2 (x^2+2)dx$ by using Trapezoid's Rule.

Area of a Trapezoid $= \frac{1}{2}h(b_1+b_2)$




$$f(x) = x^2+2; \qquad a=0; \qquad b=2;$$

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$$




$x_0 = 0 \qquad f(x_0) = f(0) = 2$

$x_1 = 0.5 \qquad f(x_1) = f(0.5) = 2.25$

$x_2 = 1 \qquad f(x_2) = f(1) = 3$

$x_3 = 1.5 \qquad f(x_3) = f(1.5) = 4.25$

$x_4 = 2 \qquad f(x_4) = f(2) = 6$

For Trapezoid #1:

Left Endpoint = $x_0 = 0$; Right Endpoint = $x_1 = 0.5$

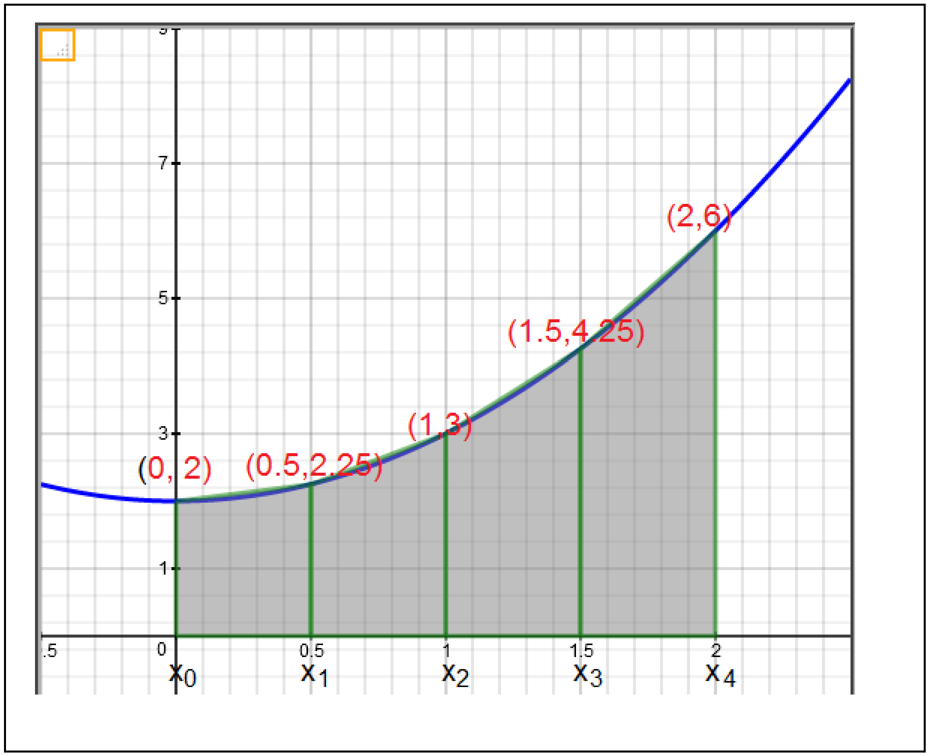
height of trapezoid = $\Delta x_1 = x_1 - x_0 = 0.5 - 0 = 0.5 = \Delta x$

base 1 of trapezoid = $f(x_0) = f(0) = 2$;

base 2 of trapezoid = $f(x_1) = f(0.5) = 2.25$

area of trapezoid = $0.5(\text{base}_1 + \text{base}_2)(\text{height}) = 1.0625$

area of trapezoid = $\frac{1}{2}[f(x_0) + f(x_1)]\Delta x_1 = 1.0625$




For Trapezoid #2:

Left Endpoint = $x_1 = 0.5$; Right Endpoint = $x_2 = 1$

height of trapezoid = $\Delta x_2 = x_2 - x_1 = 1 - 0.5 = 0.5 = \Delta x$

base 1 of trapezoid = $f(x_1) = f(0.5) = 2.25$;

base 2 of trapezoid = $f(x_2) = f(1) = 3$

area of trapezoid = 0.5(base1 + base2)(height) = 1.3125

area of trapezoid = $\frac{1}{2}[f(x_1) + f(x_2)]\Delta x_2 = 1.3125$

For Trapezoid #3:

Left Endpoint = $x_2 = 1$;  Right Endpoint = $x_3 = 1.5$

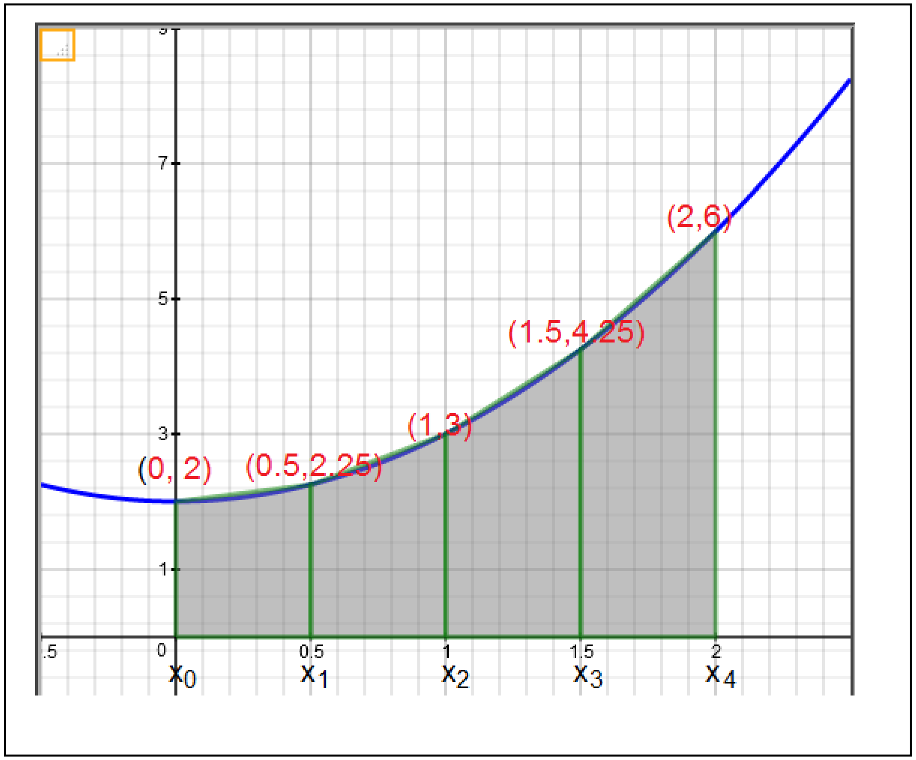
height of trapezoid = $\Delta x_3 = x_3 - x_2 = 1.5 - 1 = 0.5 = \Delta x$

base 1 of trapezoid = $f(x_2) = f(1) = 3$;

base 2 of trapezoid = $f(x_3) = f(1.5) = 4.25$

area of trapezoid = 0.5(base1 + base2)(height) = 1.8125

area of trapezoid = $\frac{1}{2}[f(x_2) + f(x_3)]\Delta x_3 = 1.8125$




For Trapezoid #4:

Left Endpoint = $x_3 = 1.5$;  Right Endpoint = $x_4 = 2$

height of trapezoid = $\Delta x_4 = x_4 - x_3 = 2 - 1.5 = 0.5 = \Delta x$

base 1 of trapezoid = $f(x_3) = f(1.5) = 4.25$;

base 2 of trapezoid = $f(x_4) = f(2) = 6$

area of trapezoid = 0.5(base1 + base2)(height) = 2.5625

area of trapezoid = $\frac{1}{2}[f(x_3) + f(x_4)]\Delta x_4 = 2.5625$

Sum of Areas of Trapezoids = $1.0625 + 1.3125 + 1.8125 + 2.5625 = 6.75$

Sum of Areas of Trapezoids = $\frac{1}{2}\left[f(x_0)+f(x_1)\right]\Delta x_1+\frac{1}{2}\left[f(x_1)+f(x_2)\right]\Delta x_2+\frac{1}{2}\left[f(x_2)+f(x_3)\right]\Delta x_3+\frac{1}{2}\left[f(x_3)+f(x_4)\right]\Delta x_4$

Sum of Areas of Trapezoids = $\frac{1}{2}\left[f(x_0)+f(x_1)\right]\Delta x+\frac{1}{2}\left[f(x_1)+f(x_2)\right]\Delta x+\frac{1}{2}\left[f(x_2)+f(x_3)\right]\Delta x+\frac{1}{2}\left[f(x_3)+f(x_4)\right]\Delta x$

Sum of Areas of Trapezoids = $\frac{1}{2}\Delta x\left[f(x_0)+f(x_1)+f(x_1)+f(x_2)+f(x_2)+f(x_3)+f(x_3)+f(x_4)\right]$

Sum of Areas of Trapezoids = $\frac{1}{2}\Delta x\left[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+f(x_4)\right]=6.75$

Sum of Areas of Trapezoids = $\frac{1}{2}\left(\frac{b-a}{n}\right)\left[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+f(x_4)\right]=6.75$

Sum of Areas of Trapezoids = $\frac{b-a}{2n}\left[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+f(x_4)\right]=6.75$

Trapezoidal Rule:

If $f$ is continuous of [a, b] and if a regular partition of [a, b] is determined by $a = x_0, x_1, x_2, \ldots, x_n$, then

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}[f(x_0)+2f(x_1)+2f(x_2)+\ldots+2f(x_{n-1})+f(x_n)]$$

Simpson's Rule:

Suppose $f$ is continuous on $[a, b]$ and $n$ is an even integer.

If a regular partition is determined by $a = x_0, x_1, x_2, \ldots, x_{n-1}, x_n = b$, then

$$\int_a^b f(x)dx \approx \frac{b-a}{3n}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\ldots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)]$$

Find the definite integral $\int_0^3 \left(x^2+4\right)dx$.

a) Using Trapezoidal Rule with $n = 4$,

approximate value of $\int_0^3 \left(x^2+4\right)dx = 21.18$

b) Using Simpson's Rule with $n = 4$,

approximate value of $\int_0^3 \left(x^2+4\right)dx = 21$

Find the definite integral $\int_0^1 \frac{1}{(x+7)^2} dx$.

a) Using Trapezoidal Rule with $n = 4$,

approximate value of $\int_0^1 \frac{1}{(x+7)^2} dx = 0.017867163338$

b) Using Simpson's Rule with $n = 4$,

approximate value of $\int_0^1 \frac{1}{(x+7)^2} dx = 0.017857157875$

Find the definite integral $\int_0^{\pi} \sin(x+5)dx.$

a) Using Trapezoidal Rule with $n = 4$,

approximate value of $\int_0^{\pi} \sin(x+5)dx = 0.537857230583$

b) Using Simpson's Rule with $n = 4$,

approximate value of $\int_0^{\pi} \sin(x+5)dx = 0.568617801099$

Find the definite integral $\int_{3}^{4} \cos^2(4x)dx.$

Note: $\int_{3}^{4} \cos^2(4x)dx = \int_{3}^{4} \left[\cos(4x)\right]^2 dx.$

a) Using Trapezoidal Rule with $n = 400$,

approximate value of $\int_{3}^{4} \cos^2(4x)dx. =$ 0.591059779758

b) Using Simpson's Rule with $n = 400$,

approximate value of $\int_{3}^{4} \cos^2(4x)dx. =$ 0.591062815286