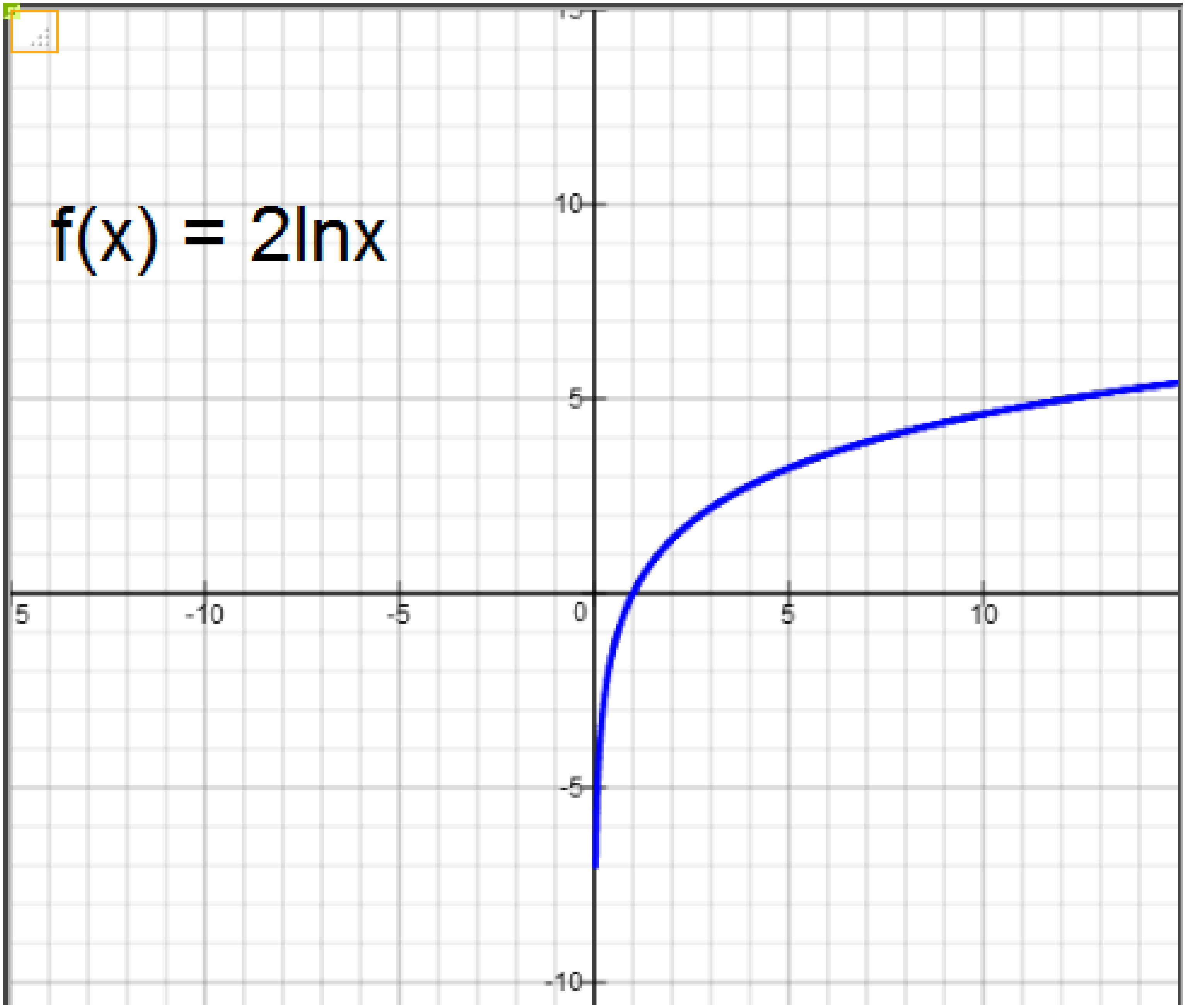
$$f(x) = 2\ln x$$

a) Domain of $f(x) = \{x : x > 0\}$

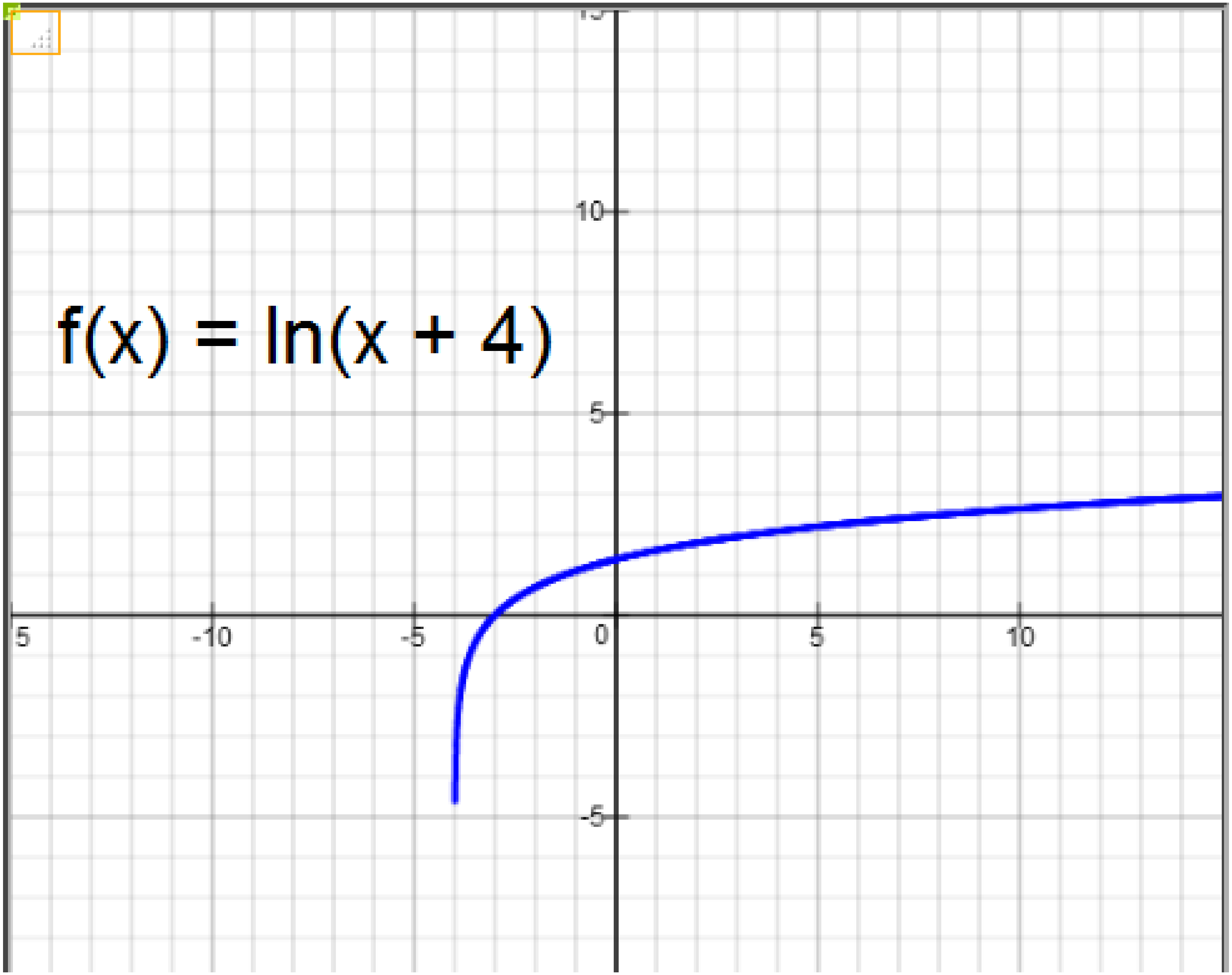
b) Range of $f(x) = (-\infty, \infty)$


f(x) = 2lnx
15
10
5
-5
-10
5
-10
-5
0
5
10

$$f(x) = \ln\left(x+4\right)$$

a) Domain of $f(x) = \left\{x : x+4 > 0\right\} = \left\{x : x > -4\right\}$

b) Range of $f(x) = \left(-\infty, \infty\right)$


f(x) = ln(x + 4)
10
5
-5
5
-10
-5
0
5
10

Properties of Logarithm:

Product Rule: $\ln(A \cdot B) = \ln A + \ln B$

Quotient Rule: $\ln(A / B) = \ln A - \ln B$

Power Rule: $\ln(x^p) = p \ln x$

Product Rule: $\ln\left(A \cdot B\right) = \ln A + \ln B$

Expand $\ln\left(4 \cdot x\right)$.

$$\ln\left(4 \cdot x\right) = \ln 4 + \ln x$$

Quotient Rule: $\ln\left(A/B\right) = \ln A - \ln B$

Expand $\ln\frac{x}{5}$.

$$\ln\frac{x}{5} = \ln x - \ln 5$$

Expand $\ln\frac{ab}{z}$.

$\boxed{\begin{array}{ll}\text{Properties of Logarithm:} & \\ \text{Product Rule:} & \ln(A\cdot B) = \ln A + \ln B \\ \text{Quotient Rule:} & \ln(A/B) = \ln A - \ln B \\ \text{Power Rule:} & \ln(x^p) = p\ln x\end{array}}$

$$\ln\frac{ab}{z} = \ln(ab) - \ln(z) = \ln(a) + \ln(b) - \ln(z)$$

Expand $\ln\left(x^2\sqrt{x^4+5}\right)$.

| |
|---|
| Properties of Logarithm: |
| Product Rule: $\ln(A \cdot B) = \ln A + \ln B$ |
| Quotient Rule: $\ln(A / B) = \ln A - \ln B$ |
| Power Rule: $\ln\left(x^p\right) = p \ln x$ |

$$\ln\left(x^2\sqrt{x^4+5}\right) = \ln\left(x^2\right) + \ln\left(\sqrt{x^4+5}\right)$$

$$= \ln\left(x^2\right) + \ln\left(\left[x^4+5\right]^{1/2}\right) \quad = 2\ln(x) + \frac{1}{2}\ln\left(x^4+5\right)$$

Expand $\ln\left(\sqrt{\dfrac{2x-4}{x}}\right) = \ln\left(\dfrac{2x-4}{x}\right)^{1/2}$

| |
|---|
| Properties of Logarithm: |
| Product Rule: $\ln(A \cdot B) = \ln A + \ln B$ |
| Quotient Rule: $\ln(A / B) = \ln A - \ln B$ |
| Power Rule: $\ln(x^p) = p \ln x$ |

$$\ln\left(\sqrt{\frac{2x-4}{x}}\right) = \ln\left(\frac{2x-4}{x}\right)^{1/2} = \frac{1}{2}\ln\left(\frac{2x-4}{x}\right)$$

$$= \frac{1}{2}\left[\ln(2x-4) - \ln x\right] = \frac{1}{2}\ln(2x-4) - \frac{1}{2}\ln(x)$$

Combine: $\ln(x-4)-\ln(x+5)$.

Properties of Logarithm:

Product Rule: $\ln(A\cdot B) = \ln A + \ln B$

Quotient Rule: $\ln(A/B) = \ln A - \ln B$

Power Rule: $\ln(x^p) = p\ln x$

$$\ln(x-4)-\ln(x+5)=\ln\left(\frac{x-4}{x+5}\right)$$

Combine: $2\ln(x^2+5)+\ln x-\ln(x-11)$

| Properties of Logarithm: | |
|---|---|
| Product Rule: | $\ln\left(A\cdot B\right) = \ln A+\ln B$ |
| Quotient Rule: | $\ln\left(A/B\right) = \ln A-\ln B$ |
| Power Rule: | $\ln\left(x^p\right)=p\ln x$ |

$$2\ln(x^2+5)+\ln x-\ln(x-11)=\ln(x^2+5)^2+\ln x-\ln(x-11)$$

$$=\ln\left(\frac{(x^2+5)^2\cdot x}{x-11}\right)$$

Combine: $2\ln 7-\frac{1}{3}\ln(x^4+1)$.

| |
|---|
| Properties of Logarithm: |
| Product Rule: $\ln\left(A\cdot B\right)=\ln A+\ln B$ |
| Quotient Rule: $\ln\left(A/B\right)=\ln A-\ln B$ |
| Power Rule: $\ln\left(x^p\right)=p\ln x$ |

$$2\ln 7-\frac{1}{3}\ln(x^4+1)=\ln 7^2-\ln(x^4+1)^{1/3}$$

$$=\ln 49-\ln(x^4+1)^{1/3}=\ln\left(\frac{49}{(x^4+1)^{1/3}}\right)=\ln\left(\frac{49}{\sqrt[3]{x^4+1}}\right)$$

# Derivative of function with logarithm

The Fundamental Theorem of Calculus:

$\int_a^b f(x)dx = F(b) - F(a)$ where $F(x)$ is antiderivative of $f(x)$.

The Second Fundamental Theorem of Calculus: $\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$

Example: Find $\frac{d}{dx}\left[\int_a^x (4t+1)\,dt\right]$. Note: The integrand $f(t) = 4t+1$

$$\int_a^x (4t+1)\,dt = 4\left(\frac{1}{2}t^2\right) + t\Big|_a^x = 4\left(\frac{1}{2}x^2\right) + x - 4\left(\frac{1}{2}a^2\right) + a = 2x^2 + x - 2a^2 - a$$

$$\frac{d}{dx}\left[\int_a^x (4t+1)\,dt\right] = \frac{d}{dx}\left[2x^2 + x - 2a^2 - a\right] = 4x + 1 = f(x)$$

The Second Fundamental Theorem of Calculus: $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$

Example: Find $\frac{d}{dx}\left[\int_a^x \left(t^3+4t\right)dt\right]$. Note: The integrand $f(t)=t^3+4t$

$$\int_a^x \left(t^3+4t\right)dt = \frac{1}{4}t^4+4\left(\frac{1}{2}t^2\right)\Bigg|_a^x = \frac{1}{4}x^4+4\left(\frac{1}{2}x^2\right)-\frac{1}{4}a^4+4\left(\frac{1}{2}a^2\right) = \frac{1}{4}x^4+2x^2-\frac{1}{4}a^4-2a^2$$

$$\frac{d}{dx}\left[\int_a^x \left(4t+1\right)dt\right] = \frac{d}{dx}\left[\frac{1}{4}x^4+2x^2-\frac{1}{4}a^4-2a^2\right] = \frac{1}{4}\left(4x^3\right)+2\left(2x\right)-0-0 = x^3+4x = f(x)$$

Note:

$\ln 1 = 0$ and $\int_1^1 \frac{1}{t}\,dt = 0$ Using Trapezoid's Rule

$\ln 2 = 0.693147181$ and $\int_1^2 \frac{1}{t}\,dt = 0.693147181$ Using Trapezoid's Rule

$\ln 3 = 1.098612289$ and $\int_1^3 \frac{1}{t}\,dt = 1.098612289$ Using Trapezoid's Rule

$\ln 4 = 1.386294361$ and $\int_1^4 \frac{1}{t}\,dt = 1.386294361$ Using Trapezoid's Rule

So $\ln x$ is defined as follows: $\ln x = \int_1^x \frac{1}{t}\,dt$

The Second Fundamental Theorem of Calculus: $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$

Hence, $\frac{d}{dx}[\ln x] = \frac{d}{dx}\left[\int_1^x \frac{1}{t}\,dt\right] = \frac{1}{x}$

Chain Rule: $\frac{d}{dx}\left[\ln(\text{expression})\right] = \frac{1}{\text{expression}} \cdot D_x(\text{expression})$

# Derivative of function with logarithm.

Chain Rule: $f(x) = Ln(\text{expression})$

$$f'(x) = \frac{1}{\text{expression}} \cdot D_x(\text{expression})$$

$$f(x) = \ln(7x)$$

$$f'(x) = \frac{1}{7x} \cdot D_x(7x) \ = \frac{1}{7x} \cdot 7 = \frac{1}{x}$$

Note: $D_x\left(\ln x\right) = \dfrac{1}{x}$

Review:

$(\ln x)^3$ is not the same as $\ln x^3 = 3\ln x$

$(\ln x)^4$ is not the same as $\ln x^4 = 4\ln x$

$(\ln x)^5$ is not the same as $\ln x^5 = 5\ln x$

$(\ln x)^6$ is not the same as $\ln x^6 = 6\ln x$

$(\ln x)^7$ is not the same as $\ln x^7 = 7\ln x$

$(\ln x)^8$ is not the same as $\ln x^8 = 8\ln x$

Let $f(x) = (\ln x)^3$. Find $f'(x) =$ ___?___

Power Rule for derivative: $f(x) = (base)^n$

$$f'(x) = n(base)^{n-1} \cdot D_x(base)$$

$$f(x) = (\ln x)^3$$

Note: $(\ln x)^3$ is not the same as $\ln x^3 = 3\ln x$

$$f'(x) = 3(\ln x)^2 \cdot D_x(\ln x) = 3(\ln x)^2 \cdot \left(\frac{1}{x}\right)$$

$$= \frac{3(\ln x)^2}{x}$$

Let $f(x) = \ln\left[2x \cdot \sqrt{x^2 - 14}\right]$. Find $f'(x) =$ ___?___

Chain Rule: $f(x) = Ln(\text{expression})$

$$\Rightarrow f'(x) = \frac{1}{\text{expression}} \cdot D_x(\text{expression})$$

$$f(x) = \ln\left[2x \cdot \sqrt{x^2 - 14}\right] = \ln[2x] + \ln\left[\sqrt{x^2 - 14}\right]$$

$$= \ln[2x] + \ln\left[\left(x^2 - 14\right)^{1/2}\right] = \ln[2x] + \frac{1}{2}\ln\left[x^2 - 14\right]$$

$$f'(x) = \frac{1}{2x} D_x\left(2x\right) + \frac{1}{2} \cdot \left[\frac{1}{x^2 - 14} D_x\left(x^2 - 14\right)\right]$$

$$f'(x) = \frac{1}{2x}\left(2\right) + \frac{1}{2} \cdot \left[\frac{1}{x^2 - 14}\left(2x\right)\right] = \frac{1}{x} + \frac{x}{x^2 - 14}$$

Let $f(x) = \ln\left(\dfrac{\ln x}{x^4}\right)$. Find $f'(x) = \underline{\quad ? \qquad\qquad}$

Chain Rule: $f(x) = Ln(\text{expression})$

$$\Rightarrow f'(x) = \frac{1}{\text{expression}} \cdot D_x(\text{expression})$$

$$f(x) = \ln\left(\frac{\ln x}{x^4}\right) = \ln\left(\ln x\right) - \ln\left(x^4\right) = \ln\left(\ln x\right) - 4\ln\left(x\right)$$

$$f'(x) = \frac{1}{\ln x} D_x\left(\ln x\right) - 4\left[\frac{1}{x}\right] = \frac{1}{\ln x}\left(\frac{1}{x}\right) - 4\left[\frac{1}{x}\right]$$

$$= \frac{1}{x\ln x} - \frac{4}{x}$$

Let $y = \ln\sqrt{\dfrac{x+2}{x-4}}$. Find $y' = \underline{\quad ? \qquad\qquad\qquad\qquad}$

$$Note: \ln\sqrt{\frac{x+2}{x-4}} = \ln\left(\frac{x+2}{x-4}\right)^{1/2} = \frac{1}{2}\left[\ln(x+2) - \ln(x-4)\right]$$

$$y' = \frac{1}{2}\left[\frac{1}{x+2} D_x\left(x+2\right) - \frac{1}{x-4} D_x\left(x-4\right)\right]$$

$$y' = \frac{1}{2}\left[\frac{1}{x+2}(1) - \frac{1}{x-4}(1)\right] = \frac{1}{2}\left[\frac{1}{x+2} - \frac{1}{x-4}\right]$$

Let $f(x) = \ln x^2 + 2$. Find Equation of Tangent Line at (1,2)

Note: $f(x) = \ln x^2 + 2 = 2\ln x + 2$

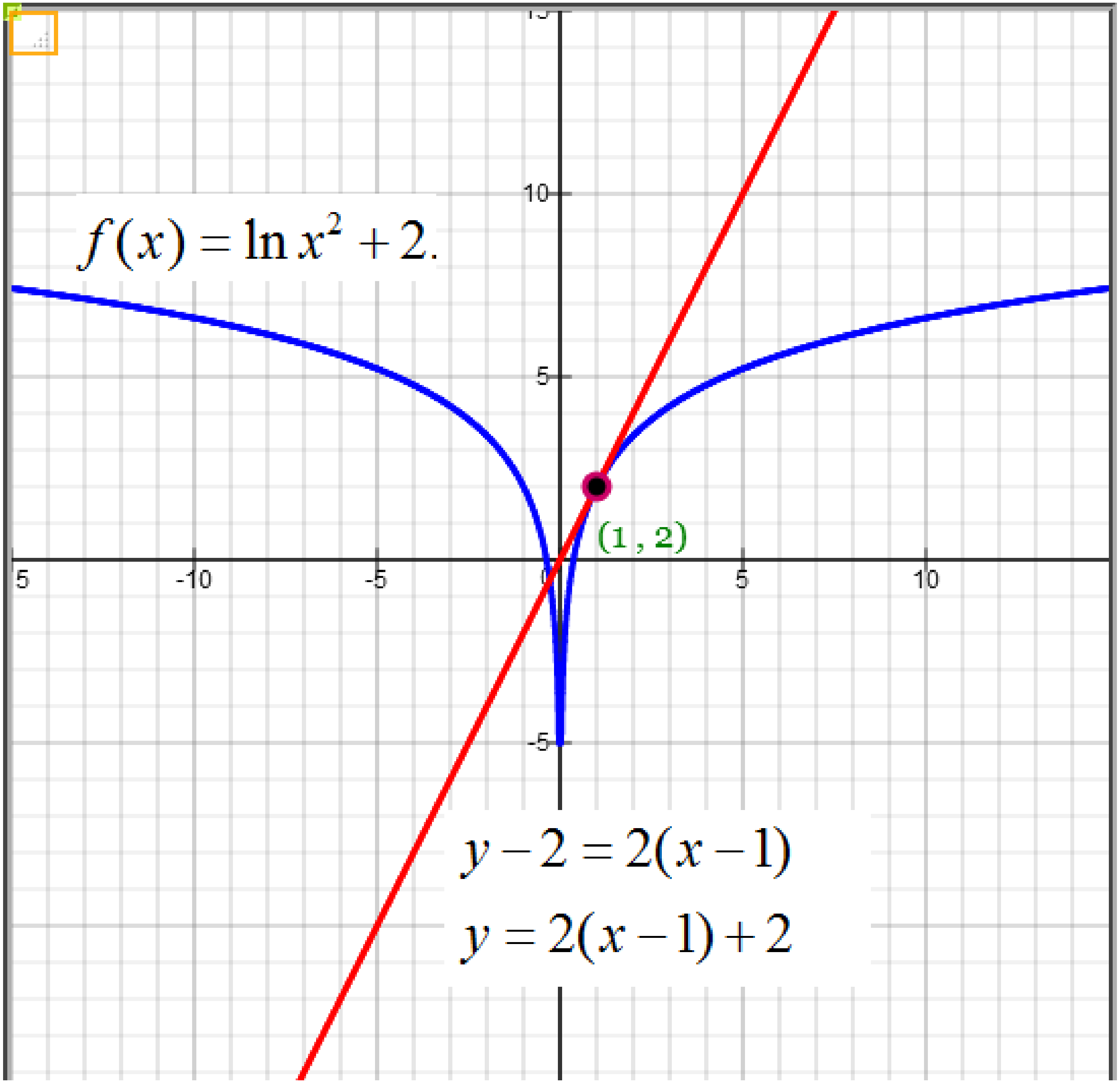
a) Find $f'(x) = 2\left(\dfrac{1}{x}\right) = \dfrac{2}{x}$

b) Slope of Tangent Line = $f'(1) = \dfrac{2}{x} = \dfrac{2}{1} = 2$

c) Equation of Tangent Line at (1,0): $y - y_1 = m(x - x_1)$

$$y - 2 = 2(x - 1)$$

$$y = 2(x - 1) + 2$$



$f(x) = \ln x^2 + 2.$
10
5
-5
5
-10
-5
5
10
(1 , 2)
$y - 2 = 2(x - 1)$
$y = 2(x - 1) + 2$

Let $f(x) = \ln\sqrt{1+\sin x}$. Find Tangent Line at $(\pi, 0)$

Note: $\ln\sqrt{1+\sin x} = \ln(1+\sin x)^{1/2} = \frac{1}{2}\ln(1+\sin x)$

a) Find $f'(x) = \frac{1}{2}\left[\frac{1}{1+\sin x}\right] D_x(1+\sin x) = \frac{1}{2}\left[\frac{1}{1+\sin x}\right](\cos x)$

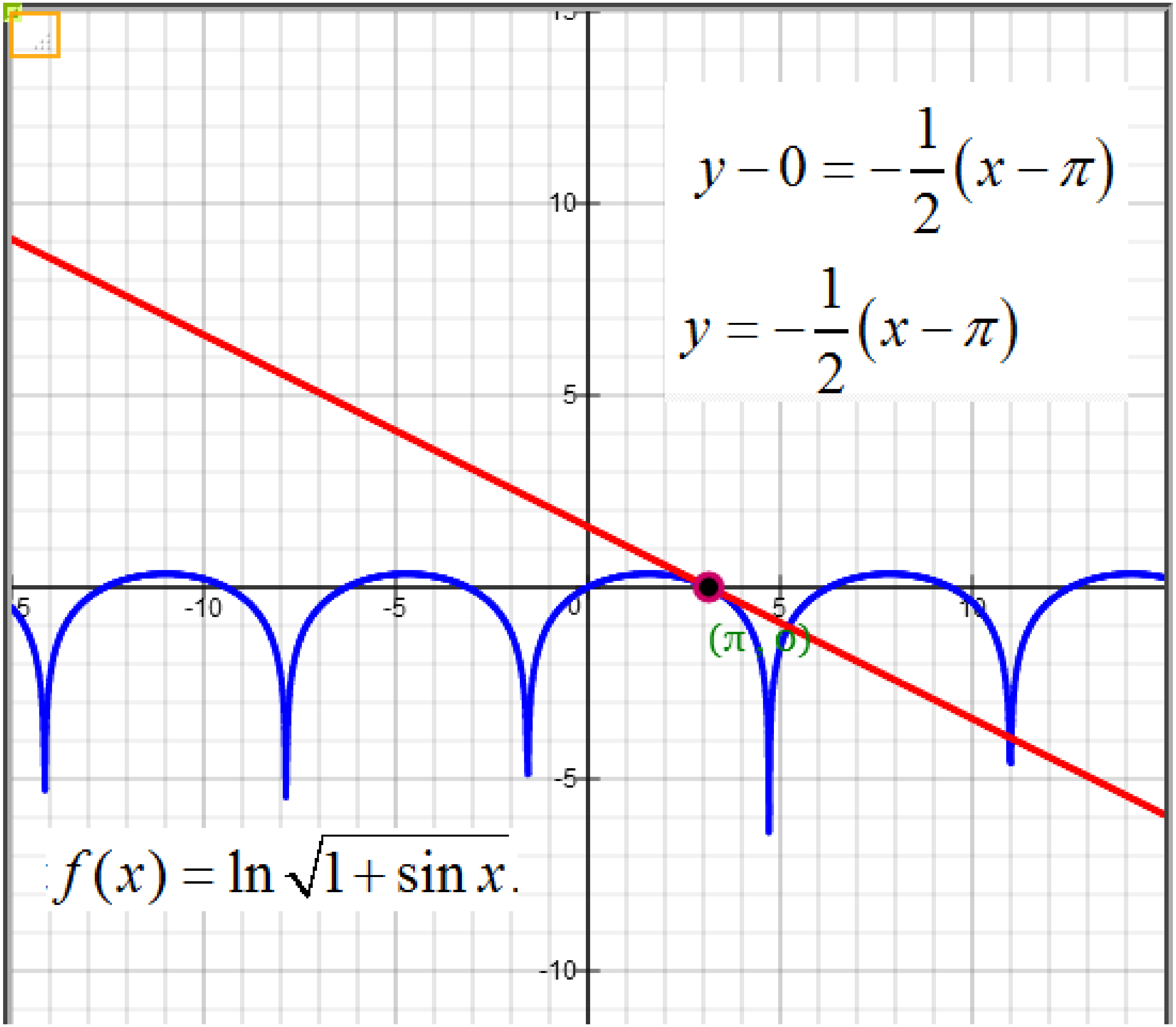
b) Slope of Tangent Line = $f'(\pi) = \frac{1}{2}\left[\frac{1}{1+\sin\pi}\right](\cos\pi) = -\frac{1}{2}$

Note: $\sin\pi = 0; \quad \cos\pi = -1$

c) Equation of Tangent Line: $y - y_1 = m(x - x_1)$

$$y - 0 = -\frac{1}{2}(x - \pi)$$

$$y = -\frac{1}{2}(x - \pi)$$



$y - 0 = -\frac{1}{2}(x - \pi)$
$y = -\frac{1}{2}(x - \pi)$
$(\pi, 0)$
$f(x) = \ln\sqrt{1 + \sin x}$.
10
5
-5
-10
-10
-5
0
5
10

Let $f(x) = 4x^2 - \ln x$. Find Extremum.

a) Find $f'(x) = 8x - \dfrac{1}{x}$

b) Finding relative extremum:

Set $8x - \dfrac{1}{x} = 0$

$$8x \cdot x - \frac{1}{x} \cdot x = 0 \cdot x \quad \text{Multiply each term by } x$$

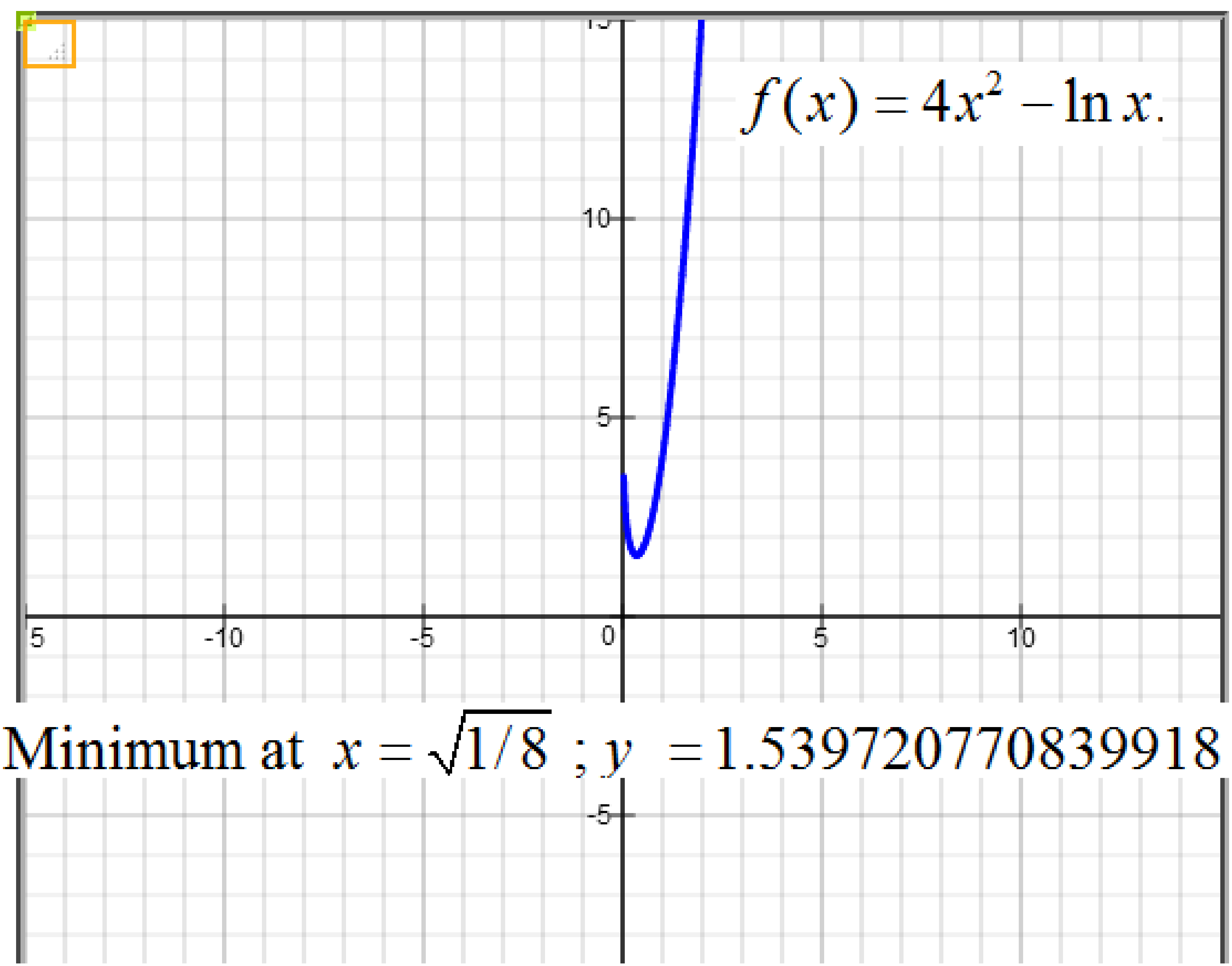
$$8x^2 - 1 = 0$$

$$8x^2 = 1 \quad \Rightarrow \quad x^2 = 1/8 \quad \Rightarrow x = \pm\sqrt{1/8}$$

Therefore, $f(x) = 4x^2 - \ln x$ has minimum at $x = \sqrt{1/8}$.

Minimum at $x = \sqrt{1/8}$ ; $y = 1.539720770839918$

$$f(x) = 4x^2 - \ln x.$$




Minimum at $x = \sqrt{1/8}$ ; $y = 1.53972077083991 8$