$$\int \frac{1}{x}\,dx = \ln|x| + C \qquad \int \frac{1}{ax \pm b}\,dx = \frac{1}{a}\ln|ax \pm b| + C$$

Find $\int \frac{5}{x}\,dx$

$$\int \frac{5}{x}\,dx = \int 5 \cdot \frac{1}{x}\,dx = 5\int \frac{1}{x}\,dx \;\; = 5\left(\ln|x|\right) + C$$

Find $\int \frac{1}{x+12}\,dx$.

Hint: $\int \frac{1}{x}\,dx = \ln|x| + \text{C}$ $\int \frac{1}{ax \pm b}\,dx = \frac{1}{a}\ln|ax \pm b| + \text{C}$

$$\int \frac{1}{x+12}\,dx = \ln|x+12| + \text{C}$$

Find $\int \frac{1}{7x+2}\,dx.$

Hint: $\int \frac{1}{x}\,dx = \ln|x| + C$ $\qquad \int \frac{1}{ax \pm b}\,dx = \frac{1}{a}\ln|ax \pm b| + C$

$$\int \frac{1}{7x+2}\,dx = \frac{1}{7}\ln|7x+2| + C$$

Find $\int \frac{9x}{x^2-15}dx = 9\int \frac{1}{x^2-15} \cdot xdx.$

Hint: $\int \frac{1}{x}dx = \ln|x| + \mathrm{C}$ $\qquad$ $\int \frac{1}{ax \pm b}dx = \frac{1}{a}\ln|ax \pm b| + \mathrm{C}$

Let $u = x^2 - 15$

a) $\frac{du}{dx} = 2x$

b) $du = 2xdx \Rightarrow \quad \frac{1}{2}du = xdx$

c) $\int \frac{9x}{x^2-15}dx = 9\int \frac{1}{x^2-15} \cdot xdx = 9\int \frac{1}{u} \cdot \frac{1}{2}du$

$$= \frac{9}{2}\int \frac{1}{u}du = \frac{9}{2}\ln|u| + C = \frac{9}{2}\ln|x^2 - 15| + C$$

Find $\int \frac{12x^2 - 1}{4x^3 - x}\,dx = \int \frac{1}{4x^3 - x}\left(12x^2 - 1\right)dx.$

Hint: Let $u = 4x^3 - x$

Hint: $\int \frac{1}{x}\,dx = \ln|x| + \mathrm{C}$ $\quad\quad$ $\int \frac{1}{ax \pm b}\,dx = \frac{1}{a}\ln|ax \pm b| + \mathrm{C}$

a) $\frac{du}{dx} = 12x^2 - 1$

b) $du = \left(12x^2 - 1\right)dx$

c) $\int \frac{12x^2 - 1}{4x^3 - x}\,dx = \int \frac{1}{4x^3 - x}\left(12x^2 - 1\right)dx = \int \frac{1}{u}\,du$

$$= \ln|u| + C = \ln\left|4x^3 - x\right| + C$$

Find $\int \frac{x^2-2}{x}\,dx.$

Hint: $\int \frac{1}{x}\,dx = \ln|x| + \text{C} \qquad \int \frac{1}{ax \pm b}\,dx = \frac{1}{a}\ln|ax \pm b| + \text{C}$

$$\int \frac{x^2-2}{x}\,dx = \int\left(\frac{x^2}{x} - \frac{2}{x}\right)dx = \int\left(x - \frac{2}{x}\right)dx$$

$$= \int (x)\,dx - \int\left(2 \cdot \frac{1}{x}\right)dx$$

$$= \int (x)\,dx - 2\int\left(\frac{1}{x}\right)dx$$

$$= \frac{x^2}{2} - 2\ln|x| + C$$

Find $\int \frac{4x^3+4x+5}{x^4+2x^2+5x}dx = \int \frac{1}{x^4+2x^2+5x}(4x^3+4x+5)dx$

Hint: Let $u = x^4+2x^2+5x$

Hint: $\int \frac{1}{x}dx = \ln|x| + \text{C}$ $\qquad \int \frac{1}{ax \pm b}dx = \frac{1}{a}\ln|ax \pm b| + \text{C}$

a) $\frac{du}{dx} = 4x^3+4x+5$

b) $du = \left(4x^3+4x+5\right)dx$

c) $\int \frac{4x^3+4x+5}{x^4+2x^2+5x}dx = \int \frac{1}{x^4+2x^2+5x}(4x^3+4x+5)dx$

$= \int \frac{1}{u}du = \ln|u| + C = \ln\left|x^4+2x^2+5x\right| + C$

Find $\int \frac{4x^2-3x+12}{x+3}\,dx.$

Hint: $\int \frac{1}{x}\,dx = \ln|x| + C$ $\qquad \int \frac{1}{ax \pm b}\,dx = \frac{1}{a}\ln|ax \pm b| + C$

a) Divide $\frac{4x^2-3x+12}{x+3}$:

$$
\begin{array}{r|l}
 & 4x - 15 \\
\hline
x+3 & 4x^2 - 3x + 12 \\
 & \underline{4x^2 + 12x \qquad} \\
 & -15x + 12 \\
 & \underline{-15x - 45} \\
 & 57
\end{array}
$$

Hence, $\dfrac{4x^2-3x+12}{x+3}=4x-15+\dfrac{57}{x+3}$

$$\int \frac{4x^2-3x+12}{x+3}\,dx=\int\left(4x-15+\frac{57}{x+3}\right)dx$$

$$=\int(4x)\,dx-\int(15)\,dx+57\int\left(\frac{1}{x+3}\right)dx$$

$$=4\left(\frac{x^2}{2}\right)-15x+57\ln\left|x+3\right|+C$$

$$=2x^2-15x+57\ln\left|x+3\right|+C$$

Find $\int \tan\left(\frac{4x}{3}\right)dx$

Hint: Let $u = \frac{4x}{3} = \frac{4}{3}x$

a) $\frac{du}{dx} = \frac{4}{3}$

b) $du = \frac{4}{3}dx \quad \Rightarrow \quad \frac{3}{4}du = \frac{3}{4}\left(\frac{3}{4}dx\right) \quad \Rightarrow \quad \frac{3}{4}du = dx$

c) $\int \tan\left(\frac{4x}{3}\right)dx = \int \tan(u)\cdot\frac{3}{4}du = \frac{3}{4}\int \tan(u)du$

$$= \frac{3}{4}\left[-\ln\left|\cos(u)\right|\right] + C = -\frac{3}{4}\left[\ln\left|\cos\left(\frac{4x}{3}\right)\right|\right] + C$$

Find $\int(\sin 3x - 11)dx = \int(\sin 3x)dx - \int(11)dx.$

For $\int(\sin 3x)dx$, let $u = 3x$

$$\frac{du}{dx} = 3 \quad \Rightarrow \quad du = 3dx \quad \Rightarrow \quad \frac{1}{3}du = dx$$

$$\int(\sin 3x)dx = \int(\sin u)\cdot\frac{1}{3}du = \frac{1}{3}[-\cos u] + C$$

$$= -\frac{1}{3}\left[\cos(3x)\right] + C$$

$$\int(\sin 3x - 11)dx = \int(\sin 3x)dx - \int(11)dx$$

$$= -\frac{1}{3}\left[\cos(3x)\right] - 11x + C$$