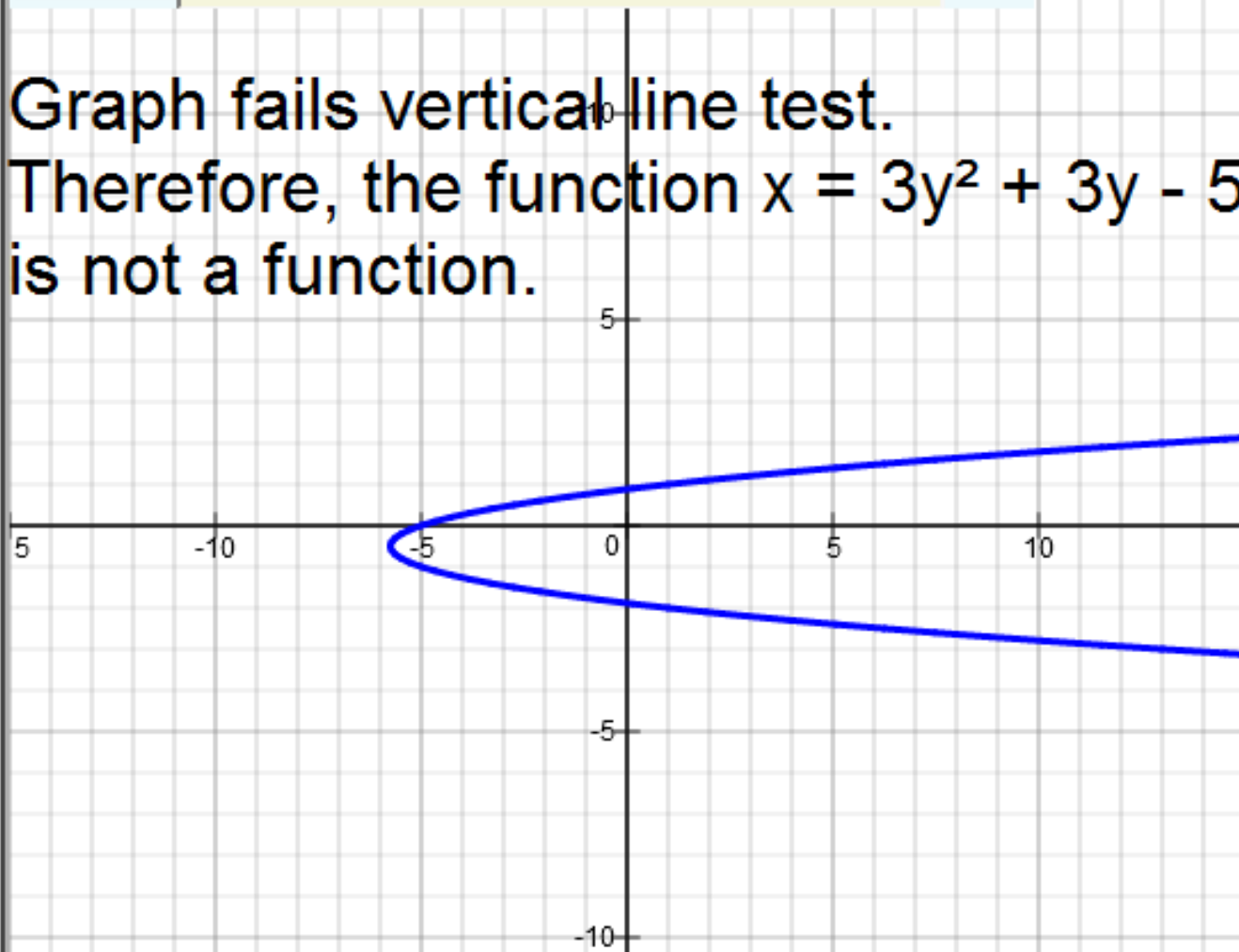


$$x = 3y^2 + 3y - 5$$

1

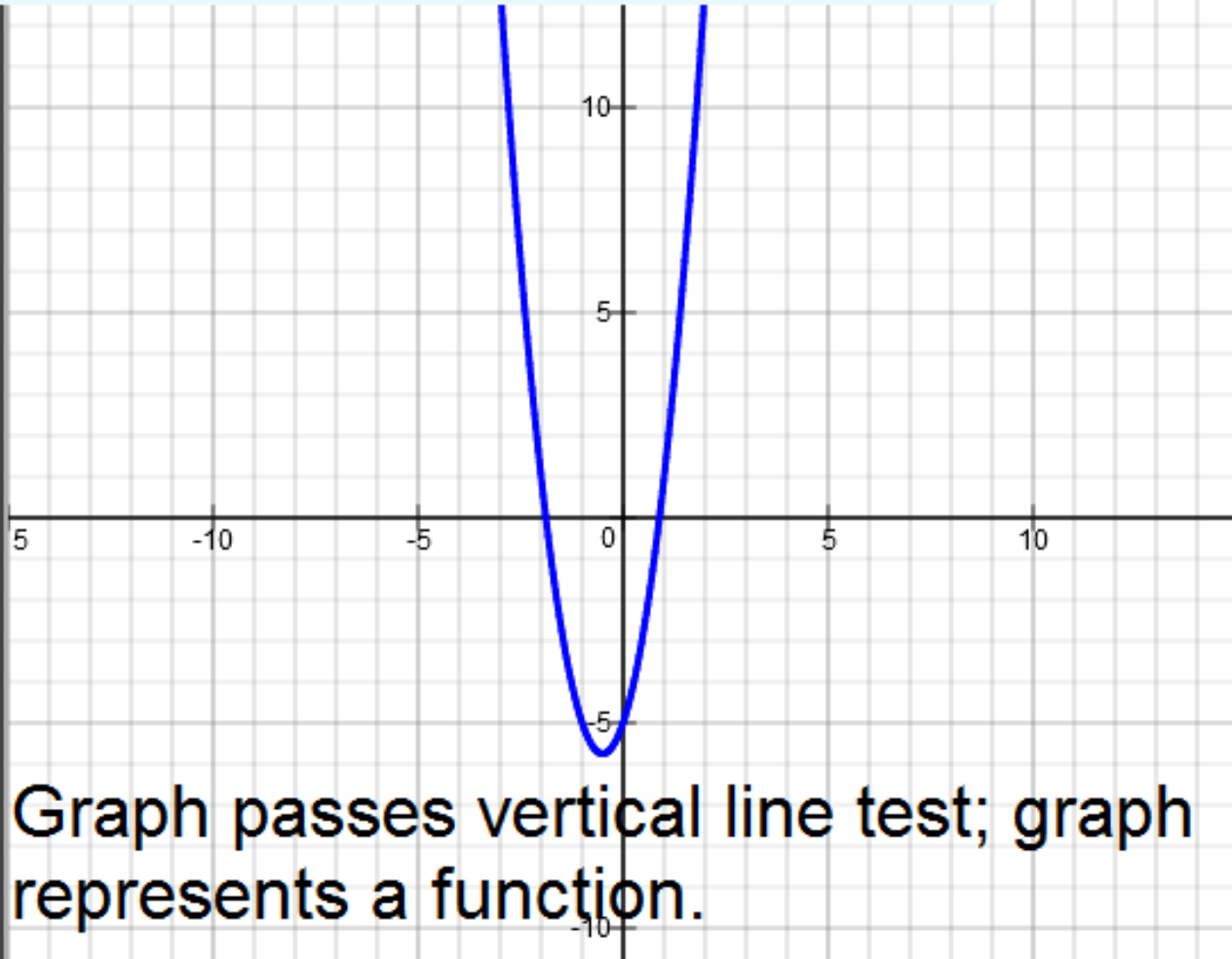
Graph fails vertical line test.

Therefore, the function $x = 3y^2 + 3y - 5$ is not a function.



$$y = 3x^2 + 3x - 5$$

1



However, graph fails horizontal line test.

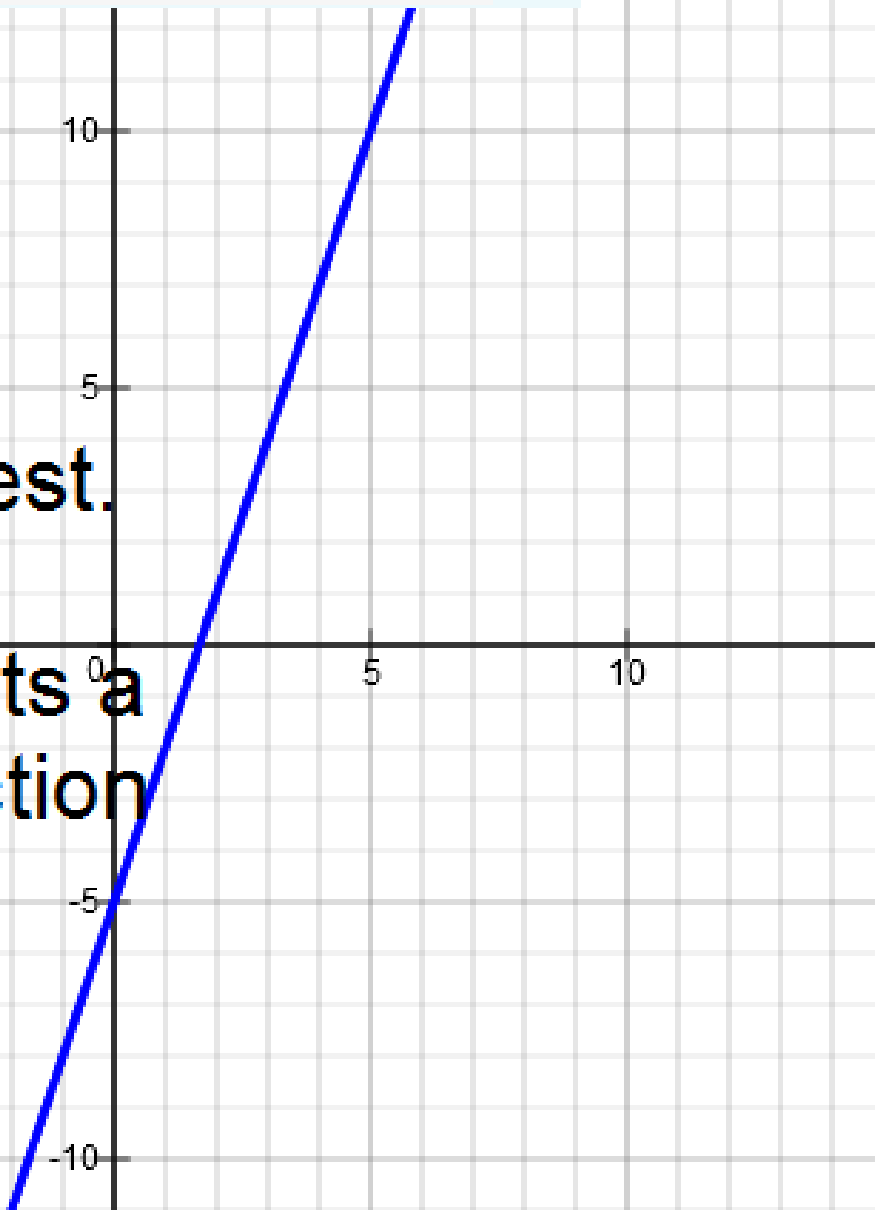
$$y = 3x - 5$$

1

Graph passes
vertical line test.

Graph passes
horizontal line test.

Graph represents a
one-to-one function



Example 1

$$\text{Let } f(x) = \frac{3}{5}x + 4.$$

a) $f(x) = \frac{3}{5}x + 4$ a one-to-one function?

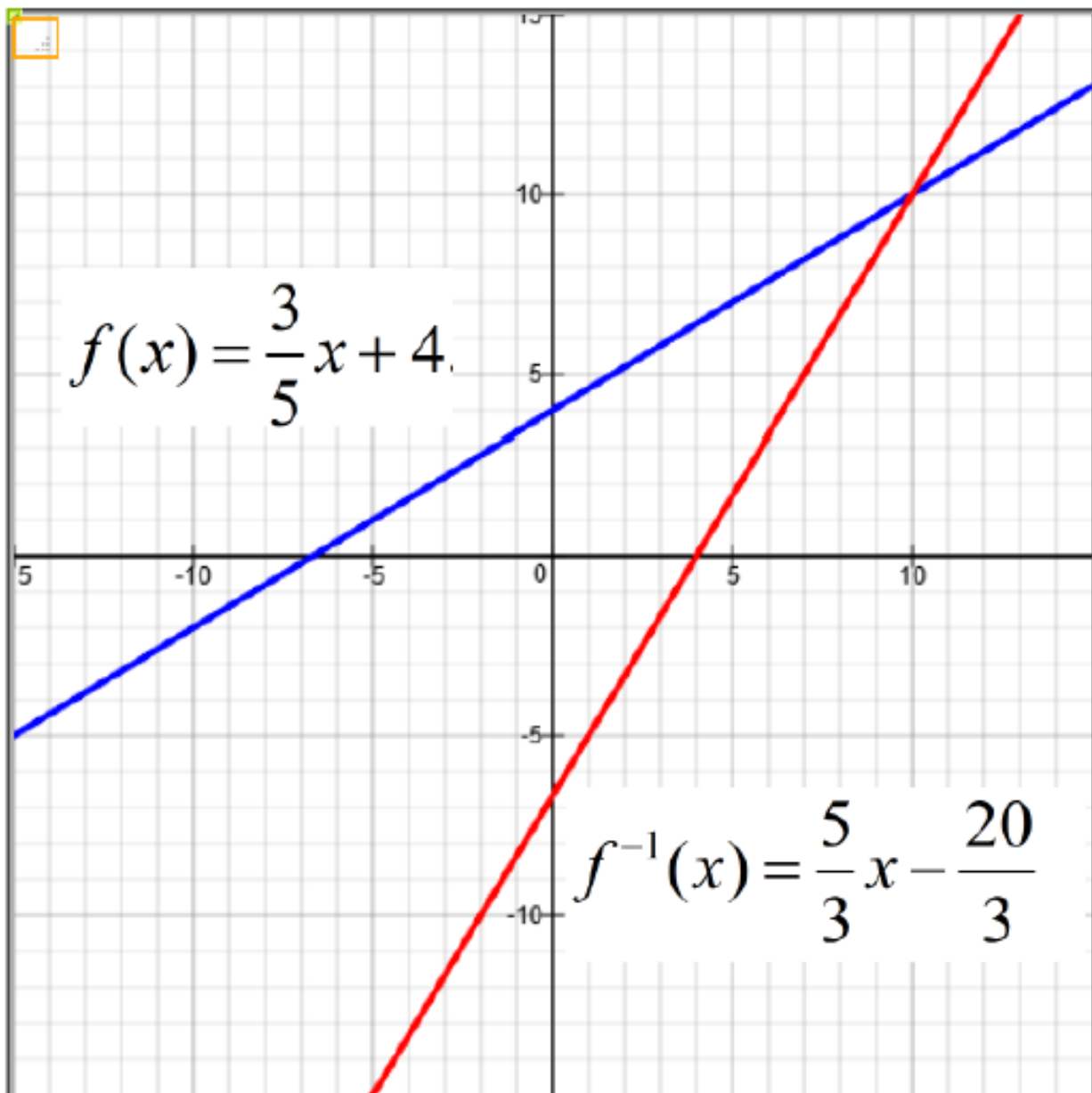
Yes, graph passes horizontal line test.

b) Find $f^{-1}(x)$:

$$x = \frac{3}{5}y + 4 \Rightarrow 5 \cdot x = 5 \cdot \frac{3}{5}y + 5 \cdot 4 \Rightarrow 5x = 3y + 20$$

$$\Rightarrow 3y + 20 = 5x \Rightarrow 3y = 5x - 20 \Rightarrow y = \frac{5}{3}x - \frac{20}{3}$$

$$f^{-1}(x) = \frac{5}{3}x - \frac{20}{3}$$



Let $f(x) = \frac{3}{5}x + 4$.

c) Domain of $f(x) = (-\infty, \infty)$

d) Range of $f(x) = (-\infty, \infty)$

e) Domain of $f^{-1}(x) = (-\infty, \infty)$

f) Range of $f^{-1}(x) = (-\infty, \infty)$

Example 2

$$f(x) = x^2 \quad x \geq 0$$

a) Is $f(x)$ a one-to-one function?

Yes, graph passes horizontal line test.

b) Find $f^{-1}(x)$.

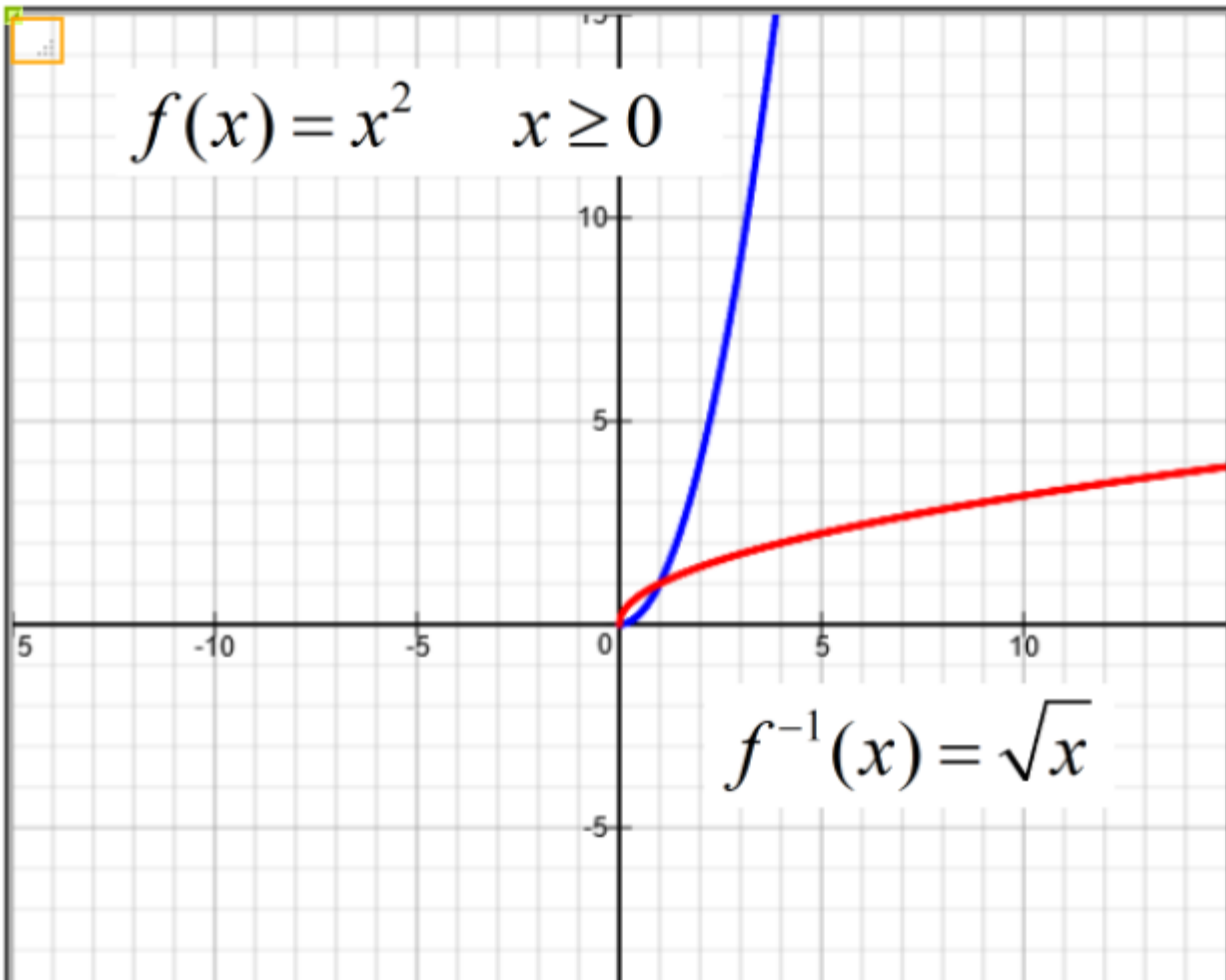
$$\text{Let } y = x^2$$

Exchange x and y :

$$x = y^2 \Rightarrow y^2 = x \Rightarrow y = \pm\sqrt{x}$$

Note that the inverse function is $y = \sqrt{x}$

$$f^{-1}(x) = \sqrt{x}$$



The image shows a coordinate plane with a grid. The x-axis is labeled with values 5, -10, -5, 0, 5, 10. The y-axis is labeled with values 5, 10, 15. A blue curve representing the function $f(x) = x^2$ for $x \geq 0$ starts at the origin (0,0) and curves upwards and to the right. A red curve representing the inverse function $f^{-1}(x) = \sqrt{x}$ also starts at the origin (0,0) and curves upwards and to the right, appearing as a mirror image of the blue curve across the line $y=x$. The two curves intersect at the origin and at the point (1,1). The text $f(x) = x^2 \quad x \geq 0$ is located in the upper left quadrant, and the text $f^{-1}(x) = \sqrt{x}$ is located in the lower right quadrant.

$$f(x) = x^2 \quad x \geq 0$$

$$f^{-1}(x) = \sqrt{x}$$

Let $y = x^2$

Domain of $f(x) = [0, \infty)$

Range of $f(x) = [0, \infty)$

Domain of $f^{-1}(x) = [0, \infty)$

Range of $f^{-1}(x) = [0, \infty)$

Example 3

$$\text{Let } f(x) = \sqrt{x+4}$$

a) Is $f(x)$ a one-to-one function?

Yes, graph passes horizontal line test.

b) Find $f^{-1}(x)$.

$$\text{Let } y = \sqrt{x+4}$$

Exchange x and y :

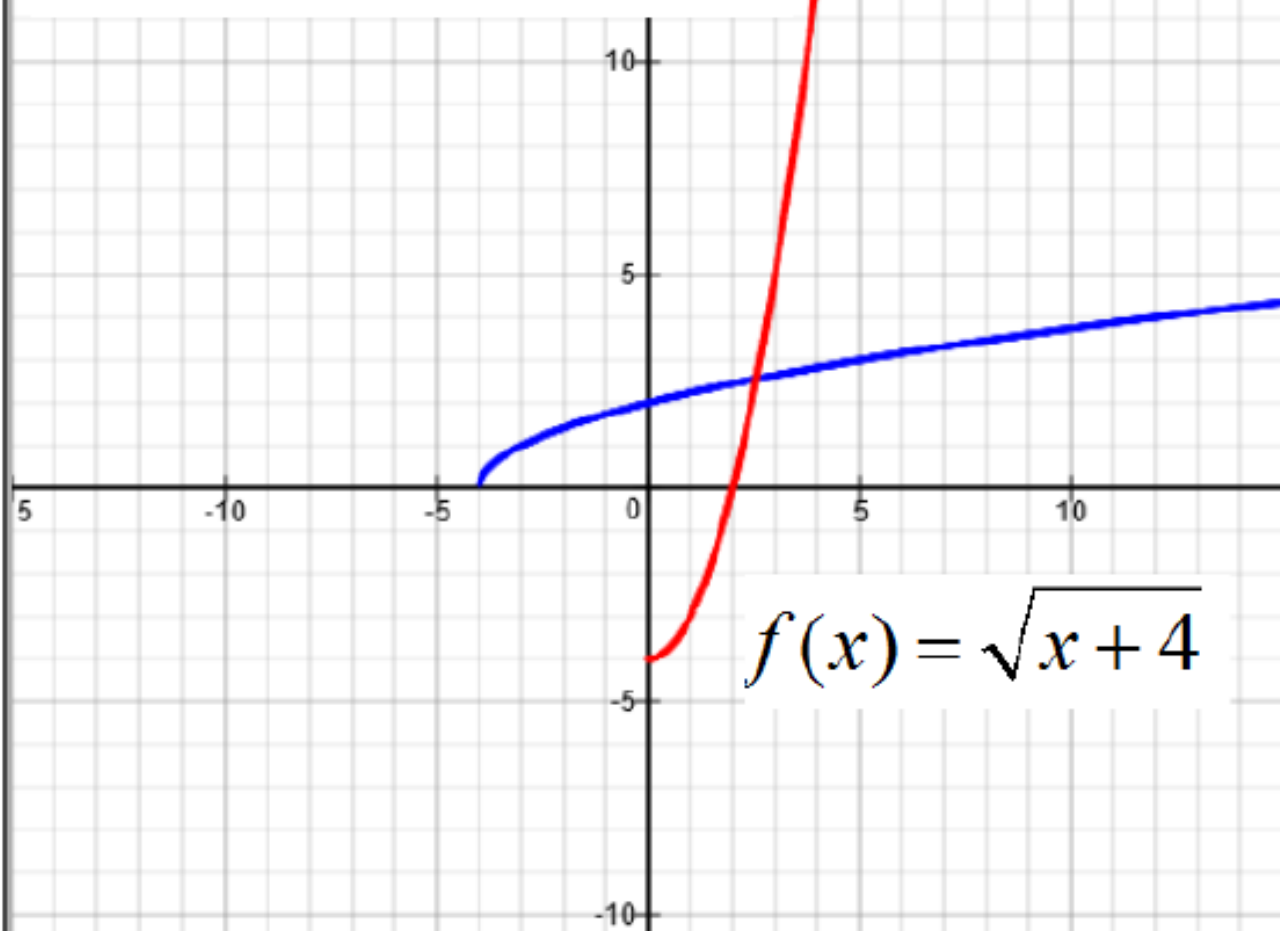
$$x = \sqrt{y+4} \Rightarrow \sqrt{y+4} = x \Rightarrow (\sqrt{y+4})^2 = (x)^2$$

$$\Rightarrow y+4 = x^2 \Rightarrow y = x^2 - 4, \quad x \geq 0$$

$$f^{-1}(x) = x^2 - 4, \quad x \geq 0$$



$$f^{-1}(x) = x^2 - 4, \quad x \geq 0$$



$$f(x) = \sqrt{x+4}$$

$$f(x) = \sqrt{x+4}$$

$$f^{-1}(x) = x^2 - 4, \quad x \geq 0$$

$$\text{Domain of } f(x) = [-4, \infty)$$

$$\text{Range of } f(x) = [0, \infty)$$

$$\text{Domain of } f^{-1}(x) = [0, \infty)$$

$$\text{Range of } f^{-1}(x) = [-4, \infty)$$

Example 4

$$f(x) = \sqrt{4-x^2}, \quad x \geq 0$$

a) Is $f(x)$ a one-to-one function?

Yes, graph passes horizontal line test.

b) Find $f^{-1}(x)$.

$$\text{Let } y = \sqrt{4-x^2}$$

Exchange x and y :


$$x = \sqrt{4-y^2} \Rightarrow \sqrt{4-y^2} = x \Rightarrow \left(\sqrt{4-y^2}\right)^2 = (x)^2$$

$$\Rightarrow 4 - y^2 = x^2 \Rightarrow -4 + y^2 = -x^2 \Rightarrow y^2 = 4 - x^2$$

$$\Rightarrow y = \pm\sqrt{4-x^2}$$

Note: Inverse function is $y = \sqrt{4-x^2}, \quad x \geq 0$

$$f^{-1}(x) = \sqrt{4-x^2}, \quad x \geq 0$$


$$f(x) = \sqrt{4 - x^2}, \quad x \geq 0$$

$$f^{-1}(x) = \sqrt{4 - x^2}, \quad x \geq 0$$

5

-10

-5

0

5

10

-5

$$f(x) = \sqrt{4 - x^2}, \quad x \geq 0$$

$$f^{-1}(x) = \sqrt{4 - x^2}, \quad x \geq 0$$

$$\text{Domain of } f(x) = [0, 2]$$

$$\text{Range of } f(x) = [0, 2]$$

$$\text{Domain of } f^{-1}(x) = [0, 2]$$

$$\text{Range of } f^{-1}(x) = [0, 2]$$

Example 5

$$f(x) = \sqrt[3]{3x - 7}$$

a) Is $f(x)$ a one-to-one function?

Yes, graph passes horizontal line test.

b) Find $f^{-1}(x)$.

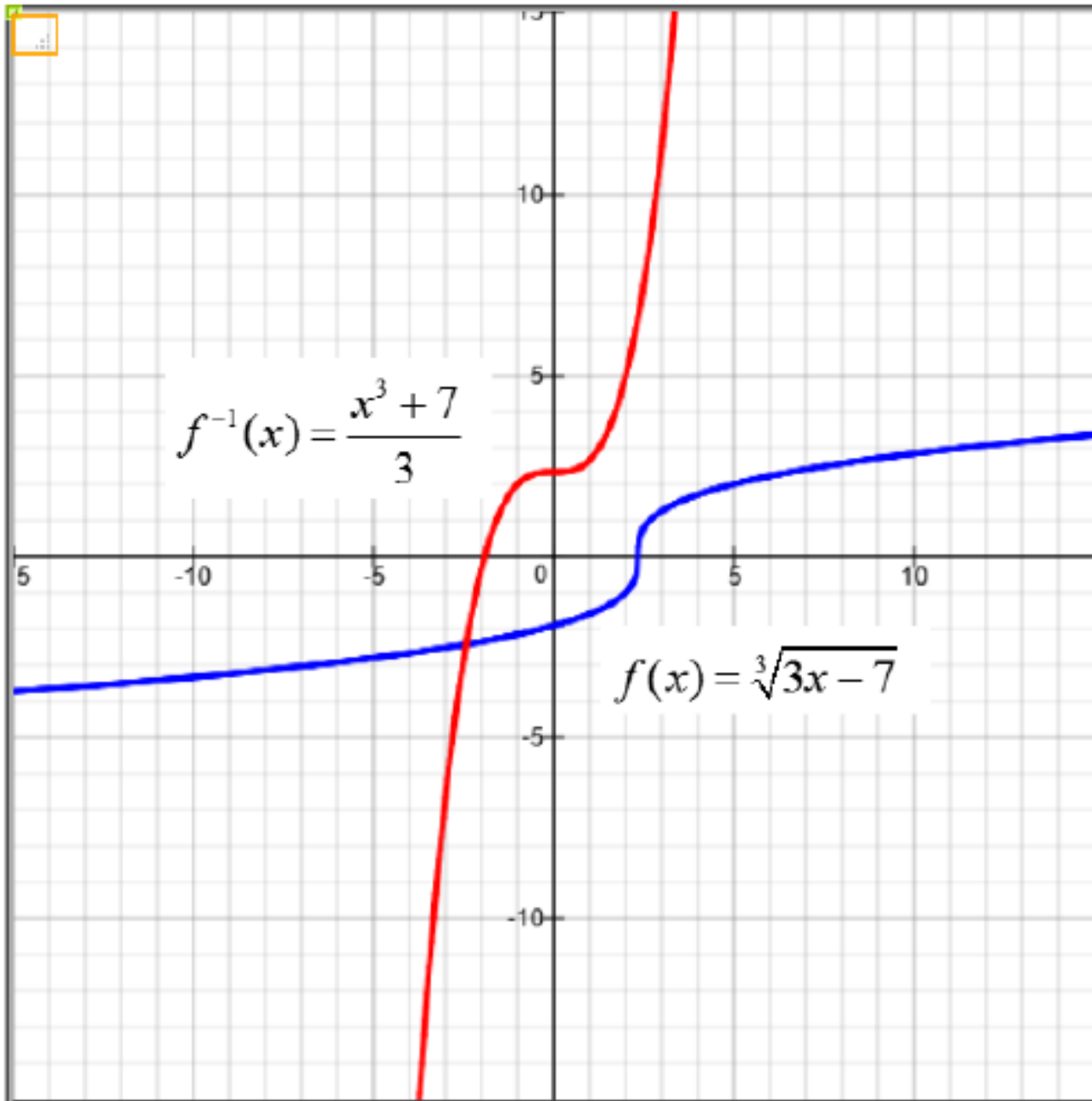
$$\text{Let } y = \sqrt[3]{3x - 7}$$

Exchange x and y :

$$x = \sqrt[3]{3y - 7} \Rightarrow \sqrt[3]{3y - 7} = x \Rightarrow \left(\sqrt[3]{3y - 7}\right)^3 = (x)^3$$

$$\Rightarrow 3y - 7 = x^3 \Rightarrow 3y = x^3 + 7 \Rightarrow y = \frac{x^3 + 7}{3}$$

$$f^{-1}(x) = \frac{x^3 + 7}{3}$$



$$f(x) = \sqrt[3]{3x - 7} \qquad f^{-1}(x) = \frac{x^3 + 7}{3}$$

$$\text{Domain of } f(x) = (-\infty, \infty)$$

$$\text{Range of } f(x) = (-\infty, \infty)$$

$$\text{Domain of } f^{-1}(x) = (-\infty, \infty)$$

$$\text{Range of } f^{-1}(x) = (-\infty, \infty)$$

Example 6

$$f(x) = \cos x \quad x \in [0, \pi]$$

a) Is $f(x)$ a one-to-one function?

Yes, graph passes horizontal line test.

b) Find $f^{-1}(x) = \cos^{-1}(x)$

c) Domain of $f(x) = \{x : 0 \leq x \leq \pi\}$

d) Range of $f(x) = [-1, 1]$

e) Domain of $f^{-1}(x) = [-1, 1]$

f) Range of $f^{-1}(x) = \{x : 0 \leq x \leq \pi\}$

