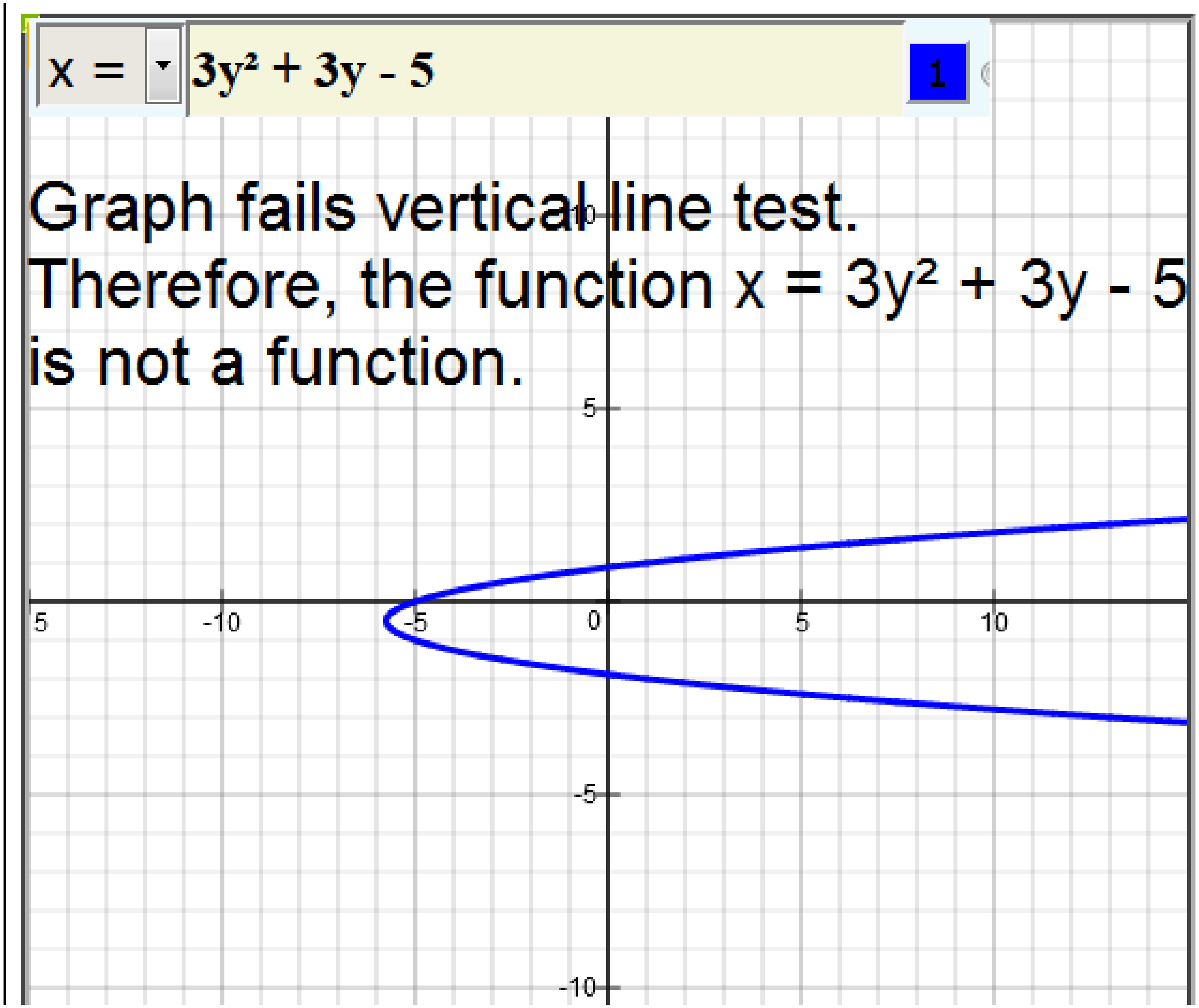
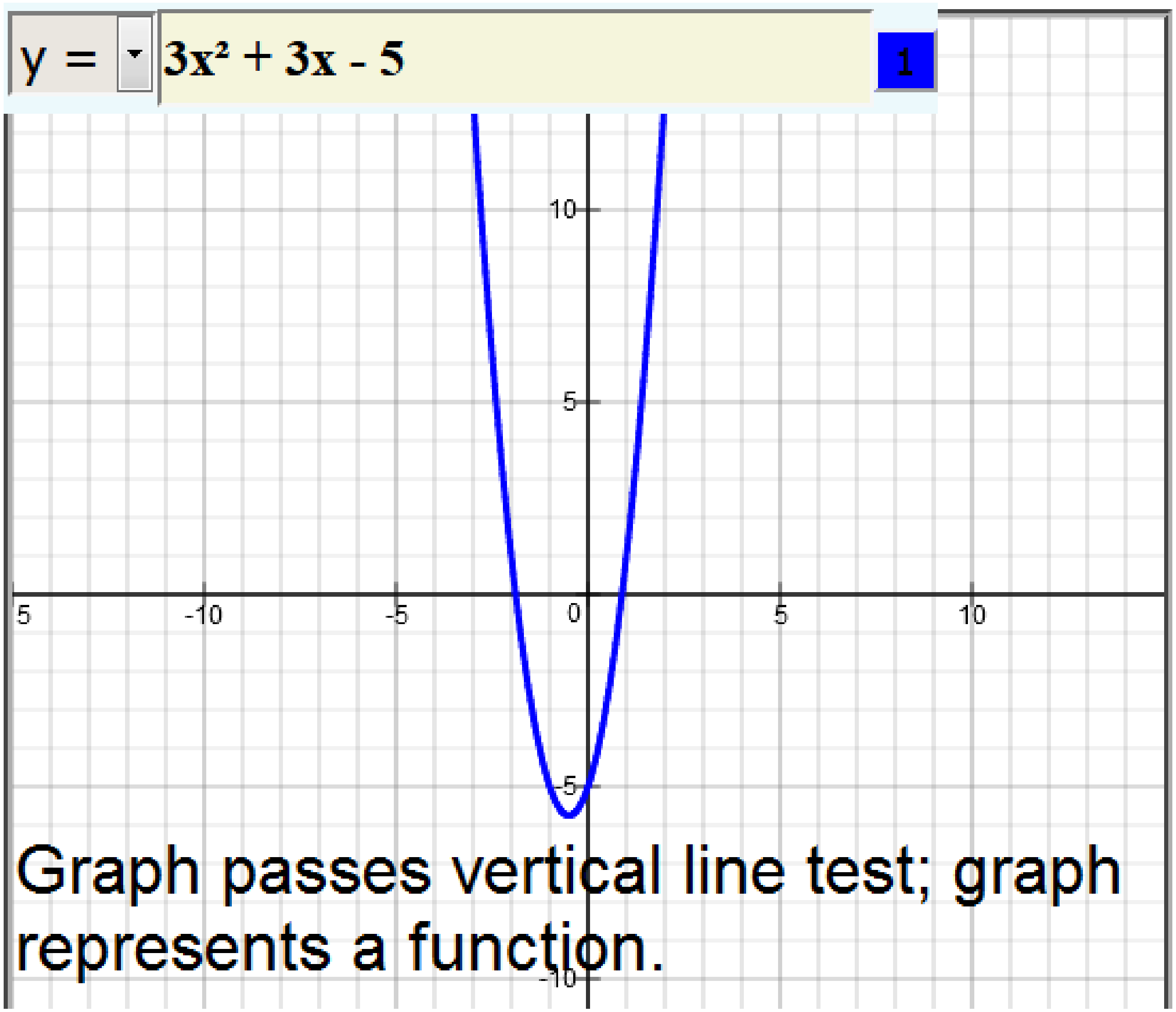
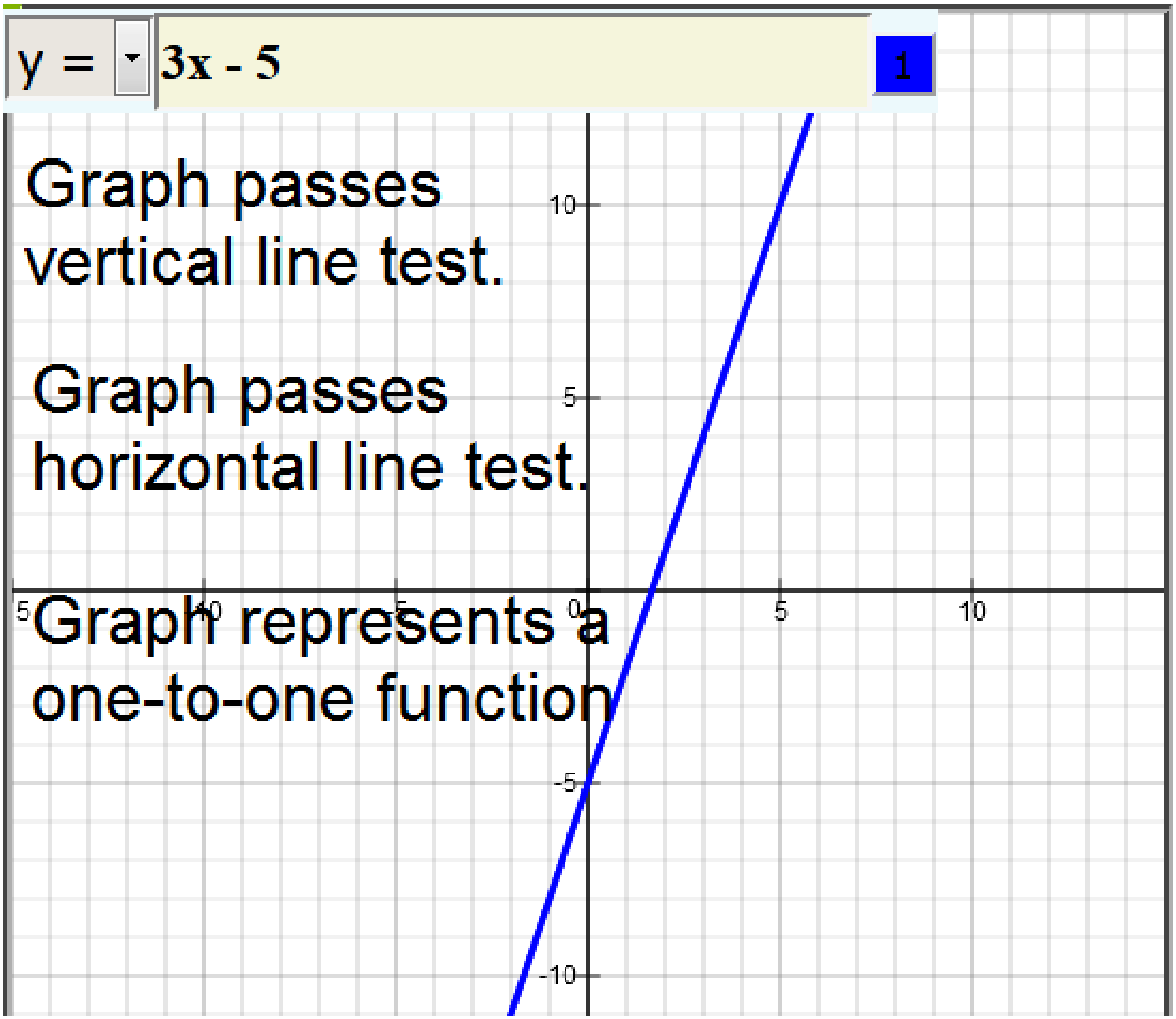

x =
3y² + 3y - 5
1
Graph fails vertical line test.
Therefore, the function x = 3y² + 3y - 5
is not a function.
10
5
0
-5
-10
5
10



However, graph fails horizontal line test.


y = 3x - 5
1
Graph passes vertical line test.
Graph passes horizontal line test.
Graph represents a one-to-one function
10
5
-5
-10
0
5
10

Example 1

Let $f(x) = \frac{3}{5}x + 4.$

a) $f(x) = \frac{3}{5}x + 4$ a one-to-one function?

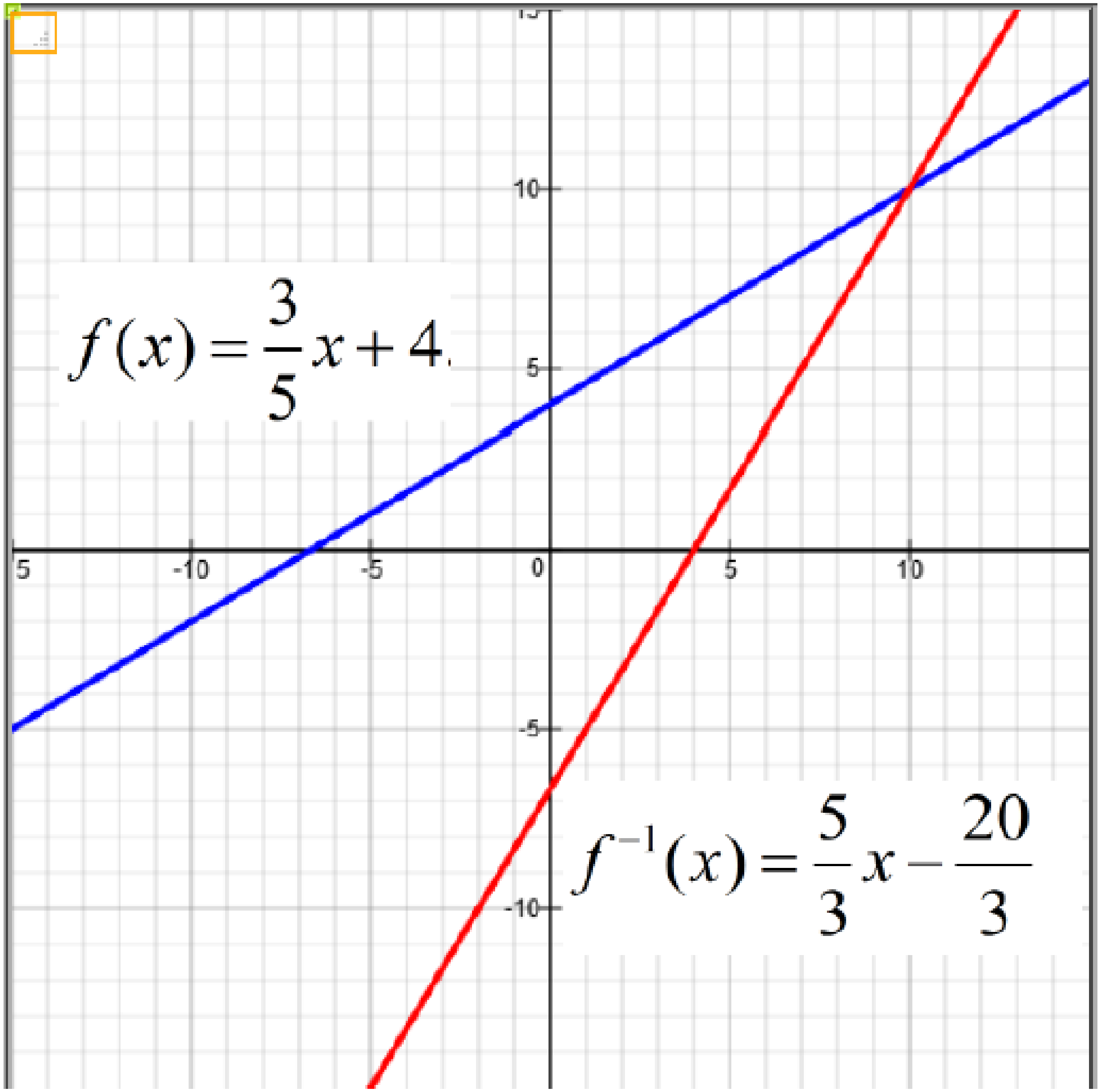
Yes, graph passes horizontal line test.

b) Find $f^{-1}(x)$:

$$x = \frac{3}{5}y + 4 \Rightarrow 5 \cdot x = 5 \cdot \frac{3}{5}y + 5 \cdot 4 \Rightarrow 5x = 3y + 20$$

$$\Rightarrow 3y + 20 = 5x \Rightarrow 3y = 5x - 20 \Rightarrow y = \frac{5}{3}x - \frac{20}{3}$$

$$f^{-1}(x) = \frac{5}{3}x - \frac{20}{3}$$


$f(x)=\frac{3}{5}x+4.$
$f^{-1}(x)=\frac{5}{3}x-\frac{20}{3}$
10
5
-5
-10
5
-10
-5
0
5
10

Let $f(x) = \frac{3}{5}x + 4.$

c) Domain of $f(x) = (-\infty, \infty)$

d) Range of $f(x) = (-\infty, \infty)$

e) Domain of $f^{-1}(x) = (-\infty, \infty)$

f) Range of $f^{-1}(x) = (-\infty, \infty)$

Example 2

$f(x) = x^2 \qquad x \geq 0$

a) Is $f(x)$ a one-to-one function?

Yes, graph passes horizontal line test.

b) Find $f^{-1}(x)$.

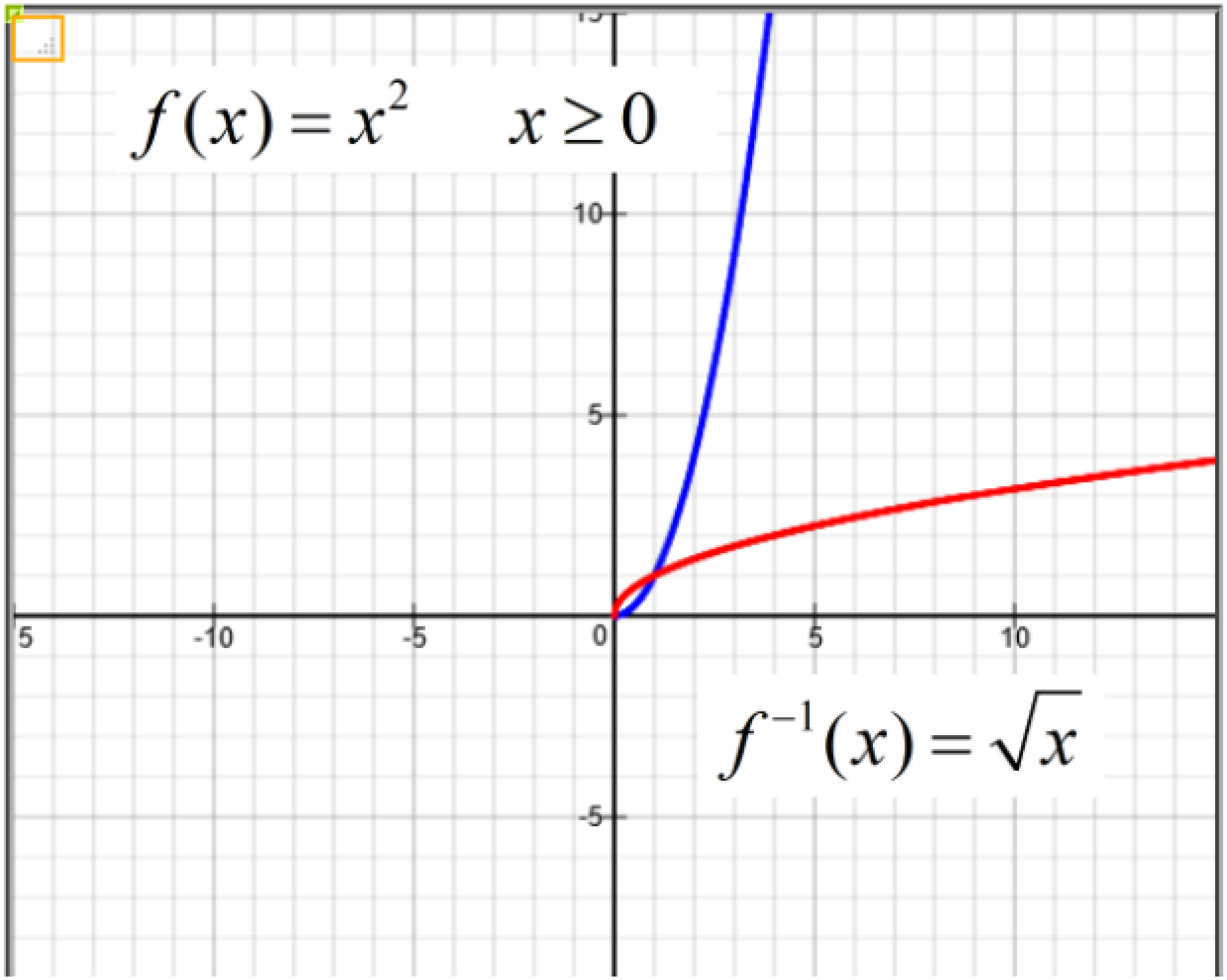
Let $y = x^2$

Exchange x and y:

$x = y^2 \Rightarrow y^2 = x \Rightarrow y = \pm\sqrt{x}$

Note that the inverse function is $y = \sqrt{x}$

$f^{-1}(x) = \sqrt{x}$



$f(x) = x^2 \quad x \geq 0$
$f^{-1}(x) = \sqrt{x}$
10
5
-5
-10
0

Let $y = x^2$

Domain of $f(x) = [0, \infty)$

Range of $f(x) = [0, \infty)$

Domain of $f^{-1}(x) = [0, \infty)$

Range of $f^{-1}(x) = [0, \infty)$

Example 3

Let $f(x) = \sqrt{x+4}$

a) Is $f(x)$ a one-to-one function?

Yes, graph passes horizontal line test.

b) Find $f^{-1}(x)$.

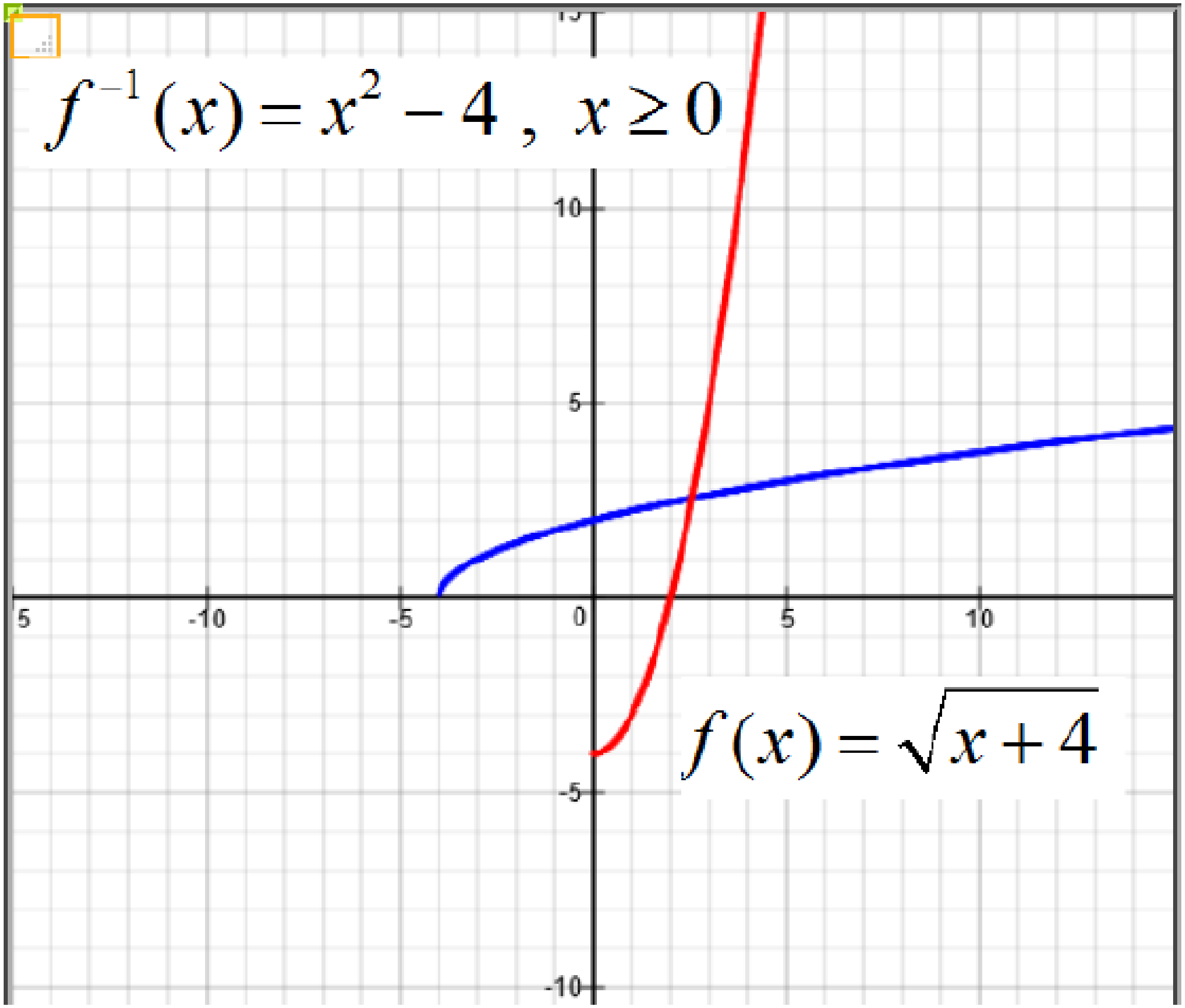
Let $y = \sqrt{x+4}$

Exchange x and y:

$$x = \sqrt{y+4} \Rightarrow \sqrt{y+4} = x \Rightarrow \left(\sqrt{y+4}\right)^2 = (x)^2$$

$$\Rightarrow y + 4 = x^2 \Rightarrow y = x^2 - 4\ ,\ \ x \geq 0$$

$$f^{-1}(x) = x^2 - 4\ ,\ \ x \geq 0$$



$f^{-1}(x) = x^2 - 4\ ,\ x \geq 0$
10
5
5
-10
-5
0
5
10
$f(x) = \sqrt{x+4}$
-5
-10

$$f(x) = \sqrt{x+4}$$

$$f^{-1}(x) = x^2 - 4 \ , \ \ x \geq 0$$

Domain of $f(x) = [-4, \infty)$

Range of $f(x) = [0, \infty)$

Domain of $f^{-1}(x) = [0, \infty)$

Range of $f^{-1}(x) = [-4, \infty)$

Example 4

$f(x)=\sqrt{4-x^2}\quad,\quad x\geq 0$

a) Is $f(x)$ a one-to-one function?

Yes, graph passes horizontal line test.

b) Find $f^{-1}(x)$.

Let $y=\sqrt{4-x^2}$

Exchange x and y:

$x=\sqrt{4-y^2}\Rightarrow\sqrt{4-y^2}=x\Rightarrow\left(\sqrt{4-y^2}\right)^2=(x)^2$

$\Rightarrow\ 4-y^2=x^2\Rightarrow -4+y^2=-x^2\Rightarrow y^2=4-x^2$

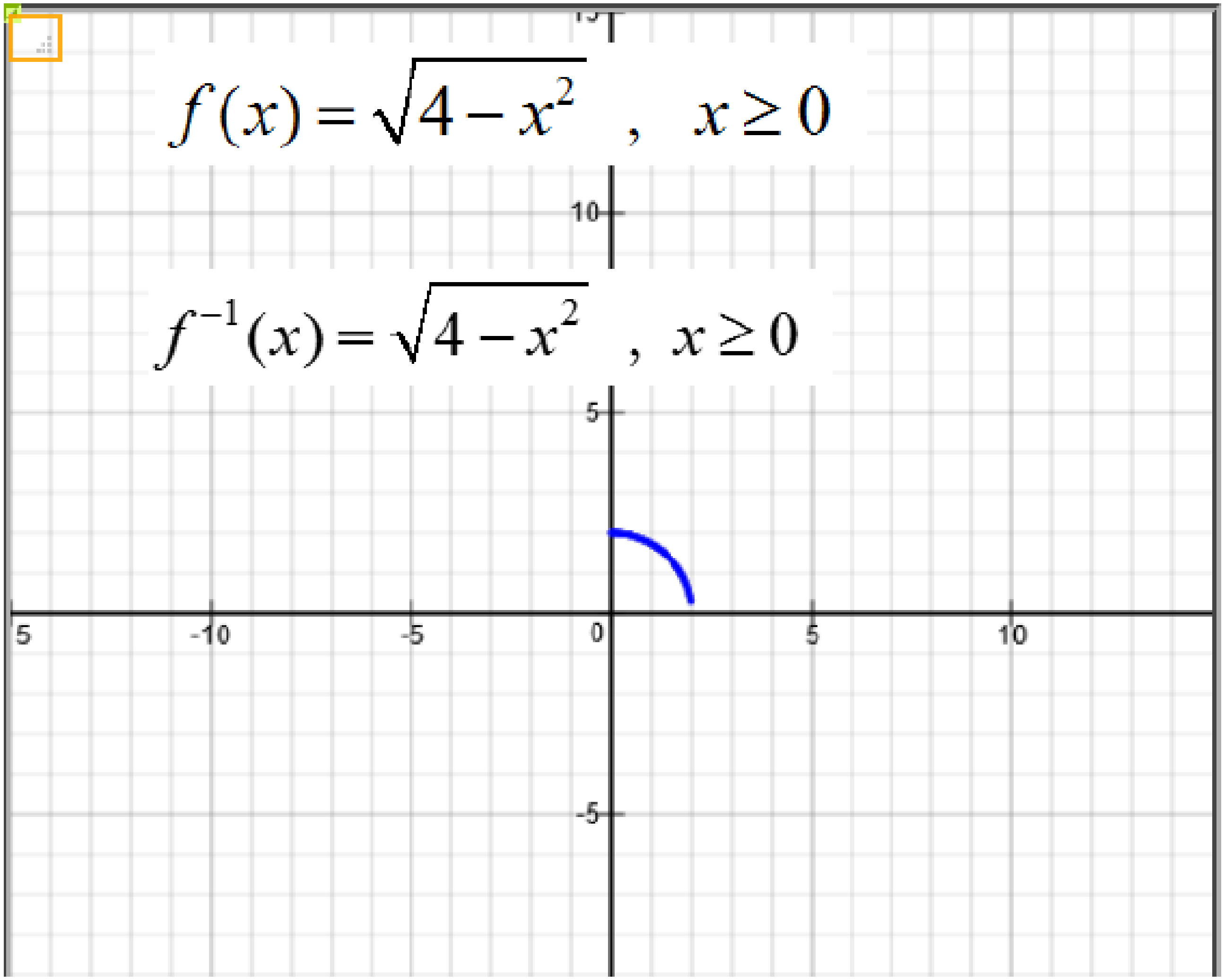
$\Rightarrow y=\pm\sqrt{4-x^2}$

Note: Inverse function is $y=\sqrt{4-x^2}\ ,\ x\geq 0$

$f^{-1}(x)=\sqrt{4-x^2}\ ,\ x\geq 0$

$$f(x)=\sqrt{4-x^2}\ ,\quad x\geq 0$$

$$f^{-1}(x)=\sqrt{4-x^2}\ ,\quad x\geq 0$$



$f(x) = \sqrt{4 - x^2}$, $x \geq 0$

$f^{-1}(x) = \sqrt{4 - x^2}$, $x \geq 0$

Domain of $f(x) = [0, 2]$

Range of $f(x) = [0, 2]$

Domain of $f^{-1}(x) = [0, 2]$

Range of $f^{-1}(x) = [0, 2]$

Example 5

$f(x) = \sqrt[3]{3x-7}$

a) Is $f(x)$ a one-to-one function?

Yes, graph passe horizontal line test.

b) Find $f^{-1}(x)$.

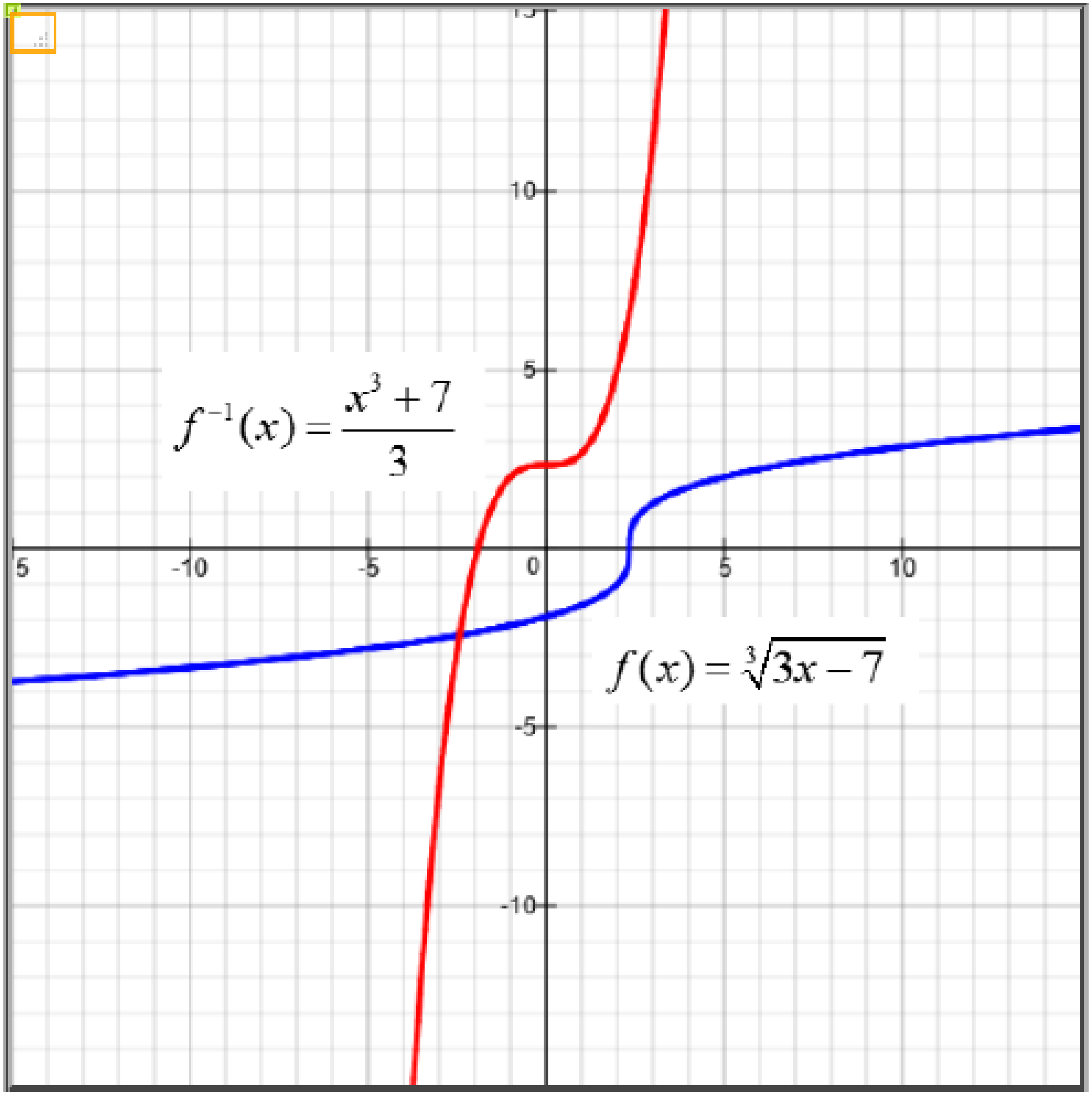
Let $y = \sqrt[3]{3x-7}$

Exchange x and y:

$$x = \sqrt[3]{3y-7} \Rightarrow \sqrt[3]{3y-7} = x \Rightarrow \left(\sqrt[3]{3y-7}\right)^3 = (x)^3$$

$$\Rightarrow 3y - 7 = x^3 \Rightarrow 3y = x^3 + 7 \Rightarrow y = \frac{x^3+7}{3}$$

$$f^{-1}(x) = \frac{x^3+7}{3}$$



$f^{-1}(x) = \frac{x^3 + 7}{3}$
$f(x) = \sqrt[3]{3x - 7}$
10
5
-5
-10
-10
-5
0
5
10

$f(x) = \sqrt[3]{3x - 7}$

$$f^{-1}(x) = \frac{x^3 + 7}{3}$$

Domain of $f(x) = (-\infty, \infty)$

Range of $f(x) = (-\infty, \infty)$

Domain of $f^{-1}(x) = (-\infty, \infty)$

Range of $f^{-1}(x) = (-\infty, \infty)$

Example 6

$f(x) = \cos x \qquad x \in [0,\ \pi]$

a) Is $f(x)$ a one-to-one function?

Yes, graph passes horizontal line test.

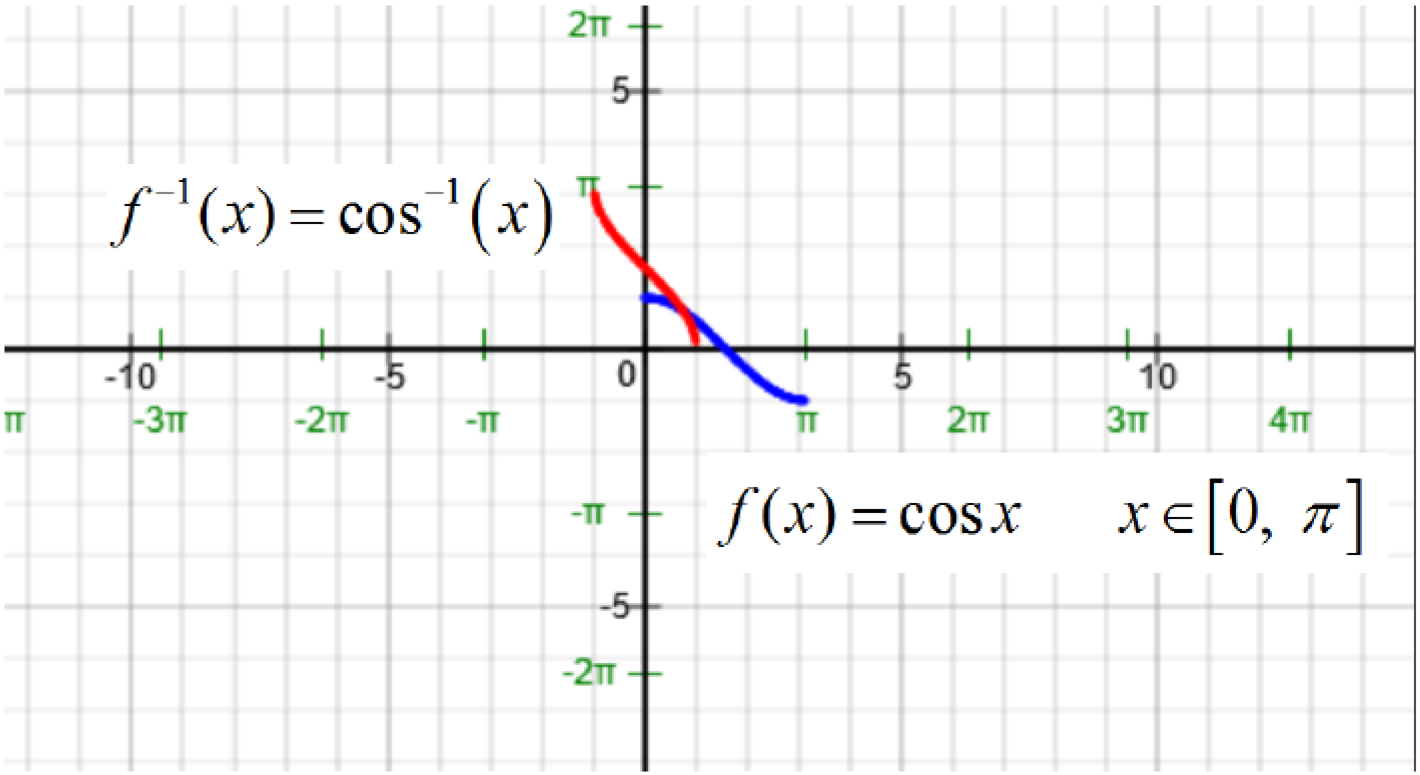
b) Find $f^{-1}(x) = \cos^{-1}(x)$

c) Domain of $f(x) = \{x : 0 \le x \le \pi\}$

d) Range of $f(x) = [-1,\ 1]$

e) Domain of $f^{-1}(x) = [-1,\ 1]$

f) Range of $f^{-1}(x) = \{x : 0 \le x \le \pi\}$



$f^{-1}(x) = \cos^{-1}(x)$
$f(x) = \cos x \quad x \in [0, \ \pi]$
2π
5
π
0
-10
-5
5
10
-3π
-2π
-π
π
2π
3π
4π
-π
-5
-2π