Solve $e^{\ln 2x} = 18.$

Recall: $e^{\ln u} = u$; $\ln e^{u} = u$; $\ln x = \log_e x$; $\ln e = 1$

$$e^{\ln 2x} = 18$$

$$2x = 18$$

$$x = 9$$

Solve $e^{3x} = 5$.

Recall: $e^{\ln u} = u$; $\ln e^{u} = u$; $\ln x = \log_e x$; $\ln e = 1$

$$e^{3x} = 5$$

$$\ln\left(e^{3x}\right) = \ln(5)$$

$$3x = \ln 5$$

$$x = \frac{\ln 5}{3}$$

Solve $12 - 3e^{x} = 6$.

Recall: $e^{\ln u} = u$; $\ln e^{u} = u$; $\ln x = \log_e x$; $\ln e = 1$

$$12 - 3e^{x} = 6$$

$$-3e^{x} = -6$$

$$e^{x} = \frac{-6}{-3}$$

$$e^{x} = 0.5$$

$$\ln\left(e^{x}\right) = \ln\left(0.5\right)$$

$$x = \ln\left(0.5\right)$$

Solve $5e^{-x} = 25$.

Hint: $e^{\ln u} = u$; $\ln e^{u} = u$; $\ln x = \log_{e} x$; $\ln e = 1$

$$5e^{-x} = 25$$

$$e^{-x} = 5$$

$$\ln\left(e^{-x}\right) = \ln(5)$$

$$-x = \ln 5$$

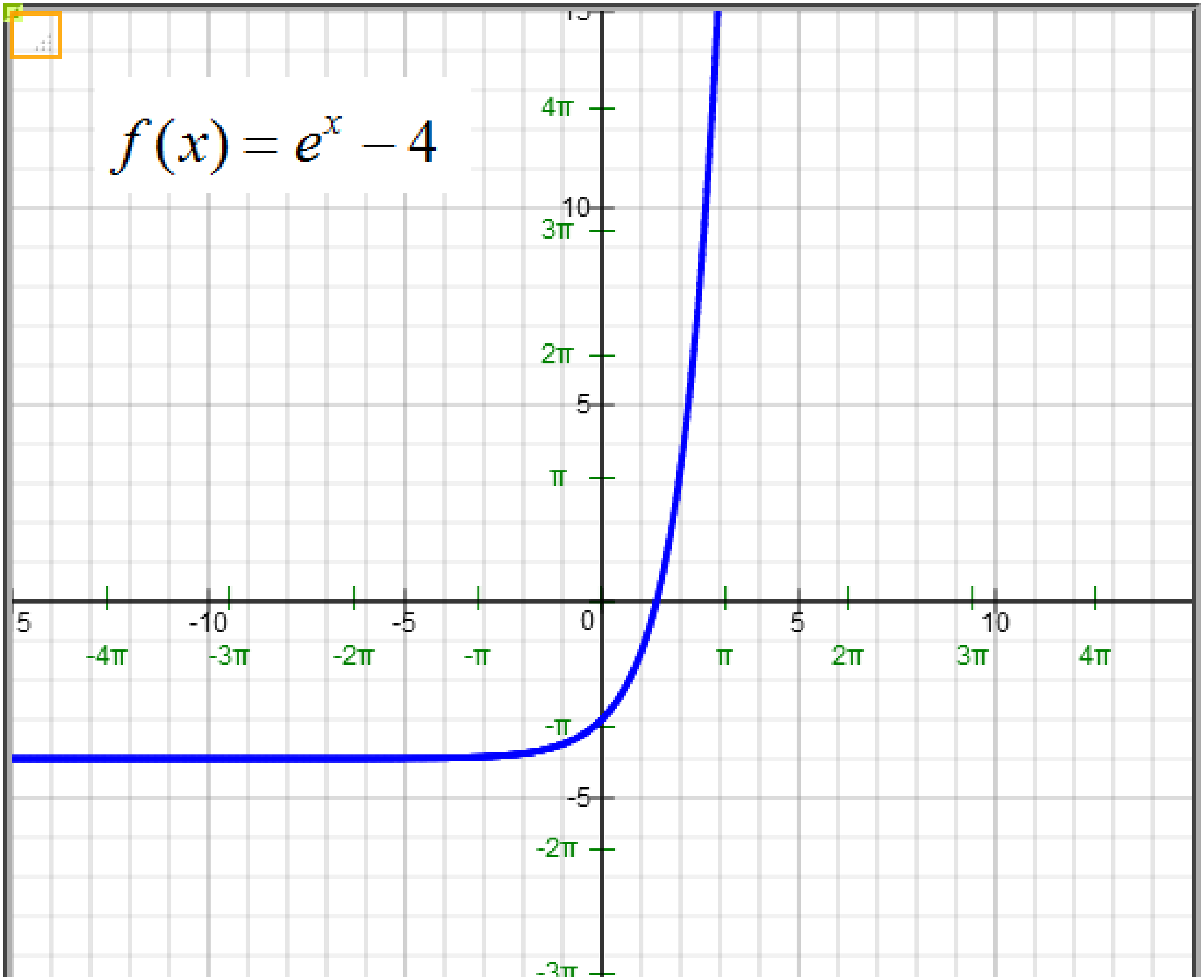
$$x = -\ln 5$$

$f(x) = e^x - 4$

Calculator Input: $y = e \wedge (x) - 4$

a) Domain of $f(x) = (-\infty, \infty)$

b) Range of $f(x) = (-4, \infty)$

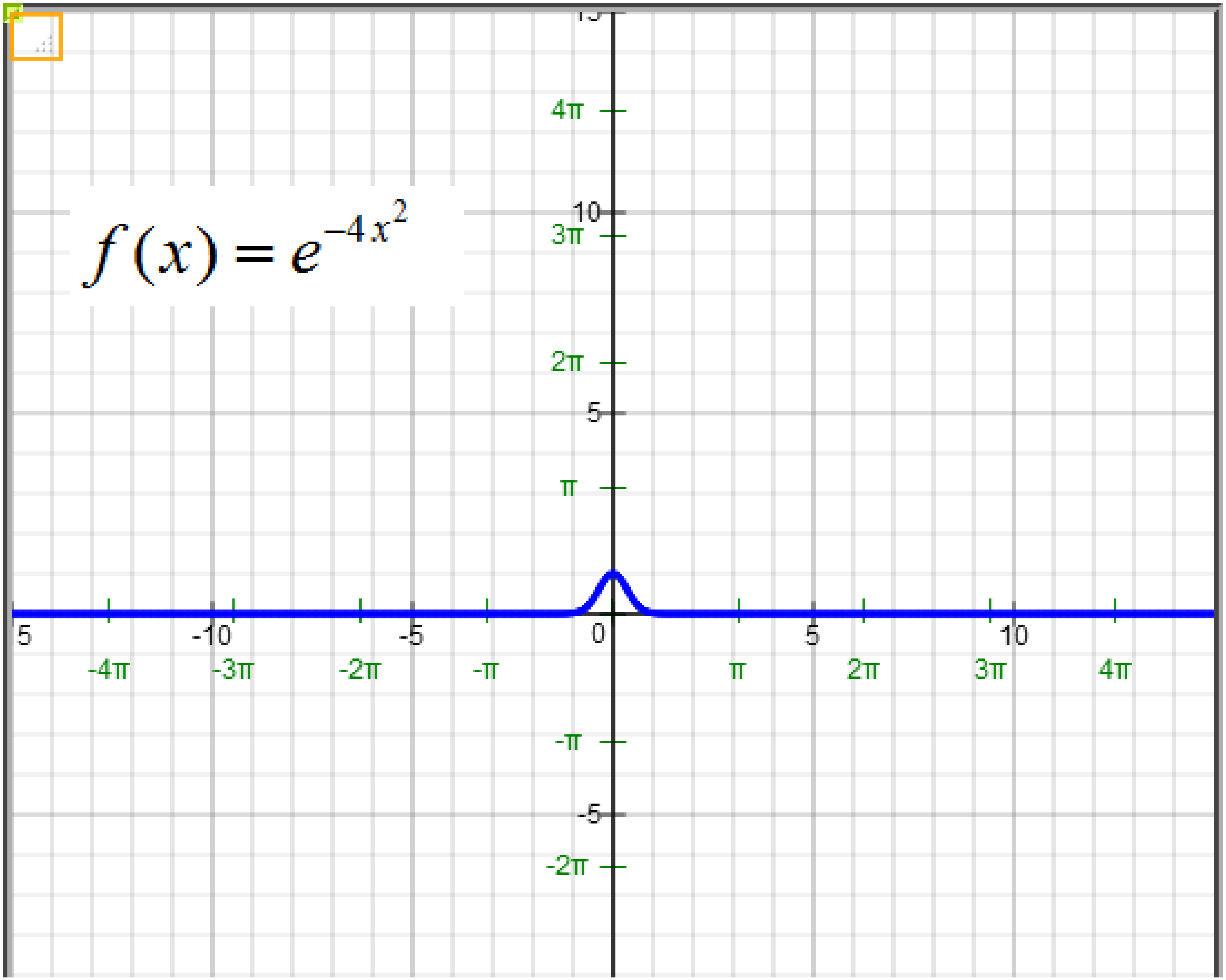

$f(x) = e^x - 4$
4π
10
3π
2π
5
π
-π
-5
-2π
5
-10
-5
0
5
10
-4π
-3π
-2π
-π
π
2π
3π
4π

$f(x) = e^{-4x^2}$

Calculator Input: $y = e \wedge \left(-4x^2\right)$

a) Domain of $f(x) = \left(-\infty, \infty\right)$

b) Range of $f(x) = (0,\ 1]$


$f(x) = e^{-4x^2}$
4π
10
3π
2π
5
π
0
-π
-5
-2π
5
-10
-5
5
10
-4π
-3π
-2π
-π
π
2π
3π
4π

Review:

$$D_x\left[\ln x\right]=\frac{1}{x} \qquad D_x\left[\ln\left(\text{expression}\right)\right]=\frac{1}{\text{expression}}D_x\left(\text{expression}\right)$$

$$\int\frac{1}{x}\,dx=\ln|x|+C \qquad \int\frac{1}{ax+b}\,dx=\frac{1}{a}\ln|ax+b|+C$$

Now we want to find $D_x\left[e^x\right]$.

Note:

$\ln e^1 = 1; \quad \ln e^2 = 2; \quad \ln e^3 = 3; \quad \ln e^x = x$

$e^{\ln 1} = 1; \quad e^{\ln 2} = 2; \quad e^{\ln 3} = 3; \quad e^{\ln x} = x;$

Hence, $\ln e^x = x$

$D_x\left[\ln e^x\right] = D_x\left[x\right]$

Recall: $D_x\left[\ln\left(\text{expression}\right)\right] = \dfrac{1}{\text{expression}} D_x\left(\text{expression}\right)$

$$\frac{1}{e^x} D_x\left(e^x\right) = 1$$

$$e^x \frac{1}{e^x} D_x\left(e^x\right) = e^x \cdot 1$$

$$D_x\left(e^x\right) = e^x$$

Chain Rule: $D_x\left(e^{\text{Expression}}\right) = e^{\text{Expression}} D_x\left[\text{Expression}\right]$

Let $y = e^{x-4}$. Find y'.

Recall: For $y = e^{\text{expression}} \Rightarrow y' = \left(e^{\text{expression}}\right) \cdot D_x\left(\text{expression}\right)$

$y' = e^{x-4} \cdot D_x\left(x-4\right) = e^{x-4} \cdot \left(1\right) = e^{x-4}$

Let $y = 5e^{4x^2+2}$. Find y'.

Recall: For $y = e^{\text{expression}} \Rightarrow y' = \left(e^{\text{expression}}\right) \cdot D_x\left(\text{expression}\right)$

$$y' = 5e^{4x^2+2} \cdot D_x\left(4x^2 + 2\right) = 5e^{4x^2+2} \cdot \left(8x\right) = 40xe^{4x^2+2}$$

Let $y = x^2 \cdot e^{2x}$. Find y'

Hint: Use Product Rule for Derivative.

Let F = First Factor; S = Second Factor

Recall: For $y = e^{\text{expression}} \Rightarrow y' = \left(e^{\text{expression}}\right) \cdot D_x\left(\text{expression}\right)$

$$y' = F \cdot D_x\left(S\right) + S \cdot D_x\left(F\right)$$

$$y' = \left(x^2\right) \cdot D_x\left(e^{2x}\right) + \left(e^{2x}\right) \cdot D_x\left(x^2\right)$$

$$y' = \left(x^2\right) \cdot \left(2e^{2x}\right) + \left(e^{2x}\right) \cdot \left(2x\right)$$

Let $y = \dfrac{2}{e^x + e^{-x}} = 2\left(e^x + e^{-x}\right)^{-1}$. Find y'.

Recall: $y = (\text{expression})^{\text{n}} \Rightarrow y = n(\text{expression})^{\text{n-1}} \cdot D_x\left(\text{expression}\right)$

Recall: $D_x(e^{power}) = e^{power} \cdot D_x(power)$

$$y' = 2\left(-1\right)\left(e^x + e^{-x}\right)^{-2} \cdot D_x(e^x + e^{-x})$$

$$y' = -2\left(e^x + e^{-x}\right)^{-2} \cdot (e^x - e^{-x})$$

$$y' = \frac{-2(e^x - e^{-x})}{\left(e^x + e^{-x}\right)^2}$$

Let $y = \dfrac{3e^x + 1}{4e^x - 1}$. Find y'

Hint: Use Quotient Rule for Derivative.

Hint: $\mathrm{D}_x(e^{power}) = e^{power} \cdot \mathrm{D}_x(power)$

Let N = Numerator Factor; D = Denominator Factor

$$y' = \frac{D \cdot D_x\left(N\right) \ + \ N \cdot D_x\left(D\right)}{D^2}$$

$$y' = \frac{\left(4e^x - 1\right) \cdot D_x\left(3e^x + 1\right) \ + \ \left(3e^x + 1\right) \cdot D_x\left(4e^x - 1\right)}{\left(4e^x - 1\right)^2}$$

$$y' = \frac{\left(4e^x - 1\right) \cdot \left(3e^x\right) \ + \ \left(3e^x + 1\right) \cdot D_x\left(4e^x\right)}{\left(4e^x - 1\right)^2}$$

Let $f(x) = e^{3x} \cdot \ln x$. Find equation of tangent line at (1, 0)

Hint: Use Product Rule for Derivative.

Let F = First Factor; S = Second Factor

$$f'(x) = F \cdot D_x(S) + S \cdot D_x(F)$$

$$f'(x) = \left(e^{3x}\right) \cdot D_x(\ln x) + (\ln x) \cdot D_x\left(e^{3x}\right)$$

$$f'(x) = \left(e^{3x}\right) \cdot \left(\frac{1}{x}\right) + (\ln x) \cdot \left(3e^{3x}\right)$$

slope of tangent line: $f'(1) = \left(e^{3x}\right) \cdot \left(\frac{1}{x}\right) + (\ln x) \cdot \left(3e^{3x}\right) = e^3$

Equation of Tangent Line: $y - y_1 = m(x - x_1)$

$$y - 0 = e^3(x - 1)$$

Let $f(x)=\dfrac{e^{x}-e^{-x}}{2}=\dfrac{1}{2}e^{x}-\dfrac{1}{2}e^{-x}$. Find extremums.

$$f'(x)=\frac{1}{2}e^{x}-\frac{1}{2}\left(-1e^{-x}\right)=\frac{1}{2}e^{x}+\frac{1}{2}\left(e^{-x}\right)$$

Set $f'(x)=0$

$$\frac{1}{2}e^{x}+\frac{1}{2}\left(e^{-x}\right)=0 \quad \Rightarrow \quad e^{x}+\left(e^{-x}\right)=0$$

Note: $e^{x}+\left(e^{-x}\right)\neq 0$

Therefore, no extremum for $f(x)=\dfrac{e^{x}-e^{-x}}{2}$.

Calculator Input: $y = 0.5e^{\wedge}(x) - 0.5e^{\wedge}(-x)$