

$$\text{Recall: } \log_b x = \frac{\log_{10} x}{\log_{10} b}; \quad b^{\log_b x} = x; \quad \ln x = \log_e x$$

$$\text{Evaluate } \log_2 \frac{3}{4}.$$

$$\log_2 \frac{3}{4} = \frac{\log_{10}(3/4)}{\log_{10} 2} = -0.4150374992788438$$

$$\text{Recall: } \log_b x = \frac{\log_{10} x}{\log_{10} b}; \quad b^{\log_b x} = x; \quad \ln x = \log_e x$$

Evaluate $\log_4 (5\pi)$.

$$\log_4 (5\pi) = \frac{\log_{10} (5\pi)}{\log_{10} 4} = 1.9867121121798406$$

$$\text{Recall: } \log_b x = y \Leftrightarrow b^y = x$$

$$\text{Recall } \log_b x = \frac{\log_{10} x}{\log_{10} b}; \quad b^{\log_b x} = x; \quad \ln x = \log_e x$$

$$\text{Solve } \log_{10} 750 = x$$

$$\log_{10} 750 = x$$

$$x = \log_{10} 750 = \frac{\log 750}{\log 10} = 2.8750612633916997$$

$$\text{Hint: } \log_b x = y \Leftrightarrow b^y = x$$

$$\text{Hint: } \log_b x = \frac{\log_{10} x}{\log_{10} b}; \quad b^{\log_b x} = x; \quad \ln x = \log_e x$$

$$\text{Solve } \log_3 18 = x$$

$$\log_3 18 = x$$

$$x = \log_3 18 = \frac{\log 18}{\log 3} = 2.6309297535714573$$

$$\text{Recall: } D_x \left(e^{\text{expr}} \right) = \left(e^{\text{expr}} \right) D_x (\text{expr})$$

$$\text{Recall: } D_x \left(a^{\text{expr}} \right) = (\ln a) \left(a^{\text{expr}} \right) D_x (\text{expr})$$

Let $f(x) = 14^x$. Find $f'(x)$.

$$f'(x) = (\ln 14) \left(14^x \right) D_x (x) = (\ln 14) \left(14^x \right) (1) = (\ln 14) \left(14^x \right)$$

Recall: $D_x \left(a^{\text{expr}} \right) = (\ln a) \left(a^{\text{expr}} \right) D_x (\text{expr})$

Let $f(x) = 5^{-7x}$. Find $f'(x)$.

$$\begin{aligned} f'(x) &= (\ln 5) \left(5^{-7x} \right) D_x (-7x) = (\ln 5) \left(5^{-7x} \right) (-7) \\ &= -7 (\ln 5) \left(5^{-7x} \right) \end{aligned}$$

Let $y = 3x \cdot 6^x$. Find y'

Hint: Use Product Rule for Derivative.

Recall: $D_x (a^{\text{expr}}) = (\ln a)(a^{\text{expr}}) D_x (\text{expr})$

Let $F = \text{First Factor}$; $S = \text{Second Factor}$

$$y' = F \cdot D_x (S) + S \cdot D_x (F)$$

$$y' = (3x) \cdot D_x (6^x) + (6^x) \cdot D_x (3x)$$

$$y' = (3x) \cdot (\ln 6 \cdot 6^x) + (6^x) \cdot (3)$$

Review:

$$D_x (\ln x) = \frac{1}{x}$$

$$D_x [\ln (\text{expression})] = \frac{1}{\text{expression}} \cdot D_x (\text{expression})$$

$$D_x (\text{Log}_a x) = \frac{1}{\text{Ln}(a)} \cdot \frac{1}{x}$$

$$D_x [\text{Log}_a (\text{expression})] = \frac{1}{\text{Ln}(a)} \frac{1}{\text{expression}} \cdot D_x (\text{expression})$$

Let $y = \log_4(2x + 5)$. Find y'

$$\text{Recall: } D_x(\text{Log}_a u) = \frac{1}{\text{Ln}(a)} \frac{1}{u} \cdot D_x(u)$$

$$y' = \frac{1}{\text{Ln}(4)} \frac{1}{(2x + 5)} \cdot D_x(2x + 5) = \frac{1}{\text{Ln}(4)} \frac{1}{(2x + 5)} \cdot (2)$$

$$\text{Let } h(x) = \log_5 (14 - x)^2 = 2 \cdot \log_5 (14 - x)$$

$$\text{Recall: } D_x (\text{Log}_a u) = \frac{1}{\text{Ln}(a)} \frac{1}{u} D_x (u)$$

$$h'(t) = 2 \left[\frac{1}{\text{Ln}(5)} \frac{1}{(14 - x)} D_x (14 - x) \right]$$

$$h'(t) = 2 \left[\frac{1}{\text{Ln}(5)} \frac{1}{(14 - x)} (-1) \right] = \frac{-2}{\text{Ln}(5)(14 - x)}$$

$$\text{Let } y = \log_3 \sqrt{x^3 - 15} = \log_3 (x^3 - 15)^{1/2} = \frac{1}{2} \cdot \log_3 (x^3 - 15)$$

$$\text{Hint: } D_x (\text{Log}_a u) = \frac{1}{\text{Ln}(a)} \frac{1}{u} D_x (u)$$

$$y' = \frac{1}{2} \cdot \left[\frac{1}{\text{Ln}(3)} \frac{1}{(x^3 - 15)} D_x (x^3 - 15) \right]$$

$$y' = \frac{1}{2} \cdot \left[\frac{1}{\text{Ln}(3)} \frac{1}{(x^3 - 15)} (3x^2) \right]$$

Let $y = 5^{-2x}$ Find equation of tangent line at $(0, 1)$

Recall: $D_x(a^u) = (\ln a)(a^u)D_x(u)$

a) $y' = (\ln 5)(5^{-2x})D_x(-2x) = (\ln 5)(5^{-2x})(-2)$

b) Slope of tangent line = $y'(0) = (\ln 5)(5^{-2x})(-2)$
 $= (\ln 5)(5^0)(-2) = -2\ln 5$

c) Equation of Tangent Line: $y - y_1 = m(x - x_1)$
 $y - 1 = (-2\ln 5)(x - 0)$

Let $y = \log_4 5x$ Find equation of tangent line at $(1/5, 0)$

Hint: $D_x (\text{Log}_a u) = \frac{1}{\text{Ln}(a)} \frac{1}{u} D_x (u)$

a) $y' = \frac{1}{\text{Ln}(4)} \frac{1}{(5x)} D_x (5x) = \frac{1}{\text{Ln}(4)} \frac{1}{(5x)} (5)$

b) Slope of tangent line = $y'(1/5) = \frac{1}{\text{Ln}(4)} \frac{1}{(5x)} (5)$

$$= \frac{1}{\text{Ln}(4)} \frac{1}{(5 \cdot (1/5))} (5) = \frac{5}{\ln 4}$$

c) Equation of Tangent Line: $y - y_1 = m(x - x_1)$

$$y - 0 = \left(\frac{5}{\ln 4} \right) (x - 1/5)$$

Let $y = x^{7/x}$. Find y'

$$y = x^{7/x} \Rightarrow \ln y = \ln(x^{7/x}) \Rightarrow \ln y = \frac{7}{x} \cdot \ln x = \frac{7 \ln x}{x}$$

Now perform implicit derivative:

$$\ln y = \frac{7 \ln x}{x}$$

$$\frac{1}{y} y' = \frac{(x) D_x (7 \ln x) - (7 \ln x) D_x (x)}{(x)^2} \quad \text{Using Quotient Rule}$$

$$\frac{1}{y} y' = \frac{(x) \left(7 \cdot \frac{1}{x} \right) - (7 \ln x)(1)}{(x)^2}$$

$$y' = y \left[\frac{7 - 7 \ln x}{x^2} \right] \quad \text{Multiply each side by } y$$

Review:

$$\int e^x du = e^x + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \frac{1}{\ln(a)} \cdot a^u + C$$

Find $\int 7^{7x} dx$

Let $u = 7x$

$$\frac{du}{dx} = 7 \quad \Rightarrow \quad du = 7dx \quad \Rightarrow \quad \frac{1}{7} du = dx$$

Recall: $\int base^u du = \frac{1}{\ln(base)} base^u + C$

$$\int 7^{7x} dx = \int 7^u \frac{1}{7} du = \frac{1}{7} \int 7^u du = \frac{1}{7} \left[\frac{1}{\ln(7)} 7^u + C \right]$$

$$= \frac{1}{7} \left[\frac{1}{\ln(7)} 7^{7x} \right] + C$$

Find $\int x^2 4^{6x^3} dx$

Let $u = 6x^3$

$$\frac{du}{dx} = 18x^2 \quad \Rightarrow \quad du = 18x^2 dx \quad \Rightarrow \quad \frac{1}{18} du = x^2 dx$$

Hint: $\int a^u du = \frac{1}{\ln(a)} a^u + C$

$$\int x^2 4^{6x^3} dx = \int 4^{6x^3} x^2 dx = \int 4^u \frac{1}{18} du$$

$$= \frac{1}{18} \int 4^u du = \frac{1}{18} \left[\frac{1}{\ln(4)} 4^u \right] = \frac{1}{18} \left[\frac{1}{\ln(4)} 4^{6x^3} \right] + C$$