Recall: $\log_b x = \dfrac{\log_{10} x}{\log_{10} b}$; $\quad b^{\log_b x} = x$; $\quad \ln x = \log_e x$

Evaluate $\log_2 \dfrac{3}{4}$.

$$\log_2 \frac{3}{4} = \frac{\log_{10}(3/4)}{\log_{10} 2} = \text{-}0.415037499278843 8$$

Recall: $\log_{\mathrm{b}} x = \dfrac{\log_{10} x}{\log_{10} b}$; $\quad \mathrm{b}^{\log_{\mathrm{b}} x} = x$; $\quad \ln x = \log_e x$

Evaluate $\log_4 (5\pi)$.

$$\log_4 (5\pi) = \frac{\log_{10} (5\pi)}{\log_{10} 4} = 1.986712112179840 6$$

Recall: $\log_{\mathrm{b}} x = y \Leftrightarrow b^{y} = x$

Recall $\log_{\mathrm{b}} x = \dfrac{\log_{10} x}{\log_{10} b}$; $\quad \mathrm{b}^{\log_{\mathrm{b}} x} = x$; $\quad \ln x = \log_{e} x$

Solve $\log_{10} 750 = x$

$\log_{10} 750 = x$

$$x = \log_{10} 750 = \frac{\log 750}{\log 10} = 2.8750612633916997$$

Hint: $\log_{\mathrm{b}} x = y \Leftrightarrow b^{y} = x$

Hint: $\log_{\mathrm{b}} x = \dfrac{\log_{10} x}{\log_{10} b}$; $\quad \mathrm{b}^{\log_{\mathrm{b}} x} = x$; $\quad \ln x = \log_{e} x$

Solve $\log_3 18 = x$

$$\log_3 18 = x$$

$$x = \log_3 18 = \frac{\log 18}{\log 3} = 2.6309297535714573$$

Recall: $D_x\left(e^{\text{expr}}\right)=\left(e^{\text{expr}}\right)D_x\left(\text{expr}\right)$

Now we want to find: $D_x\left(a^x\right)=?$

Note: $e^{\ln 1}=1; \quad e^{\ln 2}=2; \quad e^{\ln x}=x; \quad e^{\ln a}=a$

Also, $\left(b^x\right)^y=b^{xy}$

Hence, $a^x=\left(a\right)^x=\left(e^{\ln a}\right)^x=e^{(\ln a)(x)}$

$$D_x\left(a^x\right)=D_x\left(e^{(\ln a)(x)}\right)=e^{(\ln a)(x)}D_x\left[\left(\ln a\right)x\right]=a^x\left[\left(\ln a\right)\right]=\left(\ln a\right)a^x$$

Recall: $D_x\left(a^{\text{expr}}\right) = \left(\ln a\right)\left(a^{\text{expr}}\right)D_x\left(\text{expr}\right)$

Let $f(x) = 14^x$. Find $f'(x)$.

$$f'(x) = \left(\ln 14\right)\left(14^x\right)D_x\left(x\right) = \left(\ln 14\right)\left(14^x\right)\left(1\right) = \left(\ln 14\right)\left(14^x\right)$$

Recall: $D_x\left(a^{\text{expr}}\right) = (\ln a)\left(a^{\text{expr}}\right)D_x(\text{expr})$

Let $f(x) = 5^{-7x}$. Find $f'(x)$.

$$f'(x) = (\ln 5)\left(5^{-7x}\right)D_x(-7x) = (\ln 5)\left(5^{-7x}\right)(-7)$$

$$= -7(\ln 5)\left(5^{-7x}\right)$$

Let $y = 3x \cdot 6^x$.   Find y'

Hint: Use Product Rule for Derivative.

Recall: $D_x\left(a^{\text{expr}}\right) = (\ln a)\left(a^{\text{expr}}\right)D_x(\text{expr})$

Let F = First Factor;   S = Second Factor

$$y' = F \cdot D_x(S) \; + \; S \cdot D_x(F)$$

$$y' = (3x) \cdot D_x\left(6^x\right) \; + \; \left(6^x\right) \cdot D_x(3x)$$

$$y' = (3x) \cdot \left(\ln 6 \cdot 6^x\right) \; + \; \left(6^x\right) \cdot (3)$$

Review:

$$D_x(\ln x) = \frac{1}{x}$$

$$D_x\left[\ln(\text{expression})\right] = \frac{1}{\text{expression}} \cdot D_x(\text{expression})$$

Now we want to find $D_x(Log_a x) = ?$

$$D_x(Log_a x) = D_x\left[\frac{1}{ln(a)} \cdot \ln x\right] = \frac{1}{\ln a}\left(\frac{1}{x}\right)$$

Chain Rule: $D_x\left[log_a(\text{expression})\right] = \frac{1}{ln(a)}\frac{1}{\text{expression}} \cdot D_x(\text{expression})$

Let $y=\log_4\left(2x+5\right)$. Find y'

Recall: $D_x\left(Log_a u\right)=\frac{1}{Ln(a)}\frac{1}{u}\cdot D_x\left(u\right)$

$$y'=\frac{1}{Ln(4)}\frac{1}{\left(2x+5\right)}\cdot D_x\left(2x+5\right)=\frac{1}{Ln(4)}\frac{1}{\left(2x+5\right)}\cdot\left(2\right)$$

Let $h(x) = \log_5 \left(14 - x\right)^2 = 2 \cdot \log_5 \left(14 - x\right)$

Recall: $D_x \left(Log_a u\right) = \dfrac{1}{Ln(a)} \dfrac{1}{u} D_x \left(u\right)$

$$h'(t) = 2\left[\frac{1}{Ln(5)} \frac{1}{\left(14 - x\right)} D_x \left(14 - x\right) \right]$$

$$h'(t) = 2\left[\frac{1}{Ln(5)} \frac{1}{\left(14 - x\right)} \left(-1\right) \right] = \frac{-2}{Ln(5)\left(14 - x\right)}$$

Let $y = \log_3 \sqrt{x^3 - 15} = \log_3 \left(x^3 - 15\right)^{1/2} = \frac{1}{2} \cdot \log_3 \left(x^3 - 15\right)$

Hint: $D_x\left(Log_a u\right) = \frac{1}{Ln(a)} \frac{1}{u} D_x\left(u\right)$

$$y' = \frac{1}{2} \cdot \left[\frac{1}{Ln(3)} \frac{1}{\left(x^3 - 15\right)} D_x\left(x^3 - 15\right) \right]$$

$$y' = \frac{1}{2} \cdot \left[\frac{1}{Ln(3)} \frac{1}{\left(x^3 - 15\right)} \left(3x^2\right) \right]$$

Let $y = 5^{-2x}$ Find equation of tangent line at (0, 1)

Recall: $D_x\left(a^u\right) = \left(\ln a\right)\left(a^u\right)D_x\left(u\right)$

a) $y' = \left(\ln 5\right)\left(5^{-2x}\right)D_x\left(-2x\right) = \left(\ln 5\right)\left(5^{-2x}\right)\left(-2\right)$

b) Slope of tangent line = $y'(0) = \left(\ln 5\right)\left(5^{-2x}\right)\left(-2\right)$

$= \left(\ln 5\right)\left(5^0\right)\left(-2\right) = -2\ln 5$

c) Equation of Tangent Line: $y - y_1 = m(x - x_1)$

$y - 1 = \left(-2\ln 5\right)\left(x - 0\right)$

Let $y = \log_4 5x$ Find equation of tangent line at (1/5, 0)

Hint: $D_x\left(Log_a u\right) = \dfrac{1}{Ln(a)}\dfrac{1}{u}D_x\left(u\right)$

a) $y' = \dfrac{1}{Ln(4)}\dfrac{1}{\left(5x\right)}D_x\left(5x\right) = \dfrac{1}{Ln(4)}\dfrac{1}{\left(5x\right)}\left(5\right)$

b) Slope of tangent line = $y'(1/5) = \dfrac{1}{Ln(4)}\dfrac{1}{\left(5x\right)}\left(5\right)$

$$= \frac{1}{Ln(4)}\frac{1}{\left(5\cdot\left(1/5\right)\right)}\left(5\right) = \frac{5}{\ln 4}$$

c) Equation of Tangent Line: $y - y_1 = m(x - x_1)$

$$y - 0 = \left(\frac{5}{\ln 4}\right)\left(x - 1/5\right)$$

Let $y = x^{7/x}$. Find y'

$$y = x^{7/x} \Rightarrow \ln y = \ln\left(x^{7/x}\right) \Rightarrow \ln y = \frac{7}{x} \cdot \ln x = \frac{7\ln x}{x}$$

Now perform implicit derivative:

$$\ln y = \frac{7\ln x}{x}$$

$$\frac{1}{y} y' = \frac{(x) D_x(7\ln x) - (7\ln x) D_x(x)}{(x)^2} \quad \text{Using Quotient Rule}$$

$$\frac{1}{y} y' = \frac{(x)\left(7 \cdot \frac{1}{x}\right) - (7\ln x)(1)}{(x)^2}$$

$$y' = y\left[\frac{7 - 7\ln x}{x^2}\right] \quad \text{Multiply each side by } y$$

Review:

$$\int e^{x}du = e^{x} + C$$

$$\int e^{u}du = e^{u} + C$$

$$\int a^{u}du = \frac{1}{\ln(a)} \cdot a^{u} + C$$

Find $\int 7^{7x}\,dx$

Let $u = 7x$

$$\frac{du}{dx} = 7 \quad \Rightarrow \quad du = 7\,dx \quad \Rightarrow \quad \frac{1}{7}du = dx$$

Recall: $\int base^u\,du = \dfrac{1}{\ln(base)} base^u + C$

$$\int 7^{7x}\,dx = \int 7^u \frac{1}{7}du = \frac{1}{7}\int 7^u\,du = \frac{1}{7}\left[\frac{1}{\ln(7)}7^u + C\right]$$

$$= \frac{1}{7}\left[\frac{1}{\ln(7)}7^{7x}\right] + C$$

Find $\int x^2 4^{6x^3}\,dx$

Let $u = 6x^3$

$$\frac{du}{dx} = 18x^2 \quad \Rightarrow \quad du = 18x^2 dx \quad \Rightarrow \quad \frac{1}{18} du = x^2 dx$$

Hint: $\int a^u du = \frac{1}{\ln(a)} a^u + C$

$$\int x^2 4^{6x^3}\,dx = \int 4^{6x^3}\, x^2 dx = \int 4^u \,\frac{1}{18} du$$

$$= \frac{1}{18}\int 4^u\, du = \frac{1}{18}\left[\frac{1}{\ln(4)} 4^u\right] = \frac{1}{18}\left[\frac{1}{\ln(4)} 4^{6x^3}\right] + C$$