

Inverse Trigonometric Functions Review:

$$\sin(30^\circ) = \sin(\pi/6) = 1/2$$

$$\sin^{-1}(1/2) = \arcsin(1/2) = \pi/6$$

$$\sin(45^\circ) = \sin(\pi/4) = \sqrt{2}/2$$

$$\sin^{-1}(\sqrt{2}/2) = \arcsin(\sqrt{2}/2) = \pi/4$$

$$\sin(60^\circ) = \sin(\pi/3) = \sqrt{3}/2$$

$$\sin^{-1}(\sqrt{3}/2) = \pi/3$$

$$\sin(90^\circ) = \sin(\pi/2) = 1$$

$$\sin^{-1}(1) = \pi/2$$

$$\sin(0^\circ) = \sin(0) = 0$$

$$\sin^{-1}(0) = 0$$

$$\text{Prove: } D_x [\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$$

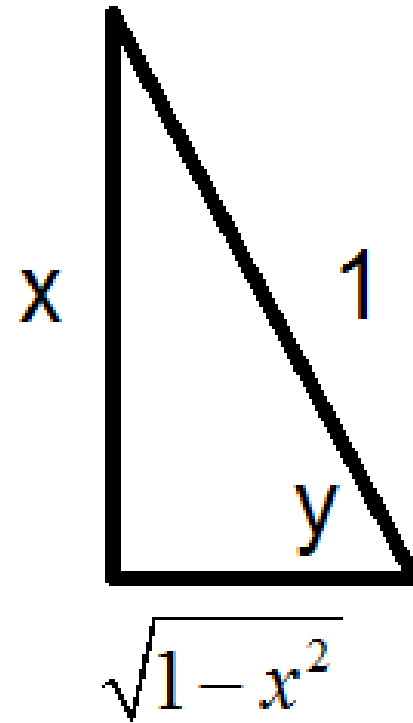
$$\text{Let } y = \sin^{-1} x$$

$$\text{Hence, } x = \sin y.$$

$$\text{Note: } \sin y = \frac{x}{1} = \frac{\text{opposite}}{\text{hypotenuse}}$$

From right triangle:

$$\cos y = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$



Prove: $D_x [\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$ con't

Let $y = \sin^{-1} x$

Hence, $x = \sin y$

Differentiating $x = \sin y$ implicitly:

$$1 = \cos y \cdot y'$$

$$y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$$

$$f(x) = 21 \arcsin(4x - 2)$$

$$\text{Chain Rule: } D_x \sin^{-1}(\text{Expr}) = \frac{1}{\sqrt{1 - (\text{Expr})^2}} D_x(\text{Expr})$$

$$f'(x) = 21 \frac{1}{\sqrt{1 - (4x - 2)^2}} D_x(4x - 2)$$

$$f'(x) = 21 \frac{1}{\sqrt{1 - (4x - 2)^2}} (4)$$

$$f'(x) = \frac{84}{\sqrt{1 - (4x - 2)^2}}$$

$$f(x) = 3 \arccos \frac{x}{5}$$

$$\text{Note: } \frac{x}{5} = \frac{1}{5}x$$

$$\text{Chain Rule: } D_x \cos^{-1}(\text{Expr}) = \frac{-1}{\sqrt{1 - (\text{Expr})^2}} D_x(\text{Expr})$$

$$f'(x) = 3 \left[\frac{-1}{\sqrt{1 - (x/5)^2}} D_x \left(\frac{1}{5}x \right) \right]$$

$$f'(x) = 3 \left[\frac{-1}{\sqrt{1 - (x/5)^2}} \left(\frac{1}{5} \right) \right] = \frac{3}{5} \left[\frac{-1}{\sqrt{1 - (x/5)^2}} \right]$$

$$f(x) = \arctan e^{2x}$$

$$\text{Chain Rule: } D_x \tan^{-1}(\text{Expr}) = \frac{1}{1 + (\text{Expr})^2} D_x(\text{Expr})$$

$$\text{Chain Rule: } D_x(e^{\text{Expr}}) = e^{\text{Expr}} \cdot D_x(\text{Expr})$$

$$f'(x) = \frac{1}{1 + (e^{2x})^2} D_x(e^{2x}) = \frac{1}{1 + (e^{2x})^2} (2e^{2x}) = \frac{2e^{2x}}{1 + e^{4x}}$$

$$g(x) = \frac{\arcsin 4x}{8x}$$

Hint: Use Quotient Rule: $D_x \left(\frac{u}{v} \right) = \frac{v \cdot u' - u \cdot v'}{v^2}$.

Hint: Chain Rule: $D_x \sin^{-1}(\text{Expr}) = \frac{1}{\sqrt{1 - (\text{Expr})^2}} D_x(\text{Expr})$

$$g'(x) = \frac{(8x) D_x(\arcsin 4x) - (\arcsin 4x) D_x(8x)}{(8x)^2}$$

Note: $D_x(\arcsin 4x) = \frac{1}{\sqrt{1 - (4x)^2}} D_x(4x) = \frac{1}{\sqrt{1 - (4x)^2}} (4)$

$$g'(x) = \frac{(8x) \left(\frac{4}{\sqrt{1 - (4x)^2}} \right) - (\arcsin 4x)(8)}{(8x)^2}$$

$$h(t) = \sin(\arctan t)$$

$$\text{Chain Rule: } D_x \tan^{-1}(\text{Expr}) = \frac{1}{1 + (\text{Expr})^2} D_x(\text{Expr})$$

$$\text{Chain Rule: } D_x \sin(\text{Expr}) = \cos(\text{Expr}) D_x(\text{Expr})$$

$$h'(t) = \cos(\arctan t) D_x(\arctan t)$$

$$\text{Note: } D_x(\arctan t) = \frac{1}{1 + (t)^2} D_t(t) = \frac{1}{1 + t^2} (1) = \frac{1}{1 + t^2}$$

$$h'(t) = \cos(\arctan t) \left(\frac{1}{1 + t^2} \right)$$

$y = 12 \arctan x$ Find equation of tangent line at $(0, 0)$

$$\text{Chain Rule: } D_x \tan^{-1}(\text{Expr}) = \frac{1}{1 + (\text{Expr})^2} D_x(\text{Expr})$$

$$\text{a) } y' = 12 \left[\frac{1}{1 + (x)^2} D_x(x) \right] = 12 \left[\frac{1}{1 + (x)^2} \right]$$

$$\text{b) Slope of tangent line} = y'(0) = 12 \left[\frac{1}{1 + (0)^2} \right] = 12$$

$$\text{c) Equation of tangent line : } y - y_1 = m(x - x_1)$$

$$y - 0 = 12(x - 0)$$

$$y = \arcsin\left(\frac{x}{4}\right) \quad \text{Find equation of tangent line at } (0,0)$$

$$\text{Chain Rule: } D_x \sin^{-1}(\text{Expr}) = \frac{1}{\sqrt{1 - (\text{Expr})^2}} D_x(\text{Expr})$$

$$\text{a) } y' = \frac{1}{\sqrt{1 - (x/4)^2}} D_x(x/4) = \frac{1}{\sqrt{1 - (x/4)^2}} (1/4)$$

$$\text{b) Slope of tangent line} = y'(0) = \frac{1}{\sqrt{1 - (0/4)^2}} (1/4) = 1/4$$

$$\text{c) Equation of tangent line : } y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{1}{4}(x - 0)$$

$y = 2x \cdot \arcsin(x - 1)$ Find equation of tangent line at (1,0)

$$\text{Chain Rule: } D_x \sin^{-1}(\text{Expr}) = \frac{1}{\sqrt{1 - (\text{Expr})^2}} D_x(\text{Expr})$$

$$y' = (2x) \left(\frac{1}{\sqrt{1 - (x - 1)^2}} \right) + (\arcsin(x - 1))(2)$$

Slope of tangent line $= y'(1)$

$$= (2(1)) \left(\frac{1}{\sqrt{1 - ((1) - 1)^2}} \right) + (\arcsin((1) - 1))(2) = (2)(1) + (0)(1) = 2$$

Equation of tangent line : $y - y_1 = m(x - x_1)$

$$y - 0 = 2(x - 1)$$