Calculus I Section 5.6 Note

Inverse Trigonometric Functions Review:

$$\sin\left(30^{o}\right) = \sin\left(\pi/6\right) = 1/2$$

$$\sin^{-1}\left(1/2\right) = \arcsin\left(1/2\right) = \pi/6$$

$$\sin\left(45^{o}\right) = \sin\left(\pi/4\right) = \sqrt{2}/2$$

$$\sin^{-1}\left(\sqrt{2}/2\right) = \arcsin\left(\sqrt{2}/2\right) = \pi/4$$

$$\sin\left(60^{o}\right) = \sin\left(\pi/3\right) = \sqrt{3}/2$$

$$\sin^{-1}\left(\sqrt{3}/2\right) = \pi/3$$

$$\sin\left(90^{\circ}\right) = \sin\left(\pi / 2\right) = 1$$

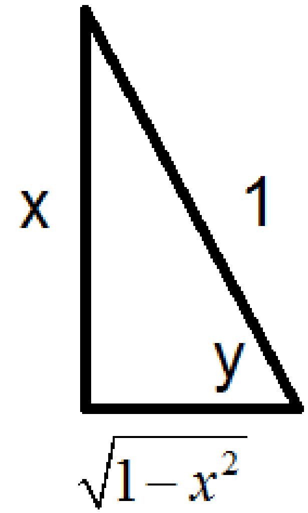
$$\sin^{-1}\left(1\right) = \pi / 2$$

$$\sin\left(0^{\circ}\right) = \sin\left(0\right) = 0$$

$$\sin^{-1}\left(0\right) = 0$$

Prove: $D_x\left[\sin^{-1}x\right]=\dfrac{1}{\sqrt{1-x^2}}$

Let $y=\sin^{-1}x$. Hence, $\sin y = x$.



Note: $\sin y = \dfrac{x}{1} = \dfrac{\text{opposite}}{\text{hypotenuse}}$

From right triangle: $\cos y = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

Differentiating $x=\sin y$ implicitly:

$\cos y \cdot y' = 1$

$y' = \dfrac{1}{\cos y} = \dfrac{1}{\sqrt{1-x^2}}$

Chain Rule: Let $y=\sin^{-1}(\text{expression})$ then $y' = \dfrac{1}{\sqrt{1-(\text{expression})^2}} D_x(\text{expression})$

$$f(x) = 21\arcsin\left(4x-2\right)$$

Chain Rule: $D_x \sin^{-1}\left(Expr\right) = \dfrac{1}{\sqrt{1-\left(Expr\right)^2}} D_x\left(Expr\right)$

$$f'(x) = 21\frac{1}{\sqrt{1-\left(4x-2\right)^2}} D_x\left(4x-2\right)$$

$$f'(x) = 21\frac{1}{\sqrt{1-\left(4x-2\right)^2}}\left(4\right)$$

$$f'(x) = \frac{84}{\sqrt{1-\left(4x-2\right)^2}}$$

$$f(x) = 3\arccos\frac{x}{5}$$

Note: $\frac{x}{5} = \frac{1}{5}x$

Chain Rule: $D_x \cos^{-1}\left(Expr\right) = \frac{-1}{\sqrt{1-\left(Expr\right)^2}} D_x\left(Expr\right)$

$$f'(x) = 3\left[\frac{-1}{\sqrt{1-\left(x/5\right)^2}} D_x\left(\frac{1}{5}x\right)\right]$$

$$f'(x) = 3\left[\frac{-1}{\sqrt{1-\left(x/5\right)^2}}\left(\frac{1}{5}\right)\right] = \frac{3}{5}\left[\frac{-1}{\sqrt{1-\left(x/5\right)^2}}\right]$$

$$f(x) = \arctan e^{2x}$$

Chain Rule: $D_x \tan^{-1}\left(Expr\right) = \dfrac{1}{1+\left(Expr\right)^2} D_x\left(Expr\right)$

Chain Rule: $D_x\left(e^{Expr}\right) = e^{Expr} \cdot D_x\left(Expr\right)$

$$f'(x) = \frac{1}{1+\left(e^{2x}\right)^2} D_x\left(e^{2x}\right) = \frac{1}{1+\left(e^{2x}\right)^2}\left(2e^{2x}\right) = \frac{2e^{2x}}{1+e^{4x}}$$

$$g(x) = \frac{\arcsin 4x}{8x}$$

Hint: Use Quotient Rule: $D_x\left(\frac{u}{v}\right) = \frac{v \cdot u' - u \cdot v'}{v^2}$.

Hint: Chain Rule: $D_x \sin^{-1}(Expr) = \frac{1}{\sqrt{1-(Expr)^2}} D_x(Expr)$

$$g'(x) = \frac{(8x)D_x(\arcsin 4x) - (\arcsin 4x)D_x(8x)}{(8x)^2}$$

Note: $D_x(\arcsin 4x) = \frac{1}{\sqrt{1-(4x)^2}} D_x(4x) = \frac{1}{\sqrt{1-(4x)^2}}(4)$

$$g'(x) = \frac{(8x)\left(\frac{4}{\sqrt{1-(4x)^2}}\right) - (\arcsin 4x)(8)}{(8x)^2}$$

$h(t) = \sin(\arctan t)$

Chain Rule: $D_x \tan^{-1}\left(Expr\right) = \dfrac{1}{1+\left(Expr\right)^2} D_x\left(Expr\right)$

Chain Rule: $D_x \sin\left(Expr\right) = \cos\left(Expr\right) D_x\left(Expr\right)$

$h'(t) = \cos\left(\arctan t\right) D_x\left(\arctan t\right)$

$Note: \quad D_x\left(\arctan t\right) = \dfrac{1}{1+\left(t\right)^2} D_t\left(t\right) = \dfrac{1}{1+t^2}(1) = \dfrac{1}{1+t^2}$

$h'(t) = \cos\left(\arctan t\right)\left(\dfrac{1}{1+t^2}\right)$

$y = 12\arctan x$ Find equation of tangent line at (0, 0)

Chain Rule: $D_x \tan^{-1}\left(Expr\right) = \dfrac{1}{1+\left(Expr\right)^2} D_x\left(Expr\right)$

a) $y' = 12\left[\dfrac{1}{1+\left(x\right)^2} D_x\left(x\right)\right] = 12\left[\dfrac{1}{1+\left(x\right)^2}\right]$

b) Slope of tangent line = $y'\left(0\right) = 12\left[\dfrac{1}{1+\left(0\right)^2}\right] = 12$

c) Equation of tangent line : $y - y_1 = m(x - x_1)$

$y - 0 = 12\left(x - 0\right)$

$y = \arcsin\left(\frac{x}{4}\right)$ Find equation of tangent line at (0,0)

Chain Rule: $D_x \sin^{-1}\left(Expr\right) = \frac{1}{\sqrt{1-\left(Expr\right)^2}} D_x\left(Expr\right)$

a) $y' = \frac{1}{\sqrt{1-\left(x/4\right)^2}} D_x\left(x/4\right) = \frac{1}{\sqrt{1-\left(x/4\right)^2}}\left(1/4\right)$

b) Slope of tangent line = $y'\left(0\right) = \frac{1}{\sqrt{1-\left(0/4\right)^2}}\left(1/4\right) = 1/4$

c) Equation of tangent line : $y - y_1 = m(x - x_1)$

$$y - 0 = \frac{1}{4}\left(x - 0\right)$$

$y = 2x \cdot \arcsin\left(x-1\right)$ Find equation of tangent line at (1,0)

Chain Rule: $D_x \sin^{-1}\left(Expr\right) = \dfrac{1}{\sqrt{1-\left(Expr\right)^2}} D_x\left(Expr\right)$

$$y' = \left(2x\right)\left(\frac{1}{\sqrt{1-\left(x-1\right)^2}}\right) + \left(\arcsin\left(x-1\right)\right)\left(2\right)$$

Slope of tangent line $= y'\left(1\right)$

$$= \left(2\left(1\right)\right)\left(\frac{1}{\sqrt{1-\left(\left(1\right)-1\right)^2}}\right) + \left(\arcsin\left(\left(1\right)-1\right)\right)\left(2\right) = \left(2\right)\left(1\right) + \left(0\right)\left(1\right) = 2$$

Equation of tangent line : $y - y_1 = m(x - x_1)$

$$y - 0 = 2\left(x-1\right)$$

$y=\sec^{-1}(4x)$ Find tangent line at $\left(\frac{\sqrt{2}}{4},\frac{\pi}{4}\right)$

$$y'=\frac{1}{|4x|\sqrt{(4x)^2+1}}D_x(4x)=\frac{1}{|4x|\sqrt{(4x)^2+1}}(4)=\frac{4}{|4x|\sqrt{(4x)^2+1}}$$

Slope of tangent line at $\left(\frac{\sqrt{2}}{4},\frac{\pi}{4}\right)=\dfrac{4}{\left|4\left(\frac{\sqrt{2}}{4}\right)\right|\sqrt{\left(4\left(\frac{\sqrt{2}}{4}\right)\right)^2+1}}=\dfrac{4}{\sqrt{2}\sqrt{3}}=\dfrac{4}{\sqrt{6}}$

Equation of Tangent Line is $y-\frac{\pi}{4}=\frac{4}{\sqrt{6}}\left(x-\frac{\sqrt{2}}{4}\right)$