Using Table of Integration Formulas

1) Find the indefinite integral: $\int \frac{2}{x^2(4+3x)^2}\,dx.$

Using Formula 13: Let $a=4; b=3;\quad u=x \quad \Rightarrow \quad du=dx$

$$\int \frac{2}{x^2(4+3x)^2}\,dx = 2\left[\int \frac{1}{x^2(4+3x)^2}\,dx.\right] = 2\left[\int \frac{1}{u^2(a+bx)^2}\,du\right]$$

$$= 2\left[-\frac{1}{4^2}\left[\frac{4+2(3)u}{u(4+3(3))}+\frac{2(3)}{4}\ln\left|\frac{u}{4+3u}\right|\right]\right]+C$$

$$= 2\left[-\frac{1}{16}\left[\frac{4+6x}{13x}+\frac{6}{4}\ln\left|\frac{x}{4+3x}\right|\right]\right]+C$$

2) Find the indefinite integral: $\int \frac{\sqrt{64-x^4}}{x}\,dx.$

Let $u = x^2 \Rightarrow u^2 = \left(x^2\right)^2 = x^4 \Rightarrow \sqrt{64-x^4} = \sqrt{64-u^2}$

Let $u = x^2 \Rightarrow \sqrt{u} = \sqrt{x^2} = x$ or $x = \sqrt{u}$

Also, $x = \sqrt{u} = u^{1/2} \Rightarrow \frac{dx}{du} = \frac{1}{2}u^{-1/2} \Rightarrow dx = \frac{1}{2}u^{-1/2}du = \frac{1}{2u^{1/2}}\,du$

$\int \frac{\sqrt{64-x^4}}{x}\,dx = \int \frac{\sqrt{64-u^2}}{u^{1/2}} \frac{1}{2u^{1/2}}\,du = \int \frac{\sqrt{64-u^2}}{2u}\,du$ Note: $u^{1/2} \cdot 2u^{1/2} = 2u$

Using Formula 39:

$$\int \frac{\sqrt{64-u^2}}{2u}\,du = \frac{1}{2}\left[\int \frac{\sqrt{64-u^2}}{u}\,du\right] = \frac{1}{2}\left[\sqrt{64-u^2} - 8\ln\left|\frac{8+\sqrt{64-u^2}}{u}\right|\right] + C$$

$$= \frac{1}{2}\left[\sqrt{64-x^4} - 8\ln\left|\frac{8+\sqrt{64-x^4}}{x^2}\right|\right] + C$$

3) Find the indefinite integral: $\int \frac{\sin^4 \sqrt{x}}{\sqrt{x}} dx.$

Let $u = \sqrt{x} = x^{1/2} \Rightarrow u^2 = \left(\sqrt{x}\right)^2 = x$

Also, $x = u^2 \Rightarrow \frac{dx}{du} = 2u \Rightarrow dx = 2udu$

$$\int \frac{\sin^4 \sqrt{x}}{\sqrt{x}} dx = \int \frac{\sin^4 u}{u} 2udu = 2\int \sin^4 udu$$

Using Formula 50 with $n = 4$:

$$2\int \sin^4 udu = 2\left[-\frac{\sin^3 u \cdot \cos u}{4} + \frac{3}{4}\left(\int \sin^2 udu\right)\right] + C$$

$$= 2\left[-\frac{\sin^3 u \cdot \cos u}{4} + \frac{3}{4}\left(\frac{1}{2}\left(u - \sin u \cos u\right)\right)\right] + C$$

$$= 2\left[-\frac{\sin^3 \sqrt{x} \cdot \cos \sqrt{x}}{4} + \frac{3}{4}\left(\frac{1}{2}\left(\sqrt{x} - \sin \sqrt{x} \cos \sqrt{x}\right)\right)\right] + C$$

4) Find the indefinite integral: $\int e^{-4x} \sin 3xdx.$

Using Formula 85 with $a = -4; b = 3; u = x; du = dx$:

$$\int e^{-4x} \sin 3xdx = \int e^{-4u} \sin 3udu$$

$$= \frac{e^{au}}{a^2 + b^2}\left(a \sin bu - b \cos bu\right) + C$$

$$= \frac{e^{-4x}}{25}\left(-4 \sin 3x - 3 \cos 3x\right) + C$$

5) Find the indefinite integral: $\int (\ln x)^3 \, dx$.

Using Formula 91 with $n = 3; u = x; du = dx$:

$$\int (\ln x)^3 \, dx = \int (\ln u)^3 \, du$$

$$= u(\ln u)^3 - 3\int (\ln u)^2 \, du$$

$$= u(\ln u)^3 - 3\left[u\left(2 - 2\ln u + (\ln u)^2\right)\right] + C$$

$$= x(\ln x)^3 - 3\left[x\left(2 - 2\ln x + (\ln x)^2\right)\right] + C$$

6) Find the indefinite integral: $\int \frac{e^x}{1+\tan e^x} \, dx$.

Let $u = e^x \Rightarrow du = e^x dx$

Using Formula 71:

$$\int \frac{e^x}{1+\tan e^x} \, dx = \int \frac{1}{1+\tan e^x} e^x dx = \int \frac{1}{1+\tan u} \, du$$

$$= \frac{1}{2}\left[u + \ln\left|\cos u + \sin u\right|\right] + C$$

$$= \frac{1}{2}\left[e^x + \ln\left|\cos e^x + \sin e^x\right|\right] + C$$

7) Find the indefinite integral: $\int \frac{e^x}{\left(1-e^{2x}\right)^{3/2}}\,dx.$

Let $u = e^x \Rightarrow u^2 = \left(e^x\right)^2 = e^{2x}$

$Also,\; u = e^x \Rightarrow\; du = e^x dx$

Using Formula 45 with $a=1$:

$$\int \frac{e^x}{\left(1-e^{2x}\right)^{3/2}}\,dx = \int \frac{u}{\left(1-u\right)^{3/2}}\,dx = \frac{u}{a^2\sqrt{a^2-u^2}} + C$$

$$= \frac{e^x}{1\sqrt{1-e^{2x}}} + C = \frac{e^x}{\sqrt{1-e^{2x}}} + C$$

8) Find the indefinite integral: $\int \frac{\cos x}{\sqrt{\sin^2 x+1}}\,dx.$

Let $u = \sin x \Rightarrow\; du = \cos x dx$

Using Formula 31 with $a=1$:

$$\int \frac{\cos x}{\sqrt{\sin^2 x+1}}\,dx = \int \frac{1}{\sqrt{\sin^2 x+1}}\cos x dx$$

$$= \int \frac{1}{\sqrt{u^2+1}}\,du$$

$$= \ln\left|u+\sqrt{u^2+a^2}\right| + C$$

$$= \ln\left|\sin x+\sqrt{\sin^2 x+1}\right| + C$$

9) Find the definite integral: $\int_0^4 \frac{x}{\sqrt{3+2x}}\,dx$

Using Formula 21 with $a=3; b=2; u=x; du=dx$:

$$\int \frac{x}{\sqrt{3+2x}}\,dx = \int \frac{u}{\sqrt{3+2u}}\,du$$

$$= \frac{-2(2a-bu)}{3b^2}\sqrt{a+bu}$$

$$= \frac{-2(6-2x)}{12}\sqrt{3+2x}\Bigg|_0^4$$

$$= \left[\frac{4}{12}\sqrt{11}\right] - \left[\frac{-12}{12}\sqrt{3}\right] = \frac{1}{3}\sqrt{11} + \sqrt{3}$$

10) Find the definite integral: $\int_0^5 \frac{x^2}{(5+2x)^2}\,dx$

Using Formula 7 with $a=5; b=2; u=x; du=dx$:

$$\int_0^5 \frac{x^2}{(5+2x)^2}\,dx = \int_0^5 \frac{u^2}{(5+2u)^2}\,du$$

$$= \frac{1}{8}\left[2x - \frac{25}{5+2x} - 10\ln|5+2x|\right]\Bigg|_0^5$$

$$= \frac{1}{8}\left[10 - \frac{25}{15} - 10\ln|15|\right] - \frac{1}{8}\left[0 - \frac{25}{5} - 10\ln|5|\right]$$