

Using Table of Integration Formulas

1) Find the indefinite integral: $\int \frac{2}{x^2(4+3x)^2} dx$.

Using Formula 34: Let $a = 4; b = 3; u = x \Rightarrow du = dx$

$$\begin{aligned}\int \frac{2}{x^2(4+3x)^2} dx &= 2 \left[\int \frac{1}{x^2(4+3x)^2} dx \right] = 2 \left[\int \frac{1}{u^2(a+bx)^2} du \right] \\ &= 2 \left[\frac{1}{4^2} \left[\frac{4+2(3)x}{x(4+3x)} + \frac{2(3)}{4} \ln \left| \frac{x}{4+3x} \right| \right] \right] + C \\ &= 2 \left[\frac{1}{16} \left[\frac{4+6x}{x(4+3x)} + \frac{6}{4} \ln \left| \frac{x}{4+3x} \right| \right] \right] + C \\ &= \frac{1}{8} \left[\frac{4+6x}{x(4+3x)} + \frac{3}{2} \ln \left| \frac{x}{4+3x} \right| \right] + C\end{aligned}$$

2) Find the indefinite integral: $\int \frac{\sqrt{64-x^4}}{x} dx$.

$$\text{Let } u = x^2 \Rightarrow u^2 = (x^2)^2 = x^4 \Rightarrow \sqrt{64-x^4} = \sqrt{64-u^2}$$

$$\text{Let } u = x^2 \Rightarrow \sqrt{u} = \sqrt{x^2} = x \text{ or } x = \sqrt{u}$$

$$\text{Also, } x = \sqrt{u} = u^{1/2} \Rightarrow \frac{dx}{du} = \frac{1}{2}u^{-1/2} \Rightarrow dx = \frac{1}{2}u^{-1/2} du = \frac{1}{2u^{1/2}} du$$

$$\int \frac{\sqrt{64-x^4}}{x} dx = \int \frac{\sqrt{64-u^2}}{u^{1/2}} \frac{1}{2u^{1/2}} du = \int \frac{\sqrt{64-u^2}}{2u} du \quad \text{Note: } u^{1/2} \cdot 2u^{1/2} = 2u$$

Using Formula 64:

$$\begin{aligned} \int \frac{\sqrt{64-u^2}}{2u} du &= \frac{1}{2} \left[\int \frac{\sqrt{64-u^2}}{u} du \right] = \frac{1}{2} \left[\sqrt{64-u^2} - 8 \ln \left| \frac{8 + \sqrt{64-u^2}}{u} \right| \right] + C \\ &= \frac{1}{2} \left[\sqrt{64-x^4} - 8 \ln \left| \frac{8 + \sqrt{64-x^4}}{x^2} \right| \right] + C \end{aligned}$$

3) Find the indefinite integral: $\int \frac{\sin^4 \sqrt{x}}{\sqrt{x}} dx$.

$$\text{Let } u = \sqrt{x} = x^{1/2} \Rightarrow u^2 = (\sqrt{x})^2 = x$$

$$\text{Also, } x = u^2 \Rightarrow \frac{dx}{du} = 2u \Rightarrow dx = 2u du$$

$$\int \frac{\sin^4 \sqrt{x}}{\sqrt{x}} dx = \int \frac{\sin^4 u}{u} 2u du = 2 \int \sin^4 u du$$

Using Formula 85 with $n = 4$:

$$\begin{aligned} 2 \int \sin^4 u du &= 2 \left[-\frac{\sin^3 u \cdot \cos u}{4} + \frac{3}{4} \left(\int \sin^2 u du \right) \right] + C \\ &= 2 \left[-\frac{\sin^3 u \cdot \cos u}{4} + \frac{3}{4} \left(\frac{1}{2} (u - \sin u \cos u) \right) \right] + C \\ &= 2 \left[-\frac{\sin^3 \sqrt{x} \cdot \cos \sqrt{x}}{4} + \frac{3}{4} \left(\frac{1}{2} (\sqrt{x} - \sin \sqrt{x} \cos \sqrt{x}) \right) \right] + C \end{aligned}$$

4) Find the indefinite integral: $\int e^{-4x} \sin 3x dx$.

Using Formula 118 with $a = -4; b = 3; u = x; du = dx$:

$$\begin{aligned}\int e^{-4x} \sin 3x dx &= \int e^{-4u} \sin 3u du \\ &= \frac{e^{au}}{a^2 + b^2} (a \sin bu - b \cos bu) + C \\ &= \frac{e^{-4x}}{25} (-4 \sin 3x - 3 \cos 3x) + C\end{aligned}$$

5) Find the indefinite integral: $\int (\ln x)^3 dx$.

Using Formula 115c with $n = 3; u = x; du = dx$:

$$\begin{aligned}\int (\ln x)^3 dx &= \int (\ln u)^3 du \\ &= u(\ln u)^3 - 3 \int (\ln u)^2 du \\ &= u(\ln u)^3 - 3 \left[u(2 - 2 \ln u + (\ln u)^2) \right] + C && \text{Using Formula \#115b} \\ &= x(\ln x)^3 - 3 \left[x(2 - 2 \ln x + (\ln x)^2) \right] + C\end{aligned}$$

6) Find the indefinite integral: $\int \frac{e^x}{1 + \tan e^x} dx$.

Let $u = e^x \Rightarrow du = e^x dx$

Using Formula 100b:

$$\begin{aligned}\int \frac{e^x}{1 + \tan e^x} dx &= \int \frac{1}{1 + \tan e^x} e^x dx = \int \frac{1}{1 + \tan u} du \\ &= \frac{1}{2} [u + \ln |\cos u + \sin u|] + C \\ &= \frac{1}{2} [e^x + \ln |\cos e^x + \sin e^x|] + C\end{aligned}$$

7) Find the indefinite integral: $\int \frac{e^x}{(1-e^{2x})^{3/2}} dx$.

$$\text{Let } u = e^x \Rightarrow u^2 = (e^x)^2 = e^{2x}$$

$$\text{Also, } u = e^x \Rightarrow du = e^x dx$$

Using Formula 70 with $a = 1$:

$$\begin{aligned} \int \frac{e^x}{(1-e^{2x})^{3/2}} dx &= \int \frac{u}{(1-u)^{3/2}} dx = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C \\ &= \frac{e^x}{1\sqrt{1-e^{2x}}} + C = \frac{e^x}{\sqrt{1-e^{2x}}} + C \end{aligned}$$

8) Find the indefinite integral: $\int \frac{\cos x}{\sqrt{\sin^2 x + 1}} dx$.

Let $u = \sin x \Rightarrow du = \cos x dx$

Using Formula 48 with $a = 1$:

$$\begin{aligned}\int \frac{\cos x}{\sqrt{\sin^2 x + 1}} dx &= \int \frac{1}{\sqrt{\sin^2 x + 1}} \cos x dx \\ &= \int \frac{1}{\sqrt{u^2 + 1}} du \\ &= \ln \left| u + \sqrt{u^2 + a^2} \right| + C \\ &= \ln \left| \sin x + \sqrt{\sin^2 x + 1} \right| + C\end{aligned}$$

9) Find the definite integral: $\int_0^4 \frac{x}{\sqrt{3+2x}} dx$

Using Formula 38 with $a = 3; b = 2; u = x; du = dx$:

$$\int \frac{x}{\sqrt{3+2x}} dx = \int \frac{u}{\sqrt{3+2u}} du$$

$$= \frac{-2(2a - bu)}{3b^2} \sqrt{a + bu}$$

$$= \frac{-2(6 - 2x)}{12} \sqrt{3 + 2x} \Big|_0^4$$

$$= \left[\frac{4}{12} \sqrt{11} \right] - \left[\frac{-12}{12} \sqrt{3} \right] = \frac{1}{3} \sqrt{11} + \sqrt{3}$$

10) Find the definite integral: $\int_0^5 \frac{x^2}{(5+2x)^2} dx$

Using Formula 30 with $a = 5; b = 2; u = x; du = dx$:

$$\int_0^5 \frac{x^2}{(5+2x)^2} dx = \int_0^5 \frac{u^2}{(5+2u)^2} du$$

$$= \frac{1}{8} \left[2x - \frac{25}{5+2x} - 10 \ln|5+2x| \right]_0^5$$

$$= \frac{1}{8} \left[10 - \frac{25}{15} - 10 \ln|15| \right] - \frac{1}{8} \left[0 - \frac{25}{5} - 10 \ln|5| \right]$$