Calculus III Section 12.5

1) Let $\mathbf{r}(t)=\langle 4t,3t\rangle$

Note: $\mathbf{r}(t)=\langle 4t,3t\rangle = \langle x(t),y(t)\rangle$

Find the length of the plane curve $\mathbf{r}(t)$ over the interval [0,4].

a) $x'(t)=\dfrac{dx}{dt}=$ _?_________

b) $y'(t)=\dfrac{dy}{dt}=$ _?_________

c) $s=$ Arc Length $=\displaystyle\int_a^b \sqrt{[x'(t)]^2+[y'(t)]^2}\,dt=\int_a^b \|\mathbf{r}'(t)\|dt=$ _?_________

2) Let $\mathbf{r}(t)=\langle t,t^2-4,t^3\rangle$

Note: $\mathbf{r}(t)=\langle t,t^2-4,t^3\rangle = \langle x(t),y(t),z(t)\rangle$

Find the length of the plane curve $\mathbf{r}(t)$ over the interval [0,1].

a) $x'(t)=\dfrac{dx}{dt}=$ _?_________

b) $y'(t)=\dfrac{dy}{dt}=$ _?_________

c) $z'(t)=\dfrac{dz}{dt}=$ _?_________

d) $s=$ Arc Length $=\displaystyle\int_a^b \sqrt{[x'(t)]^2+[y'(t)]^2+[z'(t)]^2}\,dt=\int_a^b \|\mathbf{r}'(t)\|dt=$ _?_________

3) Let $\mathbf{r}(t) = \left\langle \sqrt{t}, t^2 - 4, \sqrt{t^3} \right\rangle$

Note: $\mathbf{r}(t) = \left\langle \sqrt{t}, t^2 - 4, \sqrt{t^3} \right\rangle = \langle x(t), y(t), z(t) \rangle$

Find the length of the plane curve $\mathbf{r}(t)$ over the interval [0,1].

a) $x'(t) = \dfrac{dx}{dt} =$ __?__________

b) $y'(t) = \dfrac{dy}{dt} =$ __?__________

c) $z'(t) = \dfrac{dz}{dt} =$ __?__________

d) $s =$ Arc Length $= \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\, dt = \int_a^b \|\mathbf{r}'(t)\| dt =$ __?__________

4) Let $\mathbf{r}(t) = \left\langle e^t, e^{t^2}, t \right\rangle$

Note: $\mathbf{r}(t) = \left\langle e^t, e^{t^2}, t \right\rangle = \langle x(t), y(t), z(t) \rangle$

Find the length of the plane curve $\mathbf{r}(t)$ over the interval [0,1].

a) $x'(t) = \dfrac{dx}{dt} =$ __?__________

b) $y'(t) = \dfrac{dy}{dt} =$ __?__________

c) $z'(t) = \dfrac{dz}{dt} =$ __?__________

d) $s =$ Arc Length $= \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\, dt = \int_a^b \|\mathbf{r}'(t)\| dt =$ __?__________

5) Let $\mathbf{r}(t) = \langle t, 2t, 3t \rangle$

a) Find $\mathbf{v}(t) = \mathbf{r}'(t) =$ __?________________

b) Find $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} =$ __?________________

c) Find $\mathbf{T}'(t) =$ __?________________

d) Find the curvature K at $t = 1$: $K = \dfrac{\|\mathbf{T}'(1)\|}{\|\mathbf{r}'(1)\|} =$ __?________________

6) Let $\mathbf{r}(t) = \langle t^2, 2t^3, 3t^3 \rangle$

a) Find $\mathbf{v}(t) = \mathbf{r}'(t) =$ __?________________

b) Find $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} =$ __?________________

c) Find $\mathbf{T}'(t) =$ __?________________

d) Find the curvature K at $t = 1$: $K = \dfrac{\|\mathbf{T}'(1)\|}{\|\mathbf{r}'(1)\|} =$ __?________________

7) Let $\mathbf{r}(t) = \langle \cos t, -4\sin t, t^3 \rangle$

a) Find $\mathbf{v}(t) = \mathbf{r}'(t) =$ _?_______________

b) Find $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} =$ _?_______________

c) Find $\mathbf{T}'(t) =$ _?_______________

d) Find the curvature K at $t = 1$: $K = \dfrac{\|\mathbf{T}'(1)\|}{\|\mathbf{r}'(1)\|} =$ _?_______________