Section 13.10

1) A manufacturer of walking and running shoes finds that the total daily cost $C(x, y)$ of producing x units of walking shoes and y units of running shoes is given by $C(x, y) = 120 + 5x - .04x^2 + 5y - .06y^2$.
Find x and y so that the cost $C(x, y)$ is minimized.

2) A manufacturer of walking and running shoes finds that the total daily cost $C(x, y)$ of producing x units of walking shoes and y units of running shoes is given by $C(x, y) = 120 + 5x - .04x^2 + 5y - .06y^2$.
Due to budget constraint, the manufacturer can only produce a total of 100 pairs of shoes. Find x and y so that the cost $C(x, y)$ is minimized.

3) Minimize: $f(x, y) = 4xy$

Constraint: $g(x, y) = x + y = 4$

a) $\nabla f(x, y) =$ _?____

b) $\nabla g(x, y) =$ _?____

c) System of equations $\nabla f(x, y) = \lambda \nabla g(x, y)$: $x =$ _?___ $y =$ _?____

4) Maximize: $f(x, y) = x + 4xy + y$

Constraint: $g(x, y) = 3x + y = 14$

a) $\nabla f(x, y) =$ _?____

b) $\nabla g(x, y) =$ _?____

c) System of equations $\nabla f(x, y) = \lambda \nabla g(x, y)$: $x =$ _?___ $y =$ _?____

5) Minimize: $f(x, y) = \sqrt{4 - x^2 - y^2}$

Constraint: $g(x, y) = x + y - 1 = 0$

a) $\nabla f(x, y, z) =$ __?___

b) $\nabla g(x, y, z) =$ __?___

c) Solve the system of equations $\nabla f(x, y) = \lambda \nabla g(x, y)$: $x =$ __?___ $y =$ __?___

6) Minimize: $f(x, y, z) = x^2 + y^2 + z^2$

Constraint: $g(x, y, z) = x + y + z - 1 = 0$

a) $\nabla f(x, y, z) =$ __?___

d) $\nabla g(x, y, z) =$ __?___

c) Solve the system of equations $\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$: $x =$ __?___ $y =$ __?___ $z =$ __?___

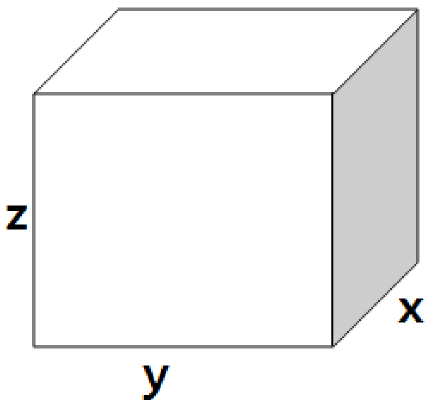
7) A box in the shape of a rectangular solid has a volume of 40 cubic feet.

The top and bottom of the box cost \$10 per square foot to construct.

The sides of the box cost \$8 per square foot to construct.

Find the dimensions of the box so that the cost of constructing is minimized.

Let $x =$ Length $y =$ width $z =$ height $C =$ cost of material



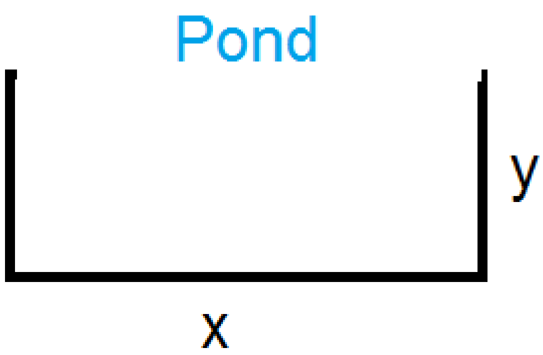
a) Cost Function in terms of x, y, z: $C(x, y, z) =$ _______?______

b) Constraint Function: $g(x, y, z) =$ Volume = $xyz =$ _______?___________

c) $\nabla C(x, y, z) =$ __?______

d) Solve the system of equations $\nabla C(x, y, z) = \lambda \nabla g(x, y, z)$: $x =$ _?_ $y =$ _?_ $z =$ _?_

8) A rancher is planning to build an enclosed area along a pond. It is to be rectangular with an area of 10,000 square feet and is to be fenced on the three sides not adjacent to the pond. Find the dimensions (x and y) such that the least amount of fencing will be needed to construct the enclosed area?



a) $x = \underline{\ ?\ }$ $\quad y = \underline{\ ?\ }$

b) Amount of Fencing $= y = \underline{\ ?\ }$