1) Evaluate $\int x\sqrt{x^2-100}\ dx.$ Hint: Let $u = x^2 - 100$

$$\frac{du}{dx} = 2x \Rightarrow \frac{1}{2}du = xdx$$

$$\int x\sqrt{x^2-100}\ dx = \int \sqrt{u}\,\frac{1}{2}du = \frac{1}{2}\frac{u^{3/2}}{3/2} = \frac{1}{2}\frac{\left(x^2-100\right)^{3/2}}{3/2} + C$$

2) Evaluate $\int \frac{x}{x^2-64}\ dx$ Hint: Let $u = x^2 - 64$

$$\frac{du}{dx} = 2x \Rightarrow \frac{1}{2}du = xdx$$

$$\int \frac{x}{x^2-64}\ dx = \int \frac{1}{x^2-64}x\ dx = \int \frac{1}{u}\frac{1}{2}\ du = \frac{1}{2}\ln|u| = \frac{1}{2}\ln|x^2-64| + C$$

3) Evaluate $\int 5x\sqrt{x-7}\ dx.$ Hint: Let $u = x - 7$

$$u = x - 7 \Rightarrow x = u + 7$$

$$\frac{du}{dx} = 1 \Rightarrow du = dx$$

$$\int 5x\sqrt{x-7}\ dx = 5\int (u+7)\sqrt{u}\ du = \int (u^{3/2} + 7u^{1/2})\ du$$

$$= \frac{u^{5/2}}{5/2} + \frac{7u^{3/2}}{3/2} = \frac{\left(x-7\right)^{5/2}}{5/2} + \frac{7\left(x-7\right)^{3/2}}{3/2} + C$$

4) Evaluate $\int \frac{\sqrt{x^2-9}}{x}\ dx$ Hint: Use Integration Tables

Using Formula 29:

$$\int \frac{\sqrt{x^2-9}}{x}\ dx = \sqrt{x^2-9} - 3\sec^{-1}\left(\frac{|x|}{3}\right) + C$$

5) Evaluate $\int_3^4 x^3\sqrt{x^2-9}\ dx.$ Hint: Let $u = x^2 - 9;\quad x^2 = u + 9$

$$\frac{du}{dx} = 2x \Rightarrow \frac{1}{2}du = xdx$$

$$\int_3^4 x^3\sqrt{x^2-9}\ dx = \int_3^4 \sqrt{x^2-9}x^2 x\ dx = \int_3^4 \sqrt{u}(u+9)\frac{1}{2}du$$

$$= \frac{1}{2}\int_3^4 (u^{3/2} + 9u^{1/2})du = \frac{1}{2}\left[\frac{u^{5/2}}{5/2} + \frac{9u^{3/2}}{3/2}\right] = \frac{1}{2}\left[\frac{\left(x^2-9\right)^{5/2}}{5/2} + \frac{9\left(x^2-9\right)^{3/2}}{3/2}\right]_3^4 = 81.4891$$

6) Use partial fraction decomposition to evaluate $\int \frac{5x - 2}{x^2 - x}\ dx.$

$$\frac{5x - 2}{x^2 - x} = \frac{2}{x} + \frac{3}{x-1}$$

$$\int \frac{5x - 2}{x^2 - x}\ dx = \int\left(\frac{2}{x} + \frac{3}{x-1}\right)dx = 2\ln|x| + 3\ln|x-1| + C$$

7) Use partial fraction decomposition to evaluate $\int \frac{x^3 - x + 3}{x^2 + x - 2}\ dx.$

Hint: Use long division to divide $\frac{x^3 - x + 3}{x^2 + x - 2}$ first.

Long Division: $\frac{x^3 - x + 3}{x^2 + x - 2} = \text{x-1} + \frac{\text{2x+1}}{\text{x}^2 + \text{x - 2}}$

Partial fraction decomposition: $\frac{\text{2x+1}}{\text{x}^2 + \text{x - 2}} = \frac{1}{x-1} + \frac{1}{x+2}$

$$\int \frac{x^3 - x + 3}{x^2 + x - 2}\ dx = \int\left(\text{x-1} + \frac{1}{x-1} + \frac{1}{x+2}\right)dx = \frac{x^2}{2} - x + \ln|x-1| + \ln|x+2| + C$$

8) Evaluate $\int \frac{15x}{(3+7x)^2}\ dx = 15\int \frac{x}{(3+7x)^2}\ dx.$ Hint: Use Integration Tables

Solution: Use Formula 5 with $n = 2$, a=3, b=7.

9) Evaluate $\int \frac{12x}{x^2+5x-10}\,dx = 12\int \frac{x}{x^2+5x-10}\,dx.$ Hint: Use Integration Tables

Solution:

Use Formula 15 with a=-10, b=5, c = 1

Then use Formula 14.

10) Find $\lim_{x\to -2} \frac{x^2-3x-10}{x+2}$

$$\lim_{x\to -2} \frac{x^2-3x-10}{x+2} = \lim_{x\to -2} \frac{2x-3}{1} = -7$$

11) Find $\lim_{x\to\infty} \frac{x^3}{e^{x^2}}$

$$\lim_{x\to\infty} \frac{x^3}{e^{x^2}} = \lim_{x\to\infty} \frac{3x^2}{e^{x^2}(2x)} = \lim_{x\to\infty} \frac{3x}{2e^{x^2}} = \lim_{x\to\infty} \frac{3}{2e^{x^2}(2x)} = \frac{3}{\infty} = 0$$

12) Evaluate $\int \frac{x^3}{\sqrt{4+x^2}}\,dx.$ Hint: Let $u = 4+x^2$; $x^2 = u-4$

$$\frac{du}{dx} = 2x \Rightarrow \frac{1}{2}du = xdx$$

$$\int \frac{x^3}{\sqrt{4+x^2}}\,dx = \int \frac{1}{\sqrt{4+x^2}} x^2 x\,dx = \int \frac{1}{\sqrt{u}}(u-4)\frac{1}{2}du = \frac{1}{2}\int \left(u^{-1/2}\right)(u-4)\,du$$

$$= \frac{1}{2}\int \left(u^{1/2} - 4u^{-1/2}\right)du = \frac{1}{2}\left[\frac{u^{3/2}}{3/2} - \frac{4u^{1/2}}{1/2}\right] = \frac{1}{2}\left[\frac{\left(4+x^2\right)^{3/2}}{3/2} - \frac{4\left(4+x^2\right)^{1/2}}{1/2}\right] + C$$

13) Determine if improper integral converges or diverges.

Evaluate the integral if integral converges.

$$\int_0^5 \frac{12}{x}\,dx = \lim_{b\to 0^-}\int_b^5 \frac{12}{x}\,dx = \lim_{b\to 0^-} 12\ln|x|\Big|_b^5$$

$$\lim_{b\to 0^-}\left(12\ln 5 - 12\ln b\right) = 12\ln 5 - (-\infty) = \infty$$

Note: As $b \to 0^-, \ln b \to -\infty$

Therefore, $\int_0^5 \frac{12}{x}\,dx$ diverges to ∞.

14) Determine if improper integral converges or diverges.

Evaluate the integral if integral converges.

$$\int_3^6 \frac{1}{\sqrt{36-x^2}}\,dx$$

$$\int_3^6 \frac{1}{\sqrt{36-x^2}}\,dx = \lim_{b\to 6^+}\int_3^b \frac{1}{\sqrt{36-x^2}}\,dx$$

Note: $\int \frac{1}{\sqrt{36-x^2}}\,dx = \sin^{-1}\left(\frac{x}{6}\right)$ Formula 41

$$\lim_{b\to 6^+}\int_3^b \frac{1}{\sqrt{36-x^2}}\,dx = \lim_{b\to 6^+}\left[\sin^{-1}\left(\frac{b}{6}\right) - \sin^{-1}\left(\frac{3}{6}\right)\right]$$

$$= \sin^{-1}\left(\frac{6}{6}\right) - \sin^{-1}\left(\frac{3}{6}\right) = \sin^{-1}(1) - \sin^{-1}(1/2) = \frac{\pi}{2} - \frac{\pi}{6}$$

So $\int_3^6 \frac{1}{\sqrt{36-x^2}}\,dx$ converges.