1) Find $\lim_{n\to\infty}\frac{5n^2}{n^2+2}$

$$\lim_{n\to\infty}\frac{5n^2}{n^2+2}=\lim_{n\to\infty}\frac{5n^2/n^2}{\left(n^2+2\right)/n^2}=\lim_{n\to\infty}\frac{5}{1+2/n^2}=\frac{5}{1+0}=5$$

2) Find $\lim_{n\to\infty}\frac{2n}{\sqrt{n^2+1}}$

$$\lim_{n\to\infty}\frac{2n}{\sqrt{n^2+1}}=\lim_{n\to\infty}\frac{2n/\sqrt{n^2}}{\sqrt{n^2+1}/\sqrt{n^2}}=\lim_{n\to\infty}\frac{2n/n}{\sqrt{\frac{n^2+1}{n^2}}}=\lim_{n\to\infty}\frac{2}{\sqrt{1+\frac{1}{n^2}}}=\frac{2}{\sqrt{1+0}}=2$$

3) $\sum_{n=1}^{\infty}\frac{n}{2n+3}$

Use the nth-term Test to explain why series diverges.

$$\lim_{n\to\infty}\frac{n}{2n+3}=1/2.$$

Therefore, by the nth-term Test, the series $\sum_{n=1}^{\infty}\frac{n}{2n+3}$ diverges.

4) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

Use the nth-term Test to explain why series diverges.

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1$$

Therefore, by the nth-term Test, the series $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$ diverges.

5) $\sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n$

Find $S_5 =$ ______________________ (sum of first 5 terms)

Find $S_{10} =$ ________________________(sum of first 10 terms)

Use the Geometric Series Test to explain why series converges.

Since $r = 2/5$ means that r is between -1 and 1.

Therefore by Geometric Series Test $\sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n$ converges

6) $\sum_{n=1}^{\infty} \frac{2}{3n+2}$ Use the Integral Test to explain why series diverges

Find $\int_1^{\infty} \frac{2}{3x+2} dx = \frac{2}{3} \ln|3x+2| \Big|_1^{\infty} = \infty$

Therefore, by the Integral Test, the series $\sum_{n=1}^{\infty} \frac{2}{3n+2}$ diverges.

7) $\sum_{n=1}^{\infty} \frac{3}{n^{5/3}}$

Use the p-series Test to explain why series converges.

Since $p = 5/3$ means that $p > 1$.

Therefore, by p-series Test, the series $\sum_{n=1}^{\infty} \frac{3}{n^{5/3}}$ converges.

8) $\sum_{n=1}^{\infty} \frac{5}{4^n + 1}$

Use Limit Comparison Test or Direct Comparison Test to explain why series converges:

Explain why series converges.

Let $a_n = \frac{5}{4^n + 1}$ and $b_n = \frac{1}{4^n} = \left(\frac{1}{4}\right)^n$

Hence, the series $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$ is a Geometric Series with $r = 1/4;$ therefore it converges.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{5}{4^n + 1}}{\frac{1}{4^n}} = \lim_{n\to\infty} \frac{5 \cdot 4^n}{4^n + 1} = 1$$

Limit Comparison Test says that either both series converge or both series diverge.

Since $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$ converges, $\sum_{n=1}^{\infty} \frac{5}{4^n + 1}$ also converges.

9) $\sum_{n=1}^{\infty} \frac{n+5}{n^3-2n+3}$

Use Limit Comparison Test or Direct Comparison Test to explain why series converges:

Explain why series converges.

Let $a_n = \frac{n+5}{n^3-2n+3}$ and $b_n = \frac{n}{n^3} = \frac{1}{n^2}$

The series $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$ is a p-series with p = 2; therefore it converges.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n+5}{n^3-2n+3}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^3+5n^2}{n^3-2n+3} = 1$$

Limit Comparison Test says that either both series converge or both series diverge.

Since $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, $\sum_{n=1}^{\infty} \frac{n+5}{n^3-2n+3}$ also converges.

10) $\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n}$

Use Alternating Series Test to explain why series converges:

$$a_n = \frac{1}{e^n} \Rightarrow a_1 = \frac{1}{e}; \quad a_2 = \frac{1}{e^2}; \quad a_3 = \frac{1}{e^3}; \ ...$$

Hence, a_n is a decreasing sequence.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{e^n} = 0$$

Therefore, by the Alternating Series Test $\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n}$ converges.

11) $\sum_{n=1}^{\infty} n\left(\frac{10}{9}\right)^n$

Use Ratio Test to explain why series diverges.

Explain why series diverges:

$a_n = n\left(\frac{10}{9}\right)^n$ and $a_{n+1} = (n+1)\left(\frac{10}{9}\right)^{n+1} = (n+1)\left(\frac{10}{9}\right)^n\left(\frac{10}{9}\right)^1$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)\left(\frac{10}{9}\right)^n\left(\frac{10}{9}\right)^1}{n\left(\frac{10}{9}\right)^n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)\left(\frac{10}{9}\right)^1}{n}\right| = 10/9 > 1$$ Use L'Hopital's Rule

Therefore, by Ratio Test, $\sum_{n=1}^{\infty} n\left(\frac{10}{9}\right)^n$ diverges.

12) $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n!}$

Use Ratio Test to explain why series converges.

Explain why series converges:

$a_n = \frac{2^n}{n!}$ and $a_{n+1} = \frac{2^{n+1}}{(n+1)!} = \frac{2^n \cdot 2^1}{(n+1)n!}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{2^n \cdot 2^1}{(n+1)n!}}{\frac{2^n}{n!}}\right| = \lim_{n\to\infty}\frac{2}{n+1} = 0 < 1$$

Therefore, by Ratio Test, $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n!}$ converges.

13) $\sum_{n=1}^{\infty} \frac{5^n}{2^n+1}$

Use Ratio Test to explain why series diverges.

Explain why series diverges:

$$a_n = \frac{5^n}{2^n+1} \quad \text{and} \quad a_{n+1} = \frac{5^{n+1}}{2^{n+1}+1} = \frac{5^n 5^1}{2^n 2^1+1}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{5^n 5^1}{2^n 2^1+1}}{\frac{5^n}{2^n+1}}\right| = \lim_{n\to\infty}\left|\frac{5^1\left(2^n+1\right)}{2^n 2^1+1}\right| = \lim_{n\to\infty}\left|\frac{5^1 2^n+5}{2^n 2^1+1}\right| = \frac{5}{2} > 1 \quad \text{Use L'Hopital's Rule}$$

Therefore, the series $\sum_{n=1}^{\infty} \frac{5^n}{2^n+1}$ diverges.

14) $f(x) = \frac{8}{\sqrt{x}}$ Hint: $\frac{8}{\sqrt{x}} = 8x^{-1/2}$

Find a second degree polynomial $P_2(x)$ to approximate $f(x)$ for values around $c = 4$.

$P_2(x) =$ ______________________________

$f(x) = 8x^{-1/2}$	$f(4) = 8(4)^{-1/2} = 4$
$f'(x) = -4x^{-3/2}$	$f'(4) = -4(4)^{-3/2} = -0.5$
$f''(x) = 6x^{-5/2}$	$f''(4) = 6(4)^{-5/2} = 0.1875$

$$P_2(x) = f(4) + f'(4)(x-4) + \frac{f''(4)(x-4)^2}{2}$$

$$P_2(x) = 4 + -0.5(x-4) + \frac{0.1875(x-4)^2}{2}$$

15) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{5^n}$

Find interval of convergence for this power series.

Find Interval of Convergence = ______________________________

Let $u_n = \frac{x^n}{5^n}$ and $u_{n+1} = \frac{x^{n+1}}{5^{n+1}} = \frac{x^n x}{5^n 5}$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{x^n x}{5^n 5}}{\frac{x^n}{5^n}}\right| = \lim_{n\to\infty}\left|\frac{x}{5}\right| = \left|\frac{x}{5}\right|$$

Set $\left|\frac{x}{5}\right| < 1 \Leftrightarrow -1 < \frac{x}{5} < 1 \Leftrightarrow -5 < x < 5$

Note: When $x = -5, \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{5^n}$ diverges.

When $x = 5, \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{5^n}$ converges.

Therefore, radius of convergence is (-5, 5].

16) $\sum_{n=0}^{\infty} \frac{(3x)^n}{(2n)!}$

Find interval of convergence for this power series.

Find $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| =$ ______________________________

Find Interval of Convergence = ______________________________

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{(3x)^{n+1}}{(2(n+1))!}}{\frac{(3x)^n}{(2n)!}}\right| = \lim_{n\to\infty}\left|\frac{3x}{(2n+2)(2n+1)}\right| = 0$$

Therefore, $\sum_{n=0}^{\infty} \frac{(3x)^n}{(2n)!}$ converges for any *x*.

Radius of convergence is (-∞,∞).

17) $f(x) = \dfrac{2}{5-x}$

Find a geometric power series centered at $c = 0$ for $f(x)$.

Power Series:________________________________

$$f(x) = \frac{2}{5-x} = \frac{2/5}{1-\frac{1}{5}x} = \frac{a}{1-r}$$

geometric power series = $\sum_{n=0}^{\infty} ar^n = \sum_{n=0}^{\infty} \frac{2}{5}\left(\frac{1}{5}x\right)^n$

18) $f(x) = \dfrac{4}{3x+2}$

Find a geometric power series centered at $c = 3$ for $f(x)$.

Power Series:________________________________

$$f(x) = \frac{4}{3x+2} = \frac{4}{2+3(x-3)+9} = \frac{4}{11+3(x-3)} = \frac{4/11}{1+\frac{3}{11}(x-3)} = \frac{a}{1-r}$$

geometric power series = $\sum_{n=0}^{\infty} ar^n = \sum_{n=0}^{\infty} (4/11)\left(\frac{3}{11}(x-3)\right)^n$