Calculus II Test 3

1) Find the vertex of the following parabola. Then draw graph of parabola.

$y^2 - 12y - 8x + 20 = 0$ Graph:

vertex=___________________

p =_______________________

$y^2 - 12y - 8x + 20 = 0$

$y^2 - 12y = 8x - 20$

$y^2 - 12y + 36 = 8x - 20 + 36$

$\left(y - 6\right)^2 = 8x + 16$

$\left(y - 6\right)^2 = 8(x + 2)$

Vertex =(-2, 6)

4p = 8 $\Rightarrow$ p=2

2) Find the center and vertices of the following ellipse. Then draw graph of ellipse.

$$\frac{(x-2)^2}{1/5} + y^2 = 1$$

Graph:

vertices =___________________

center =________________________

$$\frac{(x-2)^2}{1/5} + y^2 = 1 \quad \Leftrightarrow \frac{(x-2)^2}{1/5} + \frac{(y-0)^2}{1} = 1$$

$h = 2; k = 0$ Center(2, 0)

$a^2 = 1;$ $a = 1$

$b^2 = 1/5$

$a^2 = b^2 + c^2$

$c^2 = 4/5;$ $c = \sqrt{4/5}$

Vertices: (h,k $\pm$ a) = (2, 0 $\pm$ 1)

3) Find the center and vertices of the following hyperbola. Then draw graph of hyperbola.

$$12x^2 - 12y^2 - 12x + 24y - 45 = 0$$

vertices =___________________ Graph of hyperbola:

center =_______________________

$$12x^2 - 12y^2 - 12x + 24y - 45 = 0$$

$$\left(12x^2 - 12x\right) + \left(-12y^2 + 24y\right) = 45$$

$$12\left(x^2 - x\right) - 12\left(y^2 - 2y\right) = 45$$

$$12\left(x^2 - x + 1/4\right) - 12\left(y^2 - 2y + 1\right) = 45 + 12(1/4) - 12(1)$$

$$12\left(x^2 - x + 1/4\right) - 12\left(y^2 - 2y + 1\right) = 36$$

$$\frac{\left(x - 1/2\right)^2}{3} - \frac{\left(y - 1\right)^2}{3} = 1$$

$a^2 = 3;\quad b^2 = 3$

$a = \sqrt{3};\quad b = \sqrt{3}$

Center =(1/2, 1)

vertices =($1/2 \pm \sqrt{3}$, 1)

4) Find equation of parabola.

vertex = (2, 6)

focus = (2, 4)

Equation of parabola:___________________________________

Parabola opens down.

p = distance from vertex to focus = 2

$(x - h)^2 = -4p(y-k)$

$(x - 2)^2 = -4(2)(y-6)$

5) Find equation of ellipse.

center = (0, 0)

foci = $(0, \pm\sqrt{15})$

vertices = $(0, \pm 4)$

Equation of ellipse:___________________________________

a = distance from center to vertex =4

$c =$ distance from center to focus = $\sqrt{15}$

$a^2 = 16;$ $c^2 = 15$

$a^2 = b^2 + c^2$

$b^2 = 1$

$$\frac{(x-0)^2}{1} + \frac{(y-0)^2}{16} = 1$$

Note: Ellipse is of vertical type;

thus, a^2 is placed under $(y - k)^2$.

6) Find equation of hyperbola.

center = (0, 0)

foci = $(0, \pm\sqrt{10})$

vertices = $(0, \pm 1)$

Equation of ellipse:___________________________________

a = distance from center to vertex =1

$c =$ distance from center to focus = $\sqrt{10}$

$a^2 = 1; \qquad c^2 = 10$

$c^2 = a^2 + b^2$

$b^2 = 9$

$$\frac{(y-0)^2}{1} - \frac{(x-0)^2}{9} = 1$$

Note: Branches of hyperbola opens up and down.

7) For the following parametric equations, find a corresponding rectangular equation.

Then draw graph. Draw Graph:

$x = t - 6, \quad y = t^2$

Rectangular Equation:______________________

$x = t - 6$

$t = x + 6$

$y = t^2$

$y = (x+6)^2$ Rectangular Equation

8) For the following parametric equations, find a corresponding rectangular equation.

Then draw graph. Draw Graph:

$x = 6\cos t \quad y = 6\sin t$

Rectangular Equation:______________________

$x = 6\cos t \quad \Rightarrow \cos t = \dfrac{x}{6}$

$y = 6\sin t \quad \Rightarrow \sin t = \dfrac{y}{6}$

From Trigonometry: $\cos^2 t + \sin^2 t = 1$

$\left(\dfrac{x}{6}\right)^2 + \left(\dfrac{y}{6}\right)^2 = 1$ Rectangular Equation

9) For the following parametric equations, find dy/dx and two points with horizontal tangent lines.

$x = 10\cos t \quad y = 10\sin t$

$\frac{dy}{dx} = \frac{\text{dy/dt}}{\text{dx/dt}} = \frac{10\cos t}{-10\sin t} = -\cot t$

Horizontal Tangent Lines are at the following points: (,) and (,)

Hint: Set $\frac{dy}{dx}$=0 and solve.

Set $-\cot t = 0$

$\cot t = 0$

$t = \pi/2; \; 3\pi/2$

When $t = \pi/2$: $x = 10\cos(\pi/2) \quad y = 10\sin(\pi/2) \Rightarrow x = 0 \quad y = 10$

When $t = 3\pi/2$: $x = 10\cos(3\pi/2) \quad y = 10\sin(3\pi/2) \Rightarrow x = 0 \quad y = -10$

10) For the following polar equation, find a corresponding rectangular equation and draw graph.

Draw Graph:

$r = 4\cos\theta$

Rectangular Equation:______________________

$r = 4\cos\theta$

$rr = 4r\cos\theta$

$r^2 = 4r\cos\theta$

$x^2 + y^2 = 4x$ Rectangular Equation

11) For the following polar equation, find a corresponding rectangular equation and draw graph.

Graph:

$$r = \frac{1}{2 - \cos\theta}$$

Rectangular Equation:______________________

$$r = \frac{1}{2 - \cos\theta}$$

$$r(2 - \cos\theta) = 1$$

$$2r - r\cos\theta = 1$$

$2\sqrt{x^2 + y^2} - x = 1$ Note: $r^2 = x^2 + y^2 \Rightarrow r = \sqrt{x^2 + y^2}$

12) For the following rectangular equation, find a corresponding polar equation and draw graph.

$$x^2 + y^2 - 4x = 0$$

Polar Equation:______________________ Draw Graph:

$$x^2 + y^2 - 4x = 0$$

$$r^2 - 4r\cos\theta = 0$$

$r - 4\cos\theta = 0$ Divide by r

$r = 4\cos\theta$ Polar Equation

13) Find the area bounded by the following polar equation.

$r = 2 + \cos\theta$

Area =__________________________

Area = $\frac{1}{2}\int_0^{2\pi} r^2 d\theta = \frac{1}{2}\int_0^{2\pi} (2+\cos\theta)^2\, d\theta = 14.137166925$

14) Find the area bounded by the following polar equation.

$r^2 = 4\sin 2\theta$

Area =__________________________

Note: $r = \sqrt{4\sin 2\theta}$

Graph has two petals. For θ=0 to $\theta=\pi/2$, we will get half of a petal.

Area of half of a petal = $\frac{1}{2}\int_0^{\pi/2} r^2 d\theta = \frac{1}{2}\int_0^{\pi/2} 4\sin 2\theta d\theta = 2$

Area of two petals = 8.

15) Find eccentricity (e) and draw graph of the following ellipse.

$r = \dfrac{6}{3 + 2\cos\theta}$

$e =$ ___________

$r = \dfrac{6}{3 + 2\cos\theta} = r = \dfrac{2}{1 + \frac{2}{3}\cos\theta}$ Divide by 3

$e = \dfrac{2}{3}$

16) Find eccentricity (e) and draw graph of the following hyperbola.

$$r = \frac{8}{2 - 5\cos\theta}$$

$e =$ ____________

$$r = \frac{8}{2 - 5\cos\theta} = \frac{4}{1 - \frac{5}{4}\cos\theta}$$ Divide by 2

$e = 5/4$