# Test 1 Review

1) Find $\lim_{x \to 2} \frac{x-2}{x^2-4} = \lim_{x \to 2} \frac{(x-2)}{(x-2)(x+2)}$

$$= \lim_{x \to 2} \frac{1}{x+2} = \frac{1}{4}$$

2) Find $\lim_{x \to 4} \frac{\sqrt{x-3}+3}{x-3} = 4$

$$\lim_{x \to 4^-} \frac{\sqrt{x-3}+3}{x-3} = 4$$

$$\lim_{x \to 4^+} \frac{\sqrt{x-3}+3}{x-3} = 4$$

(3) Find $\lim_{x\to 0}\left(\dfrac{\cos x - \sin x}{\sin x}\right) = DNE$

$$\lim_{x\to 0^-}\left(\frac{\cos x - \sin x}{\sin x}\right) = -\infty$$

$$\lim_{x\to 0^+}\left(\frac{\cos x - \sin x}{\sin x}\right) = \infty$$

(4) Find $\lim_{x \to 6} \left( \frac{1}{x+7} \right) = \frac{1}{6+7} = \frac{1}{13}$

(5) $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} = 0.1666666 = 1/6$

$\lim_{x \to 9^-} \frac{\sqrt{x} - 3}{x-9} = 0.1666666 = 1/6$

$\lim_{x \to 9^+} \frac{\sqrt{x} - 3}{x-9} = 0.166666 = 1/6$

5

$$f(x) = \begin{cases} (x-1)^2 & x \le 1 \\ 1-x & x > 1 \end{cases}$$

Find $\lim\limits_{x \to 1} f(x) = 0$

$$\lim_{x \to 1^-} (x-1)^2 = (1-1)^2 = 0$$

$$\lim_{x \to 1^+} (1-x) = 1-1 = 0$$

(6) $$f(t) = \begin{cases} t^3 + 4 , & t \le 1 \\ \frac{1}{4}(t+3) , & t > 1 \end{cases}$$

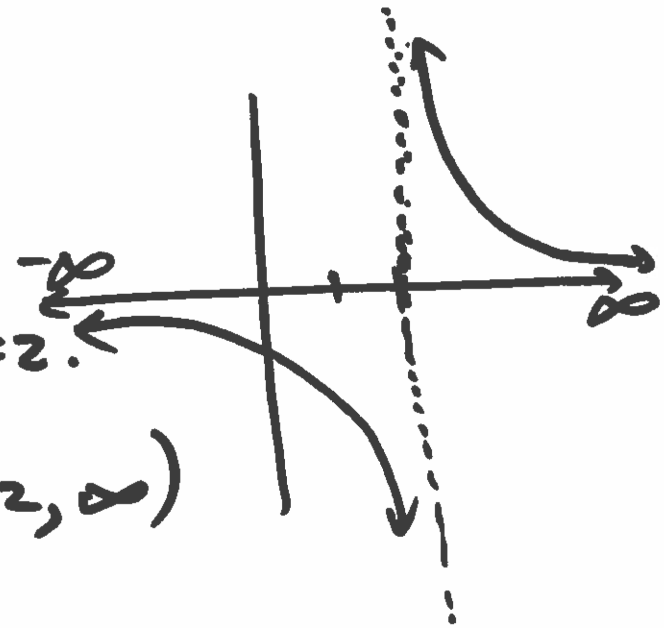
Find $\lim\limits_{t \to 1} f(t) = \text{DNE}$

$$\lim_{t \to 1^-} f(t) = \lim_{t \to 1^-} (t^3 + 4) = 1^3 + 4 = 5$$

$$\lim_{t \to 1^+} f(t) = \lim_{t \to 1^+} \left(\frac{1}{4}(t+3)\right) = \frac{1}{4}(1+3) = 1$$

(7) Let $f(x) = \dfrac{4}{x-2}$

Find where $f(x)$ is discontinuous.
Find vertical Asymptote.




Note: Graph breaks up at $x=2$.
So Graph is discontinuous at $x=2$.

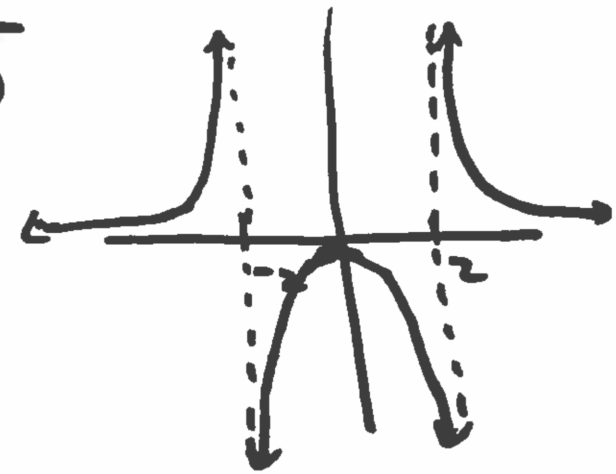
Graph is continuous on $(-\infty, 2) \cup (2, \infty)$

To find vertical asymptote, set $x-2=0$.

$\Rightarrow x=2$

V.A. is $x=2$

(8) Let $f(x) = \dfrac{x}{x^3 - 4x} = \dfrac{(x)}{(x)(x^2-4)}$




Find where $f(x)$ is discontinuous.

set $x^3 - 4x = 0$

$x \cdot (x^2-4) = 0$

$x = 0$ | $x^2 - 4 = 0$

$x^2 = 4$

$\sqrt{x^2} = \pm\sqrt{4} = \pm 2$

$x = \pm 2$

So graph is discontinuous at $x=0$, $x=2$, $x=-2$

Vertical Asymptotes are $x = 2$ and $x = -2$

Graph has a hole at $x = 0$

⑨ $F(x) = \dfrac{x+8}{x^2-64}$

Find vertical asymptotes and/or holes.

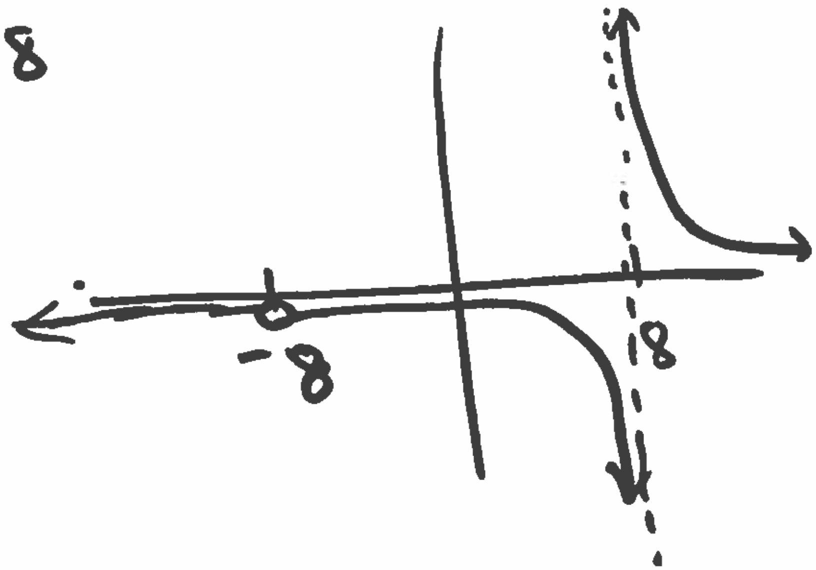
Note: $f(x) = \dfrac{(x+8)}{(x+8)(x-8)}$

set $x+8=0$ ; $x-8=0$

$x=-8$ $\qquad$ $x=8$

Graph has a hole at $x=-8$.

V. A. is $x=8$



(10) Let $f(x) = x - \frac{4}{x^2}$

Find $\lim_{x \to 0} f(x) = -\infty = \text{DNE}$

$\lim_{x \to 0^-} f(x) = -\infty$

$\lim_{x \to 0^+} f(x) = -\infty$

(11) Let $f(x) = \dfrac{\cos 3x}{4x}$

Find $\lim_{x \to 0} f(x) = \text{DNE}$

$$\lim_{x \to 0^-} f(x) = -\infty$$

$$\lim_{x \to 0^+} f(x) = \infty$$

(12) Let $f(x) = 4\sqrt{x} - 3\sqrt[3]{x}$

Find $f'(x)$.

Note: $f(x) = 4 \cdot x^{1/2} - 3 \cdot x^{1/3}$

$$f'(x) = 4 \cdot \left( \frac{1}{2} x^{-1/2} \right) - 3 \left( \frac{1}{3} \cdot x^{-2/3} \right)$$

$$f'(x) = 2 \cdot \frac{1}{x^{1/2}} - 1 \cdot \frac{1}{x^{2/3}}$$

$$f'(x) = \frac{2}{x^{1/2}} - \frac{1}{x^{2/3}}$$

(13) Let $f(x) = 3x^3 - 4$

Find tangent line passing through $(0, -4)$.

$f'(x) = 3 \cdot (3x^2) - 0 = 9x^2$

Slope of tangent line $= f'(x) = 9x^2 = 9 \cdot (0)^2 = 0$

Equation of tangent line: $y - y_1 = m(x - x_1)$

$y - -4 = 0(x - 0)$

$y + 4 = 0$

(14) $f(x) = \sin x + \cos x$

Find tangent line at $(\pi, -1)$

$f'(x) = \cos x - \sin x$

Slope of tangent line $= f'(x) = \cos(\pi) - \sin(\pi) = -1 - 0 = -1$

Equation of tangent line: $y - y_1 = m(x - x_1)$

$y - -1 = -1(x - \pi)$

$y + 1 = -1(x - \pi)$

(15) Let $f(x) = \sqrt{x} \cdot \cos x$ Find $f'(x)$.

Using Product Rule:

$$f'(x) = \sqrt{x} \cdot D_x(\cos x) + \cos x \cdot D_x(\sqrt{x})$$

$$= \sqrt{x} \cdot (-\sin x) + \cos x \cdot \left(\frac{1}{2\sqrt{x}}\right)$$

$$f'(x) = -\sqrt{x} \cdot \sin x + \frac{\cos x}{2\sqrt{x}}$$

(16) Let $f(x) = \dfrac{x^2 + 4x + 3}{x^2 - 4 + \sqrt{x}}$ Find $f'(x)$.

Using Quotient Rule:

$$f'(x) = \frac{(x^2 - 4 - \sqrt{x}) \cdot D_x(x^2 + 4x + 3) - (x^2 + 4x + 3) \cdot D_x(x^2 - 4 + \sqrt{x})}{(x^2 - 4 + \sqrt{x})^2}$$

$$f'(x) = \frac{(x^2 - 4 - \sqrt{x}) \cdot (2x + 4) - (x^2 + 4x + 3) \cdot \left(2x + \frac{1}{2\sqrt{x}}\right)}{(x^2 - 4 + \sqrt{x})^2}$$

(17) Let $f(x) = \dfrac{x^3}{\sin x}$ Find $f'(x)$.

Using Quotient Rule:

$$f'(x) = \frac{(\sin x) \cdot D_x(x^3) - (x^3) \cdot D_x(\sin x)}{(\sin x)^2}$$

$$f'(x) = \frac{\sin x \cdot (3x^2) - x^3 \cdot \cos x}{(\sin x)^2}$$

(18) Let $f(x) = (x+1)\cdot(x^2-1)$ Find tangent line at $(2, 9)$.

Using Product Rule:

$$f'(x) = (x+1)\cdot D_x(x^2-1) + (x^2-1)\cdot D_x(x+1)$$

$$f'(x) = (x+1)\cdot(2x) \quad + (x^2-1)\cdot(1)$$

Slope of tangent line $= f'(x) = (2+1)(2\cdot 2)+(2^2-1)(1)$
$= 15$

Equation of tangent line: $y - y_1 = m(x - x_1)$

$$y - 9 = 15(x-2)$$

(19) Let $f(x) = \dfrac{x+2}{x-2}$ Find tangent line at $(-2, 0)$

Using Quotient Rule:

$$f'(x) = \frac{(x-2)\cdot D_x(x+2) - (x+2)\cdot D_x(x-2)}{(x-2)^2}$$

$$f'(x) = \frac{(x-2)\cdot(1) - (x+2)(1)}{(x-2)^2} = \frac{(x-2) - (x+2)}{(x-2)^2}$$

Slope of tangent line $= f'(-2) = \dfrac{(-2-2) - (-2+2)}{(-2-2)^2}$

$$= \frac{-4}{16} = -\frac{1}{4}$$

Equation of tangent line:

$$y - 0 = -\frac{1}{4}(x - -2)$$

$$y = -\frac{1}{4}(x+2)$$

(20) Let $f(x) = 14 \cdot x^{2/3}$ Find $f'(x), f''(x)$.

$$f'(x) = 14 \cdot \left( \frac{2}{3} x^{-1/3} \right) = \frac{28}{3} \cdot x^{-1/3}$$

$$f''(x) = \frac{28}{3} \cdot \left( -\frac{1}{3} x^{-4/3} \right) = -\frac{28}{9} x^{-4/3}$$

(21) Let $f(x) = 4 \cdot \sin(x^2 + 5x)$ Find $f'(x)$.

$$f'(x) = 4 \cdot \cos(x^2 + 5x) \cdot Dx(x^2 + 5x)$$

$$f'(x) = 4 \cdot \cos(x^2 + 5x) \cdot (2x + 5)$$

(22) Let $y = \frac{x}{4} - \frac{\cos 4x}{4}$ Find $y'$

Note: $y = \frac{1}{4} \cdot x - \frac{1}{4} \cdot \cos 4x$

$$y' = \frac{1}{4}(1) - \frac{1}{4} \cdot \left(-\sin 4x \cdot D_x(4x)\right)$$

$$y' = \frac{1}{4} + \frac{1}{4} \cdot (\sin 4x) \cdot (4)$$

$$y' = \frac{1}{4} + \sin 4x$$

(23) $x^2 + y^2 = 16$ Find $y'$ using implicit differentiation.

$$2 \cdot x \cdot \frac{dx}{dx} + 2y \cdot \frac{dy}{dx} = 0$$

$$2x + 2y \cdot y' = 0$$

$$2y \cdot y' = -2x$$

$$y' = \frac{-2x}{2y} = -\frac{x}{y}$$

$$y' = \frac{dy}{dx} = \frac{-x}{y}$$

(24) $\underbrace{x^2 \cdot y}_{\text{Product}} - \underbrace{x \cdot y^2}_{\text{Product}} = 10$ Find $y'$ by using implicit differentiation.

$$\left[x^2 \cdot D_x(y) + y \cdot D_x(x^2)\right] - \left[x \cdot D_x(y^2) + y^2 \cdot D_x(x)\right] = 0$$

$$\left[x^2 \cdot 1 \cdot \frac{dy}{dx} + y \cdot 2x\frac{dx}{dx}\right] - \left[x \cdot 2y \cdot \frac{dy}{dx} + y^2 \cdot 1\frac{dx}{dx}\right] = 0$$

$$\left[x^2 \cdot y' + 2xy\right] - \left[2xy \cdot y' + y^2\right] = 0$$

$$x^2 \cdot y' + 2xy - 2xy \cdot y' - y^2 = 0$$

$$x^2 \cdot y' - 2xy \cdot y' = -2xy + y^2$$

$$y' \cdot (x^2 - 2xy) = -2xy + y^2$$

$$y' = \frac{-2xy + y^2}{(x^2 - 2xy)}$$

(25) $x^2 \cdot y - x \cdot y^2 = -2$ Find tangent line at $(1, 2)$

From Example 24:

$$y' = \frac{-2xy + y^2}{x^2 - 2xy}$$

Slope of tangent line $= y' = \dfrac{-2(1)(2) + (2)^2}{(1)^2 - 2(1)(2)}$

$$= \frac{0}{-3} = 0$$

Equation of tangent line: $y - 2 = 0(x-1)$

$y - 2 = 0$