Test 1 Review

19 Chapter 1, p. 91

$$\lim_{t\to 2} \frac{t+2}{t^2-4} = \lim_{t\to 2} \frac{\cancel{(t+2)}}{\cancel{(t+2)}(t-2)}$$

$$= \lim_{t\to 2} \frac{1}{t-2} = \text{DNE}$$

21 p. 91

$$\lim_{x \to 4} \frac{\sqrt{x-4} - 1}{x-4} = \text{DNE}$$

$$\lim_{x \to 4^-} \frac{\sqrt{x-4} - 1}{x-4} = \text{DNE}$$

$$\lim_{x \to 4^+} \frac{\sqrt{x-4} - 1}{x-4} = -\infty$$

#25, p.91

$$\lim_{x \to 0} \left(\frac{1 - \cos x}{\sin x} \right) = 0$$

$$\lim_{x \to 0^-} \left(\frac{1 - \cos x}{\sin x} \right) = 0$$

$$\lim_{x \to 0^+} \left(\frac{1 - \cos x}{\sin x} \right) = 0$$

39, p. 91

$$\lim_{x \to 3^+} \left(\frac{1}{x+3} \right) = \frac{1}{6}$$

41, p. 91

$$\lim_{x \to 4^-} \left(\frac{\sqrt{x} - 2}{x - 4} \right) = 0.25$$

Using Graphing Method

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#43, p. 91

$$f(x) = \begin{cases} (x-2)^2, & x \le 2 \\ 2-x, & x > 2 \end{cases}$$

Find $\lim_{x \to 2} f(x) = 0$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x-2)^2 = 0$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (2-x) = 0$$

#45, p. 91

$$h(t) = \begin{cases} t^3 + 1, & t < 1 \\ \frac{1}{2}(t+1), & t \geq 1 \end{cases}$$

$$\lim_{t \to 1} h(t) = \text{DNE}$$

$$\lim_{t \to 1^-} h(t) = \lim_{t \to 1^-} (t^3 + 1) = 2$$

$$\lim_{t \to 1^+} h(t) = \lim_{t \to 1^+} \left(\frac{1}{2}(t+1) \right) = 1$$

#51, p. 92

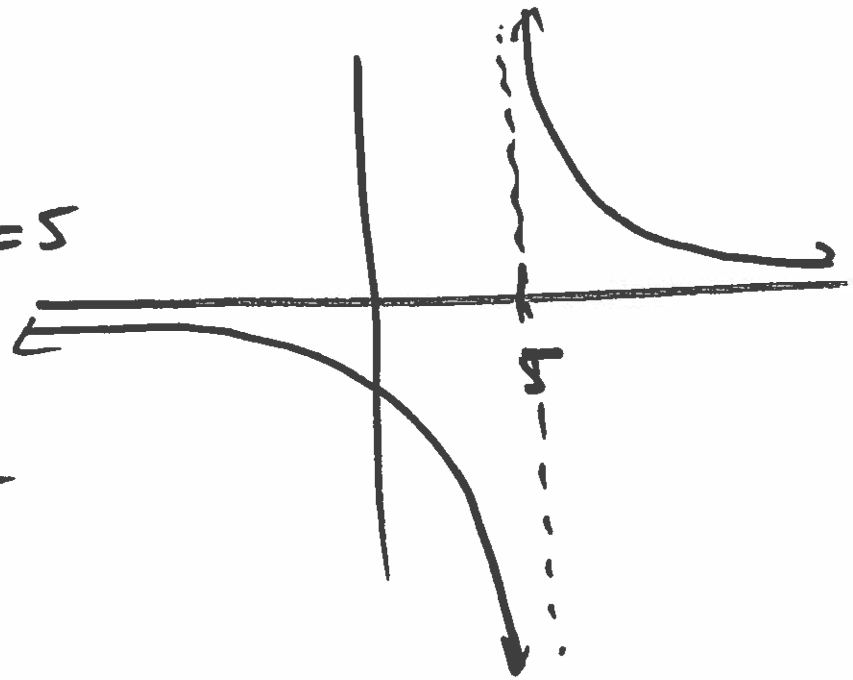
$$f(x) = \frac{4}{x-5}$$

To find where $f(x)$ is discontinuous,

set $x - 5 = 0$

$x = 5$

So, $f(x)$ is discon't at $x=5$



$f(x)$ has a nonremovable discontinuity at $x=5$

#53, p. 92 $\quad f(x) = \dfrac{x}{x^3 - x}$

To find where $f(x)$ is discontinuous,

set $x^3 - x = 0$

$(x)(x^2 - 1) = 0$

$(x) = 0$	$x^2 - 1 = 0$
$x = 0$	$x^2 = 1$
	$\sqrt{x^2} = \pm\sqrt{1}$
	$x = \pm 1$

$f(x)$ is discon't at $x = 0$, $x = 1$, $x = -1$

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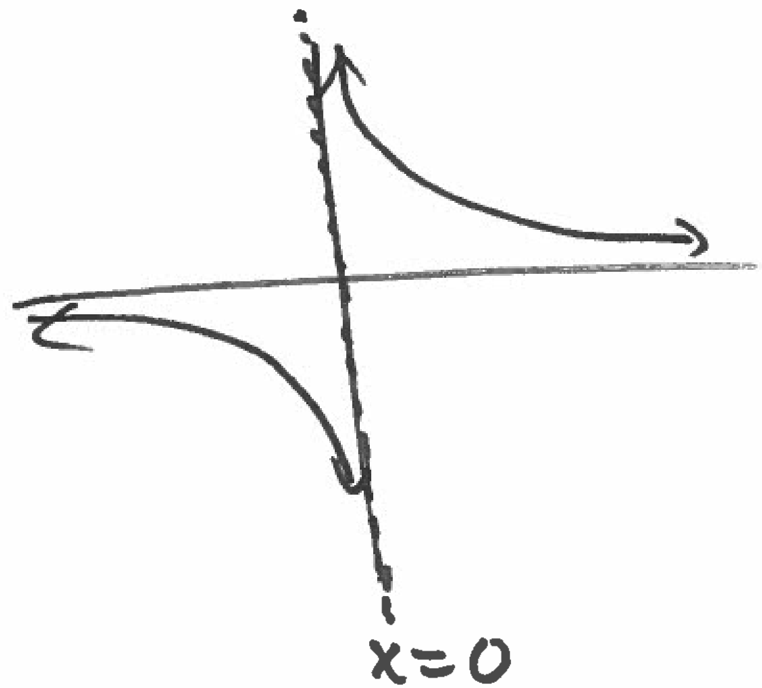
67, p. 92 $\quad f(x) = \dfrac{3}{x}$

Find Vertical Asymptotes.

set $x = 0$ $\quad$ Vertical Asymptote



#69, p. 92 $f(x) = \dfrac{2x+1}{x^2-64}$ Find Vertical Asymptotes

Set $x^2-64=0$

$$x^2 = 64$$

$$\sqrt{x^2} = \pm\sqrt{64}$$

$$x = \pm 8$$

Vertical Asymptotes are: $x=8$; $x=-8$

#77 $\lim_{x \to 0^+} \left(x - \frac{1}{x^3} \right) = -\infty = \text{DNE}$

#79 $\lim_{x \to 0^+} \left(\frac{\sin 4x}{5x} \right) = 0.8$

Chapter 2, pp. 157 - 158

(13) $h(x) = 6\sqrt{x} + 3\sqrt[3]{x}$

$h(x) = 6 \cdot x^{1/2} + 3 \cdot x^{1/3}$

$h'(x) = 6\left(\frac{1}{2}x^{-1/2}\right) + 3\left(\frac{1}{3}x^{-2/3}\right)$

$h'(x) = 3x^{-1/2} + x^{-2/3}$

#23, p.157

$f(x) = 2x^4 - 8$

Find tangent line at $(0, -8)$

$f'(x) = 2\cdot(4x^3) - 0$

$f'(x) = 8x^3$

Slope of tangent line at $(0, -8)$

$= f'(0)$

$= 8\cdot(0)^3$

$= 0$

Equation of tangent line:

$y - -8 = 0(x - 0)$

$y + 8 = 0$

31, p. 157

$$f(x) = \sqrt{x} \cdot \sin x$$

$$f'(x) = \left(\sqrt{x}\right) \cdot D_x\left(\sin x\right) + \left(\sin x\right) \cdot D_x\left(\sqrt{x}\right)$$

$$= \left(x^{1/2}\right) \cdot \left(\cos x\right) + \left(\sin x\right) \cdot \left(\frac{1}{2} x^{-1/2}\right)$$

#33, p.157

$$f(x) = \frac{x^2 + x - 1}{x^2 - 1}$$

$$f'(x) = \frac{(x^2-1) \cdot D_x(x^2+x-1) - (x^2+x-1) \cdot D_x(x^2-1)}{(x^2-1)^2}$$

$$f'(x) = \frac{(x^2-1)(2x+1) - (x^2+x-1)(2x)}{(x^2-1)^2}$$

#35, p. 157 $\quad y = \dfrac{x^4}{\cos x}$

$$y' = \frac{(\cos x) \cdot D_x(x^4) - (x^4) \cdot D_x(\cos x)}{(\cos x)^2}$$

$$y' = \frac{(\cos x)(4x^3) - (x^4)(-\sin x)}{(\cos x)^2}$$

#41, p. 157 $f(x) = (x+2)\cdot(x^2+5)$ Find tangent line at $(-1, 6)$

$$f'(x) = (x+2)\cdot D_x(x^2+5) + (x^2+5)\cdot D_x(x+2)$$
$$= (x+2)(2x) + (x^2+5)(1)$$

slope of tangent line at $(-1, 6)$

$$= f'(-1)$$
$$= (-1+2)(2(-1)) + ((-1)^2+5)(1) = 4$$

Equ. of tangent line:

$$Y - 6 = 4(X - -1)$$
$$Y - 6 = 4(X+1)$$

\# 43, p. 157

$$f(x) = \frac{x+1}{x-1}$$

Find tangent line at $(1/2, -3)$

$$f'(x) = \frac{(x-1)\cdot D_x(x+1) - (x+1)\cdot D_x(x-1)}{(x-1)^2}$$

$$f'(x) = \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2} = \frac{(x-1) - (x+1)}{(x-1)^2}$$

Slope of tangent line at $(1/2, -3)$

$$= f'(1/2)$$

$$= \frac{(1/2 - 1) - (1/2 + 1)}{(1/2 - 1)^2} = -8$$

Equ. of tangent line: $y - -3 = -8(x - 1/2)$

$$y + 3 = -8(x - 1/2)$$

\# 47, p. 157 $f(x) = 15 \cdot x^{3/2}$

$$f'(x) = 15 \cdot \left(\frac{3}{2} x^{1/2}\right) = \frac{45}{2} \cdot x^{1/2}$$

$$f''(x) = \frac{45}{2} \cdot \left(\frac{1}{2} x^{-1/2}\right) = \frac{45}{4} x^{-1/2}$$

#57, p. 157 $\quad f(x) = 5 \cdot \cos(9x+1)$

Recall: $\quad D_x\left[\boxed{\cos}(\text{expression})\right] = -\sin(\text{expression}) \cdot D_x(\text{expression})$

$$f'(x) = 5 \cdot \left[-\sin(9x+1) \cdot D_x(9x+1)\right]$$

$$f'(x) = 5\left[-\sin(9x+1) \cdot (9)\right] = -45 \cdot \sin(9x+1)$$

#59, p. 157

$$y = \frac{x}{2} - \frac{\sin 2x}{4}$$

$$y = \frac{1}{2} \cdot x - \frac{1}{4} \cdot \sin(2x)$$

Recall: $D_x(\sin(\text{expression})) = \cos(\text{expression}) \cdot D_x(\text{expression})$

$$y' = \frac{1}{2} - \frac{1}{4} \cdot \left[\cos(2x) \cdot D_x(2x) \right]$$

$$y' = \frac{1}{2} - \frac{1}{4} \left[\cos 2x \cdot 2 \right]$$

$$y' = \frac{1}{2} - \frac{1}{4} \left[2 \cdot \cos 2x \right]$$

$$y' = \frac{1}{2} - \frac{1}{2} \cos 2x$$

77, p. 158

$$x^2 + y^2 = 64 \qquad \text{Find } y' = \frac{dy}{dx}$$

Using Implicit Differentiation:

$$2x + 2y \cdot y' = 0$$

$$2y \cdot y' = -2x$$

$$y' = \frac{-2x}{2y} = -\frac{x}{y} \quad \text{Answer}$$

#79 $\quad x^3 \cdot y - x \cdot y^3 = 4$

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Implicit Derivative:

$$(x^3) \cdot D_x(y) + (y) \cdot D_x(x^3)$$

$$+ (-x) \cdot D_x(y^3) + (y^3) \cdot D_x(-x) = 0$$

$$\Rightarrow \quad x^3 \cdot (1 \cdot y') + (y)(3x^2) + (-x) \cdot (3y^2 \cdot y') + (y^3)(-1) = 0$$

$$x^3 \cdot y' + 3x^2 y - 3xy^2 \cdot y' - y^3 = 0$$

$$x^3 \cdot y' - 3xy^2 \cdot y' = -3x^2 y + y^3$$

$$y' \cdot (x^3 - 3xy^2) = -3x^2 y + y^3$$

$$y' = (-3x^2 y + y^3) / (x^3 - 3xy^2) \quad \text{Answer}$$

#83, p. 158. $x^2 + y^2 = 10$ Find tangent line at $(3, 1)$

Implicit Derivative:

$$2x + 2y \cdot y' = 0$$

$$2y \cdot y' = -2x$$

$$y' = \frac{-2x}{2y} = \frac{-x}{y}$$

slope of tangent line at $(3, 1) = y'(3,1)$

$$y'(3,1) = -\frac{x}{y} = -\frac{3}{1} = -3$$

Equ. of tangent line: $y - 1 = -3(x - 3)$