Test 1 Review

1) Find $I = \int x\sqrt{x^2 - 36}\,dx = \int \sqrt{x^2 - 36}\cdot x\,dx$

Let $u = x^2 - 36$

$$\frac{du}{dx} = 2x \Rightarrow du = 2x\,dx \Rightarrow \frac{1}{2}\,du = x\,dx$$

$$I = \int \sqrt{u}\cdot\frac{1}{2}\,du = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\,u^{3/2}\cdot\frac{2}{3} = \frac{1}{3}u^{3/2}$$

$$I = \frac{1}{3}\left(x^2 - 36\right)^{3/2} + C$$

2) Find $I = \int x \cdot e^{x^2-4}\,dx = \int e^{x^2-4} \cdot x\,dx$

Let $u = x^2 - 4$

$\frac{du}{dx} = 2x \Rightarrow du = 2x \cdot dx \Rightarrow \frac{1}{2}du = x\,dx$

$I = \int e^u \cdot \frac{1}{2}du = \frac{1}{2}\int e^u du = \frac{1}{2}e^u$

$I = \frac{1}{2}e^{x^2-4} + C$

(3) Find $I = \int \cos^4(\pi x - 1)\,dx$

Let $u = \pi x - 1$

$$\frac{du}{dx} = \pi \Rightarrow du = \pi \cdot dx \Rightarrow \frac{1}{\pi}du = dx$$

$$I = \int \cos^4(u) \cdot \frac{1}{\pi}du = \frac{1}{\pi}\underbrace{\int \cos^4 u\,du}_{I_2}$$

Formula #86:

$$\int \cos^n u\,du = \frac{1}{n}\cos^{n-1}u\sin u + \frac{n-1}{n}\int \cos^{n-2}u\,du$$

$$I_2 = \frac{1}{4}\cos^3 u\sin u + \frac{3}{4}\underbrace{\int \cos^2 u\,du}_{\text{Use Formula \#82}}$$

$$I_2 = \frac{1}{4}\cos^3 u\sin u + \frac{3}{4}\left[\frac{1}{2}u + \frac{1}{4}\sin 2u\right]$$

$$I = \frac{1}{\pi}\left[\frac{1}{4}\cos^3(\pi x - 1)\sin(\pi x - 1) + \frac{3}{4}\left[\frac{1}{2}(\pi x - 1) + \frac{1}{4}\sin(2(\pi x - 1))\right]\right]$$

(4) Find $I = \int \dfrac{x^2}{x^2+4x-5}\,dx$

Divide $\dfrac{x^2}{x^2+4x-5} = 1 + \dfrac{-4x+5}{x^2+4x-5}$

$$\begin{array}{r|l} & \quad 1 \\ \hline x^2+4x-5 & x^2 + 0x + 0 \\ - & x^2 + 4x - 5 \\ \hline & \quad -4x + 5 \end{array}$$

$\dfrac{x^2}{x^2} = 1$

$1 \cdot (x^2+4x-5)$

$= x^2+4x-5$

Decompose $\dfrac{-4x+5}{x^2+4x-5} = \dfrac{-4x+5}{(x+5)(x-1)} = \dfrac{A}{x+5} + \dfrac{B}{x-1}$

$$\frac{-4x+5}{(x+5)(x-1)} = \frac{A(x-1)+B(x+5)}{(x+5)(x-1)}$$

set $-4x+5 = A(x-1) + B(x+5)$

Let $x=1$, $\quad -4(1)+5 = A(1-1) + B(1+5)$

$$1 = 6B$$

$$B = 1/6$$

Let $x=-5$, $\quad -4(-5)+5 = A(-5-1) + B(-5+5)$

$$25 = -6A$$

$$A = \frac{-25}{6}$$

$$I = \int \left(1 + \frac{-25/6}{x+5} + \frac{1/6}{x-1} \right) dx$$

$$I = \int 1\,dx - \frac{25}{6}\int \frac{1}{x+5}\,dx + \frac{1}{6}\int \frac{1}{x-1}\,dx$$

$$I = x - \frac{25}{6}\ln|x+5| + \frac{1}{6}\ln|x-1| + C$$

(5) Find $I = \int \frac{x}{(2+4x)^2}\,dx$

Formula 29:

$$\int \frac{u}{(a+bu)^2}\,du = \frac{1}{b^2}\left[\frac{a}{a+bu} + \ln|a+bu|\right]$$

Let $u = x$, $a = 2$, $b = 4$

$$I = \frac{1}{4^2}\left[\frac{2}{2+4x} + \ln|2+4x|\right]$$

$$I = \frac{1}{16}\left[\frac{2}{2+4x} + \ln|2+4x|\right] + C$$

(6) Find $I = \int \frac{x}{\sqrt{3+2x}}\,dx$

Formula 38:

$$\int \frac{u}{\sqrt{a+bu}}\,du = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu}.$$

Let $u = x$, $a = 3$, $b = 2$

$$I = \frac{2}{3(2)^2}\left(2x - 2(3)\right)\sqrt{3+2x}.$$

$$I = \frac{2}{12}(2x-6)\sqrt{3+2x}$$

(7) Find $I = \int_0^1 \frac{4x}{1+e^{x^2}}\,dx = 4\int \frac{1}{1+e^{x^2}}\, x\cdot dx$

Let $u = x^2$

$\frac{du}{dx} = 2x \Longrightarrow du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$I = 4\int \frac{1}{1+e^u}\cdot\frac{1}{2}du = 2\int \frac{1}{1+e^u}\,du$

Formula 119b : $\int \frac{1}{1+e^u}\,du = u - \ln(1+e^u)$

$I = 2\left[u - \ln(1+e^u)\right] = 2\left[x^2 - \ln(1+e^{x^2})\right]\Big|_0^1$

$I = 2\left[1 - \ln(1+e)\right] - 2\left[0 - \ln(2)\right] \approx \cancel{3.399}$ -2.0128

(8) Find $I = \int \frac{x}{x^2 + 4x + 5}\,dx$

Formula 34C :

$$\int \frac{u}{a+bu+cu^2}\,du = \frac{1}{2c}\left(\ln|a+bu+cu^2| - b\int \frac{1}{a+bu+cu^2}\,du\right)$$

Note : $a=5$, $b=4$, $c=1$

$$I = \frac{1}{2(1)}\left(\ln|5+4x+x^2| - 4\underbrace{\int \frac{1}{5+4x+x^2}\,dx}_{\text{Use Formula 34b}}\right)$$

$b^2 < 4ac$

$4^2 < 4(1)(5)$

$$I = \frac{1}{2}\left(\ln\left|5+4x+x^2\right| - 4\left[\frac{2}{\sqrt{20-16}}\tan^{-1}\left(\frac{2x+4}{\sqrt{20-16}}\right)\right]\right)$$

$$I = \frac{1}{2}\left(\ln\left|5+4x+x^2\right| - 4\left[\frac{2}{2}\tan^{-1}\left(\frac{2x+4}{2}\right)\right]\right)$$

$$I = \frac{1}{2}\left(\ln\left|5+4x+x^2\right| - 4\left[\tan^{-1}(x+2)\right]\right) + C$$

⑨ Find $I = \int \frac{5}{x\sqrt{9x^2-1}}\,dx = 5\int \frac{1}{x\sqrt{9x^2-1}}\,dx$

Formula 20:

$$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\left(\frac{|u|}{a}\right)$$

Let $u^2 = 9x^2$; $a^2 = 1$

$u = 3x$ $a = 1$

Note: $u = 3x$
$x = \frac{1}{3}u$

$$\frac{du}{dx} = 3 \Rightarrow du = 3dx \Rightarrow \frac{1}{3}du = dx$$

$$I = 5\int \frac{1}{\cancel{\frac{1}{3}}u\sqrt{u^2-a^2}}\,\cancel{\frac{1}{3}}du = 5\int \frac{1}{u\sqrt{u^2-a^2}}\,du$$

$$I = 5\left[\frac{1}{a}\sec^{-1}\frac{|u|}{a}\right] = 5\left[\frac{1}{1}\sec^{-1}\frac{|3x|}{1}\right] = 5\sec^{-1}|3x| + c$$

(10) Find $I = \int_1^2 \frac{\ln x}{x}\,dx = \int \ln x \cdot \frac{1}{x}\,dx$

Let $u = \ln x$

$$\frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{1}{x}dx$$

$$I = \int u \cdot du = u^2 \cdot \frac{1}{2} = \frac{1}{2}u^2 = \frac{1}{2}(\ln x)^2 \Big|_1^2$$

$$I = \frac{1}{2}(\ln 2)^2 - \frac{1}{2}(\ln 1)^2$$

Note: $\ln 1 = 0$

$$\underline{I} = \frac{1}{2}(\ln 2)^2$$

(11) Find $I = \int_0^4 x\sqrt{6-x}\, dx$

Formula 35:

$$\int u\sqrt{a+bu}\, du = \frac{2}{15b^2}(3bu - 2a)(a+bu)^{3/2}$$

Let $u = x$, $a = 6$, $b = -1$

$$I = \frac{2}{15(-1)^2}\left(3(-1)x - 2(6)\right)(6-x)^{3/2}$$

$$I = \frac{2}{15}(-3x - 12)(6-x)^{3/2} \Big|_0^4$$

$$I = \frac{2}{15}(-24)(2)^{3/2} - \frac{2}{15}(-12)(6)^{3/2} = 14.46$$

Note: $\ln 1 = 0$

(12) Find $L = \lim_{x \to 1} \frac{(\ln x)^2}{x-1} = \frac{0}{0}$

So we can apply L'Hôpital's Rule

$$L = \lim_{x\to 1} \frac{D_x\left(\ln(x) \right)^2}{D_x\left(x-1 \right)} = \lim_{x\to} \frac{2(\ln x)^1 \cdot D_x(\ln x)}{1}$$

$$= \lim_{x\to 1} 2 \cdot (\ln x) \cdot \frac{1}{x} = 2(\ln 1) \cdot \left(\frac{1}{1}\right)$$

$$= 2 \cdot 0 \cdot 1 = 0$$

(13) Find $L = \lim_{x \to \frac{1}{2}} \dfrac{\cos \pi x}{\cos 5\pi x} = \dfrac{\cos(\frac{1}{2}\pi)}{\cos(\frac{5}{2}\pi)} = \dfrac{0}{0}$

Apply L'Hôpital's Rule:

$$L = \lim_{x \to \frac{1}{2}} \frac{D_x(\cos \pi x)}{D_x(\cos 5\pi x)} = \lim_{x \to \frac{1}{2}} \frac{-\sin \pi x \cdot \pi}{-\sin 5\pi x \cdot 5\pi}$$

$$L = \frac{1}{5} \frac{(-\sin \frac{1}{2}\pi)}{(-\sin \frac{5}{2}\pi)} = \frac{1}{5} \frac{(-1)}{(-1)} = \frac{1}{5}$$

(14) Find $L = \lim_{x \to \infty} \dfrac{e^{4x}}{x^2} = \dfrac{\infty}{\infty}$

So we can apply L'Hôpital's Rule

$$L = \lim_{x \to \infty} \frac{D_x(e^{4x})}{D_x(x^2)} = \lim_{x \to \infty} \frac{e^{4x} \cdot 4}{2x}$$

$$L = \lim_{x \to \infty} \frac{2e^{4x}}{x} = \frac{\infty}{\infty}$$ Apply L'Hôpital's Rule again

$$L = \lim_{x \to \infty} \frac{2 \cdot e^{4x} \cdot 4}{1} = \frac{2 \cdot e^{\infty} \cdot 4}{1} = \frac{\infty}{1} = \infty$$

(15) Find $I = \int_1^{\infty} \frac{1}{x+4}\,dx$

$$= \lim_{c\to\infty} \underbrace{\int_1^{c} \frac{1}{x+4}\,dx}_{I_2}$$

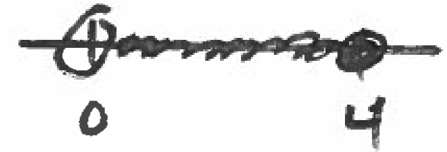
$$I_2 = \ln|x+4| \Big|_1^{c}$$

$$I_2 = \ln|c+4| - \ln|1+4|$$

$$I = \lim_{c\to\infty}\left[\ln|c+4| - \ln 5\right] = \ln(\infty) - \ln(5)$$

$$= \infty - \ln(5) = \infty$$

(16) Find $I = \int_0^4 \frac{1}{x^2}\,dx$



$$I = \lim_{c \to 0^+} \underbrace{\int_c^4 \frac{1}{x^2}\,dx}_{I_2}$$

Note: $\frac{1}{x^2}$ is defined when x is to the right of 0.

$$I_2 = \int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = x^{-1} \cdot (-1) = \frac{-1}{x} \Big|_c^4$$

$$I_2 = -\frac{1}{4} - -\frac{1}{c} = -\frac{1}{4} + \frac{1}{c}$$

$$I = \lim_{c \to 0^+} \left(\frac{-1}{4} + \frac{1}{c} \right) = -\frac{1}{4} + \frac{1}{\to 0^+} = \frac{-1}{4} + \infty$$

$$= \infty$$