Test 1 Review: Chapter 8

#1, p. 579

$$I = \int x\sqrt{x^2-36}\,dx$$

Hint: $u = x^2 - 36$

$du = 2x\,dx$

$\frac{1}{2}du = x \cdot dx$

$$I = \int \sqrt{x^2-36} \cdot x\,dx$$

$$I = \int \sqrt{u} \cdot \tfrac{1}{2}du$$

$$I = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right]$$

$$= \frac{1}{2}\cdot\frac{2}{3}\left[(x^2-36)^{3/2}\right]$$

$$= \frac{1}{3}\left[(x^2-36)^{3/2}\right] + C$$

#2, p. 579

$I = \int x \cdot e^{x^2-1}\,dx$

Let $u = x^2 - 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$I = \int e^u \cdot \frac{1}{2}du$

$I = \frac{1}{2}\int e^u\,du$

$I = \frac{1}{2}e^u = \frac{1}{2}e^{x^2-1} + C$ Answer

#17, p. 579

Hint: Let $u = \pi x - 1$

$du = \pi \cdot dx$

$\frac{1}{\pi} du = dx$

$$I = \int \cos^3(\pi x - 1)\, dx$$

$$I = \int \cos^3(u) \cdot \frac{1}{\pi} du$$

$$I = \frac{1}{\pi} \int \cos^3 u \, du$$

Now use Formula 51.

$$I = \frac{1}{\pi}\left[\frac{\cos^2 u \cdot \sin u}{3} + \frac{2}{3} \int \cos u \, du \right]$$

$$I = \frac{1}{\pi}\left[\frac{\cos^2 u \cdot \sin u}{3} + \frac{2}{3}(-\sin u) \right]$$

$$I = \frac{1}{\pi}\left[\frac{\cos^2(\pi x - 1) \cdot \sin(\pi x - 1)}{3} + -\frac{2}{3} \sin(\pi x - 1) \right] + C$$

#37, p. 579

$$I = \int \frac{x^2}{x^2+5x-24}\,dx$$

First divide : $\frac{x^2}{x^2+5x-24} = 1 + \frac{-5x+24}{x^2+5x-24}$

$$\begin{array}{r|l} & 1 \\ \hline x^2+5x-24 & x^2 \\ & \underline{-\;x^2+5x-24} \\ & -5x+24 \end{array}$$

Remainder

$$I = \int \left(1 + \frac{-5x+24}{x^2+5x-24}\right) dx$$

$$I = \int 1\,dx + \underbrace{\int \frac{-5x+24}{x^2+5x-24}\,dx}_{I_2} = x + I_2$$

#37, P. 579 $I_2 = \int \frac{-5x + 24}{x^2 + 5x - 24} dx$

Decompose : $\frac{-5x + 24}{x^2 + 5x - 24} = \frac{-5x + 24}{(x+8)(x-3)} = \frac{A}{x+8} + \frac{B}{x-3}$

$$\frac{-5x + 24}{(x+8)(x-3)} = \frac{A(x-3) + B(x+8)}{(x+8)(x-3)}$$

$\Rightarrow \quad -5x + 24 = A(x-3) + B(x+8)$

let $x = 3 \Rightarrow 9 = 11B \Rightarrow B = 9/11$

let $x = -8 \Rightarrow 64 = -11A \Rightarrow A = -64/11$

$$I_2 = \int \frac{-64/11}{x+8} dx + \int \frac{9/11}{x-3} dx$$

$$I_2 = -\frac{64}{11}\int \frac{1}{x+8}\,dx + \frac{9}{11}\int \frac{1}{x-3}\,dx$$

#37, p.579

$$I_2 = -\frac{64}{11}\ln|x+8| + \frac{9}{11}\ln|x-3|$$

$$I = x + -\frac{64}{11}\ln|x+8| + \frac{9}{11}\ln|x-3| + C$$

#39, p. 579 $I = \int \frac{x}{(4+5x)^2}\,dx$

Use Formula #4

$$\int \frac{u}{(a+bu)^2}\,du$$

$$I = \int \frac{u}{(a+bx)^2}\,du$$

Let $a = 4$

$b = 5$

$u = x$

$du = dx$

$$I = \frac{1}{b^2}\left(\frac{a}{a+bu} + \ln|a+bu|\right) + C$$

$$I = \frac{1}{25}\left(\frac{4}{4+5x} + \ln|4+5x|\right) + C$$

#40, p. 579 $I = \int \frac{x}{\sqrt{4+5x}}\,dx$

Use Formula 21

$$\int \frac{u}{\sqrt{a+bu}}\,du$$

let $a = 4$

$b = 5$

$u = x$

$du = dx$

$$I = \int \frac{u}{\sqrt{a+bu}}\,du$$

$$I = -\frac{2(2a-bu)}{3b^2}\sqrt{a+bu} + C$$

$$I = \frac{-2(8-5x)}{75}\sqrt{4+5x} + C$$

#42 p. 579

$$I = \int_0^1 \frac{x}{1+e^{x^2}}\,dx$$

Hint: let $u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$I = \int \frac{1}{1+e^u} \cdot \frac{1}{2}du$$

$$I = \frac{1}{2}\int \frac{1}{1+e^u}du$$

Now use Formula 84:

$$I = \frac{1}{2}\left[u - \ln|1+e^u|\right]$$

$$I = \frac{1}{2}\left[x^2 - \ln|1+e^{x^2}|\right] + C$$

#43 p. 579

$$I = \int \frac{x}{x^2+4x+8}\,dx$$

Use Formula 15

$$\int \frac{u}{a+bu+cu^2}\,du$$

$$I = \int \frac{u}{a+bu+cu^2}\,du$$

$$I = \frac{1}{2c}\left(\ln|a+bu+cu^2| - b\int \frac{1}{a+bu+cu^2}\,du\right)$$

Let $a = 8$
$b = 4$
$c = 1$
$u = x$
$du = dx$

$$I = \frac{1}{2}\left(\ln|8+4x+x^2| - 4\int \frac{1}{8+4x+x^2}\,dx\right)$$

Now Use Formula 15

$$I = \frac{1}{2}\left(\ln|8+4x+x^2| - 4\left[\frac{1}{2}\tan^{-1}\left(\frac{2x+4}{4}\right)\right]\right) + C$$

#44 p. 579

Use Formula 33

$$\int \frac{1}{u\sqrt{u^2-a^2}}\,du$$

$$I = \int \frac{3}{2x\sqrt{9x^2-1}}\,dx$$

$u^2 = 9x^2$

$a^2 = 1$

$u = 3x \quad | \quad a = 1$

$du = 3dx$

$x = \frac{1}{3}u$

$2x = 2 \cdot \frac{1}{3}u = \frac{2}{3}u$

$$I = \int \frac{1}{\frac{2}{3}u\sqrt{u^2-a^2}}\,du$$

$$I = \frac{3}{2}\int \frac{1}{u\sqrt{u^2-a^2}}\,du$$

$$I = \frac{3}{2}\left[\frac{1}{a}\sec^{-1}\frac{|u|}{a}\right] + C$$

$$I = \frac{3}{2}\left[\sec^{-1}|3x|\right] + C$$ Answer

#63 p. 579 $I = \int_1^4 \frac{\ln x}{x} dx$

Hint: $u = \ln x$

$du = \frac{1}{x} dx$

$$I = \int \ln x \cdot \frac{1}{x} dx$$

$$I = \int u \cdot du$$

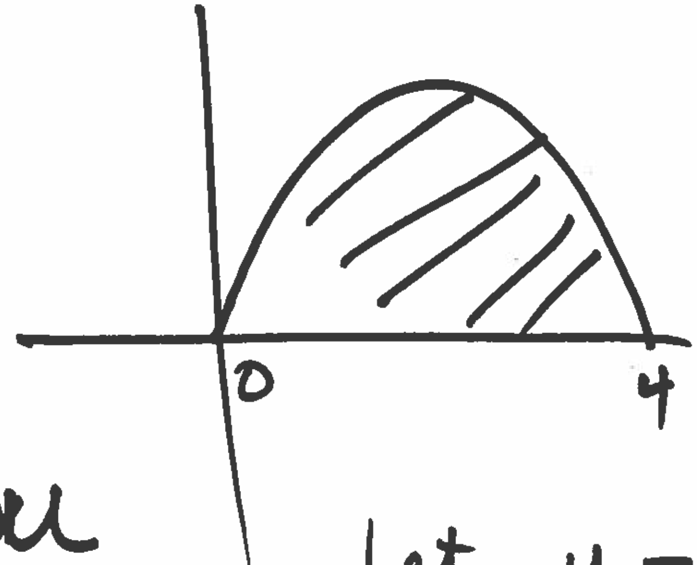
$$I = \frac{u^2}{2} = \left. \frac{(\ln x)^2}{2} \right|_1^4$$

$$= \frac{(\ln 4)^2}{2} - \frac{(\ln 1)^2}{2}$$

Note: $\ln 1 = 0$

$$= \frac{(\ln 4)^2}{2}$$

67 $y = x\sqrt{4-x}$



Area $= \int_0^4 x\cdot\sqrt{4-x}\,dx$

$= \int (4-u)\sqrt{u}\cdot(-1)\,du$

Let $u = 4-x \Rightarrow x=4-$

$\frac{du}{dx} = -1$

$du = -1dx$

$-1du = dx$

$= -1\int\left(4u^{1/2} - u^{3/2}\right)du$

$= -1\left[4\cdot\frac{2}{3}\cdot u^{3/2} - \frac{2}{5}\cdot u^{5/2}\right]$

$= -1\left[\frac{8}{3}(4-x)^{3/2} - \frac{2}{5}(4-x)^{5/2}\right]\Big|_0^4$

$u\cdot\sqrt{u}$

$= u^1\cdot u^{1/2} = u^{3/2}$

$0 \; +1\left[\frac{8}{3}(4)^{3/2} - \frac{2}{5}(4)^{5/2}\right] =$

#73, p 579:

Note: $\ln 1 = 0$

$$\lim_{x\to 1} \frac{(\ln x)^2}{x-1} = \frac{0}{0} = \text{Use L'Hôpital's Rule}$$

$$= \lim_{x\to 1} \frac{2\cdot(\ln x)\cdot D_x(\ln x)}{1}$$

$$= \lim_{x\to 1} 2\cdot \ln x \cdot \frac{1}{x} = \lim_{x\to 1} \frac{2\ln x}{x} = \frac{0}{1} = 0$$

\# 74 P. 579 $\lim\limits_{x \to 0} \dfrac{\sin \pi x}{\sin 5\pi x} = \dfrac{0}{0}$

Use L'Hôpital's Rule

$$= \lim_{x \to 0} \left(\frac{\cos(\pi x) \cdot D_x(\pi x)}{\cos(5\pi x) \cdot D_x(5\pi x)} \right)$$

$$= \lim_{x \to 0} \left(\frac{\cos \pi x \cdot \pi}{\cos(5\pi x) \cdot 5\pi} \right) =$$

$$= \frac{1 \cdot \pi}{1 \cdot 5\pi} = \frac{1}{5}$$

#75 · p. 579

$$\lim_{x\to\infty}\left(\frac{e^{2x}}{x^2}\right) = \frac{\infty}{\infty} = \text{Use L'Hôpital's Rule}$$

$$= \lim_{x\to\infty}\left(\frac{e^{2x}\cdot 2}{2x}\right)$$

$$= \lim_{x\to\infty}\left(\frac{e^{2x}}{x}\right) = \frac{\infty}{\infty} = \text{Use L'Hôpital's Rule again}$$

$$= \lim_{x\to\infty}\left(\frac{e^{2x}\cdot 2}{1}\right) = \frac{\infty}{1} = \infty$$