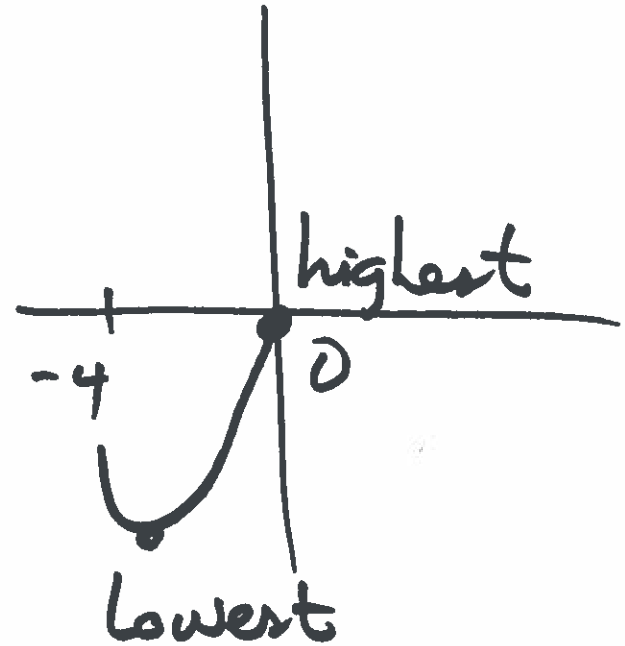
Test 2 Review

Ch. 3 pp 238 - 239

① $f(x) = x^2 + 5x \quad [-4, 0]$



$f'(x) = 2x + 5$

set $f'(x) = 0$

$2x + 5 = 0$

$x = -2.5$

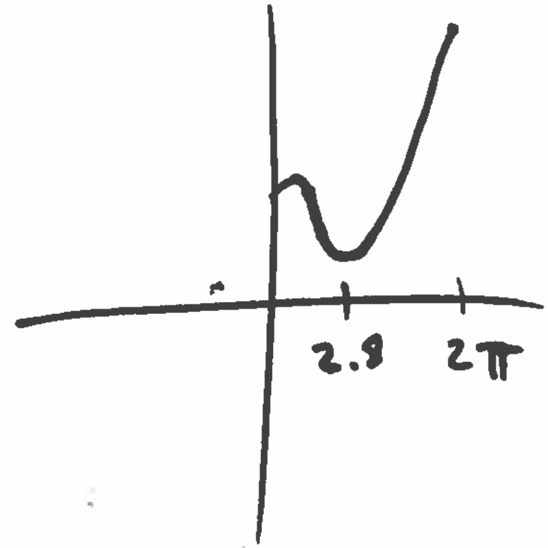
When $x = -2.5$, $y = (-2.5)^2 + 5(-2.5) = -6.25$

$x = -4$, $y = -4$

$x = 0$, $y = 0$

hightest at $(0, 0)$; lowest at $(-2.5, -6.25)$

⑦ $g(x) = 2x + 5\cos x \quad [0, 2\pi]$



$g'(x) = 2 + 5(-\sin x) = \underbrace{2 - 5\sin x}$

set $g'(x) = 0$

$2 - 5\sin x = 0$

$-5\sin x = -2$

$\sin x = \frac{-2}{-5} = 0.4$

$x = \sin^{-1}(0.4) = \arcsin(0.4) = \text{Asin}(0.4)$

$x = 0.4115$

$x = 2.73$

$x = 0.4115 \qquad y = 2(0.4115) + 5\cos(0.4115) = 5.4$

$x = 2.73 \qquad y = 0.877$ Lowest

$x = 0 \qquad y = 5$

$x = 2\pi \qquad y = 17.566$ highest

(9) $f(x) = 2x^2 - 7$ $\qquad [\overset{a}{0}, \overset{b}{4}]$

Does Rolle's Thm. apply?

a) Condition 1: $f(x)$ is con't on $[0, 4]$? Yes

b) Condition 2: $f(a) = f(b)$? No

$$f(0) = 2(0)^2 - 7 = -7$$

$$f(4) = 2(4)^2 - 7 = 25$$

Rolle's Thm. does not apply.

(11) $f(x) = \dfrac{x^2}{1-x^2}$ $[-2, 2]$

Does Rolle's Thm apply?

a) $f(x)$ is con't on $[-2, 2]$? No

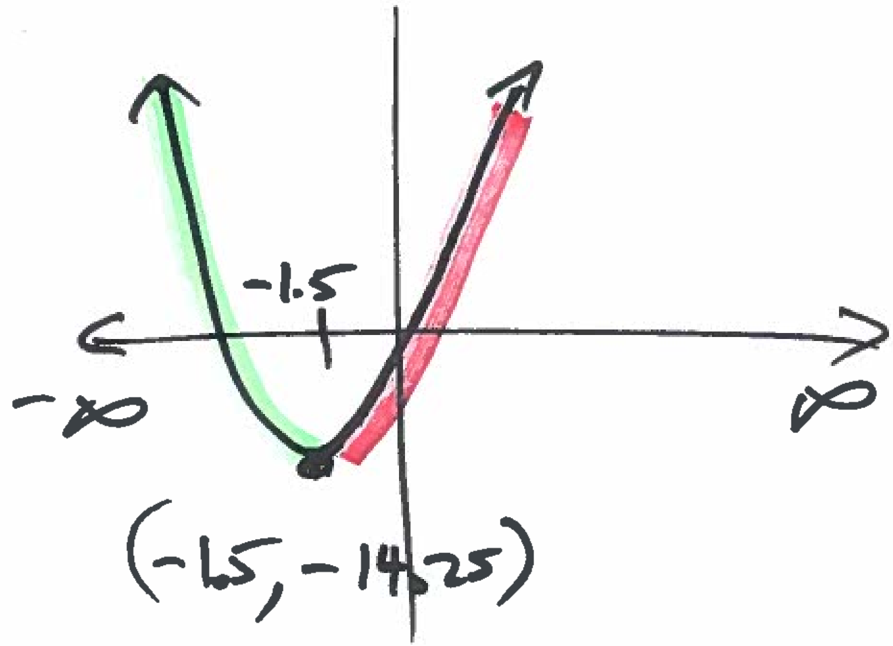
b) $f(a) = f(b)$? Yes

$$f(-2) = \frac{(-2)^2}{1-(-2)^2} = -4/3$$

$$f(2) = -4/3$$

Rolle's Thm. does not apply.

21 $f(x) = x^2 + 3x - 12$



$f'(x) = 2x + 3$

set $f'(x) = 0$

$2x + 3 = 0$

$x = -3/2 = -1.5$

When $x = -1.5$, $y = (-1.5)^2 + 3(-1.5) - 12 = -14.25$

Graph decreases on $(-\infty, -1.5)$

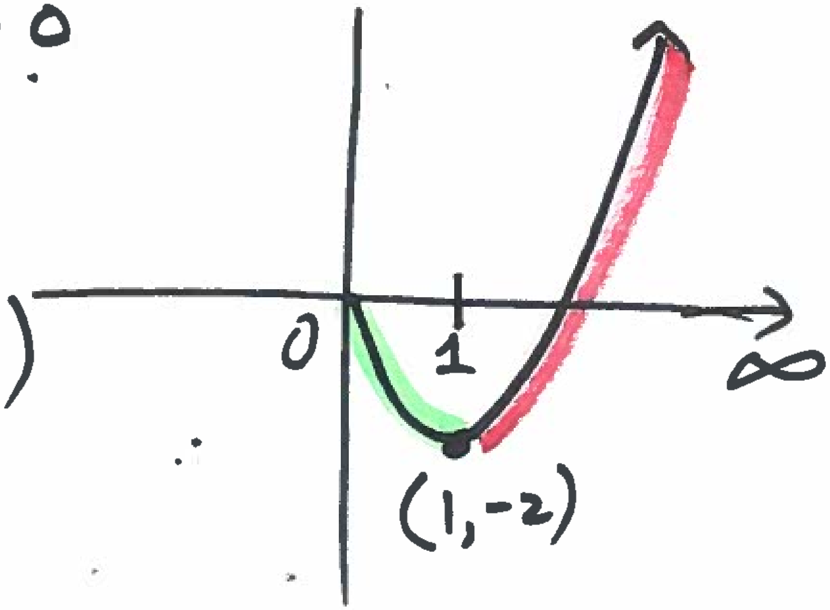
Graph increases on $(-1.5, \infty)$

Graph decreases on $(0,1)$

Graph increase on $(1,\infty)$

(25) $h(x) = \sqrt{x} \cdot (x-3) \qquad x > 0$

$= x^{1/2} \cdot (x-3)$



$h'(x) = F \cdot Dx(S) + S \cdot Dx(F)$

$= (x^{1/2})(1) + (x-3)(\frac{1}{2}x^{-1/2})$

set $h'(x) = 0$

$x^{1/2} + (x-3) \cdot \frac{1}{2} \cdot \frac{1}{x^{1/2}} = 0$

$\frac{x^{1/2}}{1} + \frac{x-3}{2x^{1/2}} = 0$

$\frac{2x + x - 3}{2x^{1/2}} = 0$

$\Rightarrow 2x + x - 3 = 0$

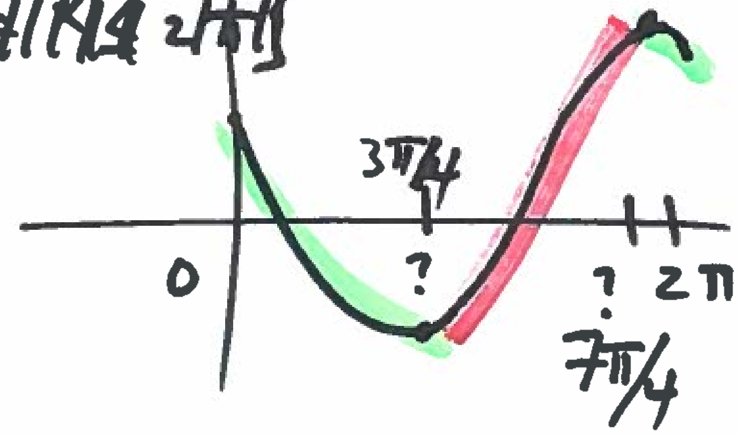
$3x - 3 = 0$

$x = 1 \; ; \quad Y = \sqrt{x}(x-3) = \sqrt{1}(1-3) = -2$

Graph decreases on $(0, 3\pi/4)$ $(7\pi/4, 2\pi)$

Graph increases on $(3\pi/4, 7\pi/4)$

(33) $f(x) = (\cos(x) - \sin(x))$ $[0, 2\pi)$



$f'(x) = (-\sin x) - (\cos x)$

set $f'(x) = 0$

$-\sin x - \cos x = 0$

$-\sin x = \cos x$

$-\dfrac{\sin x}{\cos x} = 1$

$-\tan x = 1$

$\tan x = -1 \Rightarrow x = \tan^{-1}(-1)$

$x = -\pi/4, \ 3\pi/4$

$x = 7\pi/4, \ 3\pi/4$

(35) $f(x) = x^3 - 9x^2$

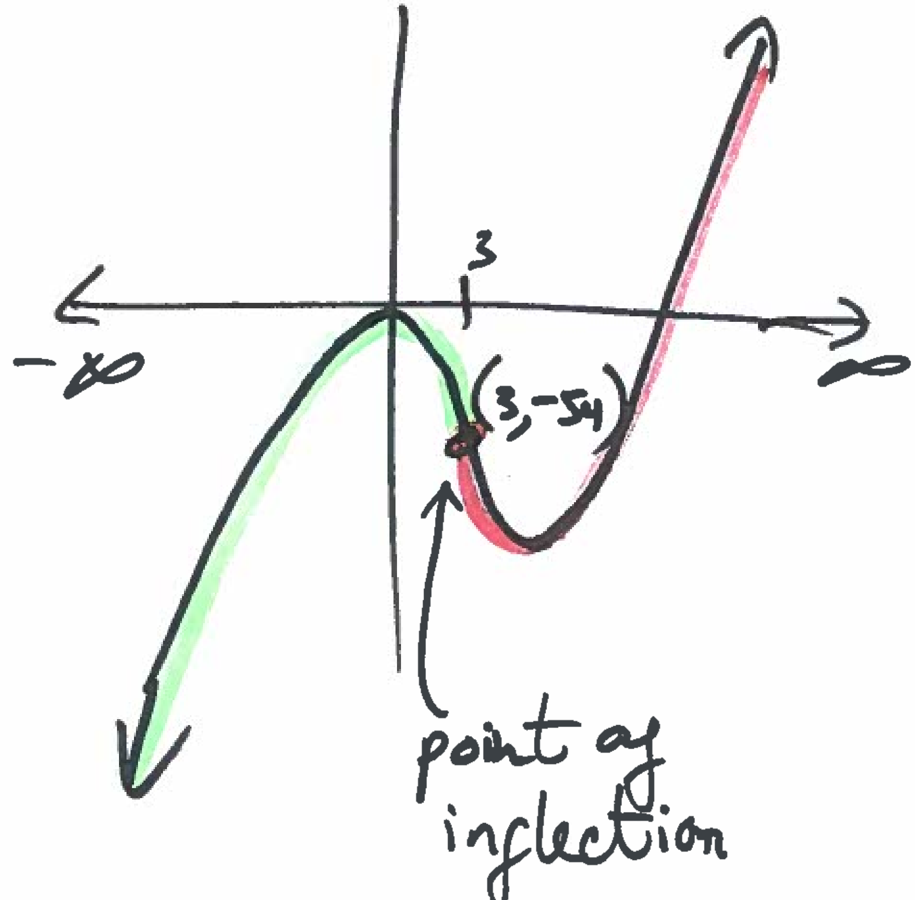
$f'(x) = 3x^2 - 18x$

$f''(x) = 6x - 18$

set $f''(x) = 0$

$6x - 18 = 0$

$x = 3$



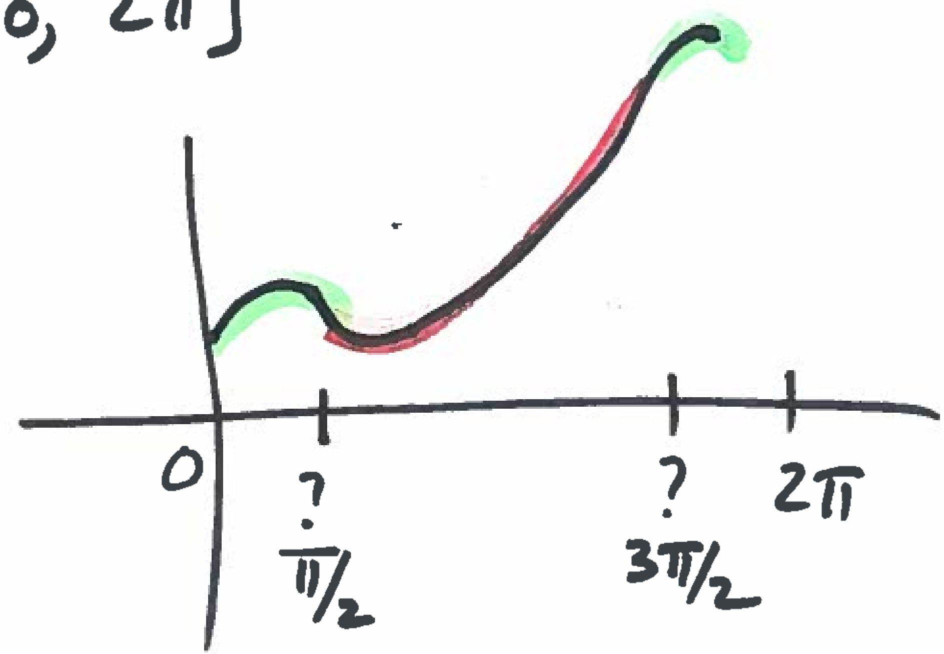
$f''(x) = 0$

When $x = 3$, $y = (3)^3 - 9(3)^2 = -54$

Graph concaves down on $(-\infty, 3)$

Graph concaves up on $(3, \infty)$

(39) $f(x) = x + \cos x \quad [0, 2\pi]$



$f'(x) = 1 + (-\sin x)$

$f''(x) = 0 + (-\cos x)$

set $f''(x) = 0$

$-\cos x = 0$

$\cos x = 0 \Rightarrow x = \cos^{-1}(0)$

$x = \pi/2 \, ; \, 3\pi/2$

Graph concaves down on $(0, \pi/2)$; $(\frac{3\pi}{2}, 2\pi)$

Graph concaves up on $(\pi/2, 3\pi/2)$

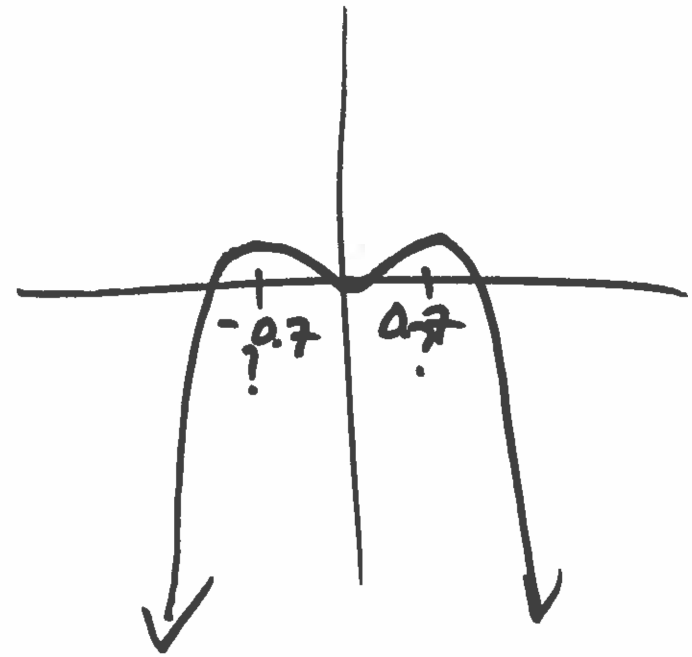
(43) $g(x) = 2x^2 \cdot (1-x^2) = 2x^2 - 2x^4$

$g'(x) = 4x - 8x^3$

set $g'(x) = 0$

$4x - 8x^3 = 0$

$(4x)(1 - 2x^2) = 0$



$4x = 0$ | $1 - 2x^2 = 0$

$x = 0$ | $1 = 2x^2$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{1/2} = \pm 0.707$

Graph has max. at $x = \pm 0.707$

Graph has min. at $x = 0$

(53) $\lim_{x \to \infty} \left(8 + \frac{1}{x} \right) = 8 + \frac{1}{\infty} = 8 + 0 = 8$

(57) $\lim_{x \to -\infty} \left(\frac{3x^2}{x+5} \right)$

$= \lim_{x \to -\infty} \left(\frac{3x^2/x}{x/x + 5/x} \right)$

$= \lim_{x \to -\infty} \left(\frac{3x}{1 + 5/x} \right) = \frac{-\infty}{1 + 5/-\infty} = \frac{-\infty}{1}$

$= -\infty$

(85) $f(x) = x^3 - 3x - 1$

$f'(x) = 3x^2 - 3$

Use Newton's Program with initial guesses: -2; -0.8, 1.1

X-intercepts are -1.532, -0.347, 1.879385

(87) $f(x) = x^4 + x^3 - 3x^2 + 2$

$f'(x) = 4x^3 + 3x^2 - 6x$

Use Newton's Program with initial guesses: -3; -1

x-intercepts are: -2.18227, -0.7951977

Ch. 4 pp. 312 - 313

① Find $\int (x-6)\,dx$

$$= \int x^1\,dx - \int 6\,dx$$

$$= \frac{x^2}{2} - 6x + C$$

(5) $\displaystyle \int \frac{x^4+8}{x^3}\,dx$

$$= \int \left(\frac{x^4}{x^3} + \frac{8}{x^3} \right) dx$$

$$= \int (x + 8 \cdot x^{-3}) dx$$

$$= \frac{x^2}{2} + 8 \cdot \left(\frac{x^{-2}}{-2} \right) + C$$

$$= \frac{x^2}{2} + -4x^{-2} + C$$

7 $\int (2x - 9\sin x)\,dx$

$= 2 \cdot \int x\,dx - 9 \cdot \int \sin x\,dx$

$= 2 \cdot \left(\frac{x^2}{2}\right) - 9(-\cos x)$

$= x^2 + 9\cos x + C$

25 $\displaystyle\sum_{i=1}^{20} 2i = 2(1) + 2(2) + 2(3) + \dots + 2(20)$

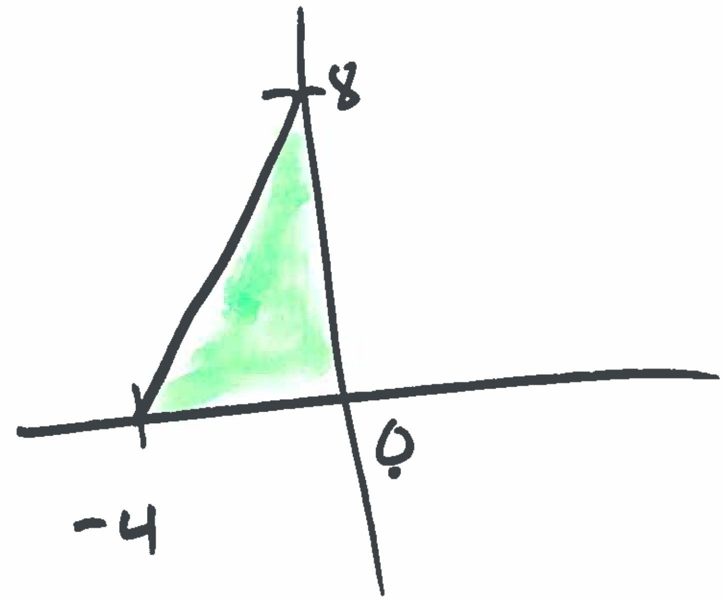
$= 420$

27 $\sum_{i=1}^{20} (i+1)^2 = (2)^2 + (3)^2 + (4)^2 + \cdots + (21)^2 = 3310$

31 Shaded Area $= \int_0^3 (8-2x)\,dx = \sum_{i=1}^{\infty} f(x_i)\,\Delta x_i = 15$

33 Shaded Area $= \int_{-2}^{1} (5-x^2)\,dx = \sum_{i=1}^{\infty} f(x_i)\,\Delta x_i = 12$

37 $f(x) = 2x + 8 \qquad [-4, 0]$



$$\text{Shaded Area} = \int_{-4}^{0} f(x)\,dx$$

$$= \int_{-4}^{0} (2x+8)\,dx$$

$$= F(0) - F(-4) = 0 - (-16) = 16$$

$$F(x) = \int (2x+8)\,dx = 2\cdot\left(\frac{x^2}{2}\right) + 8x = x^2 + 8x$$

$$F(0) = (0)^2 + 8(0) = 0$$

$$F(-4) = (-4)^2 + 8(-4) = -16$$

(38) Shaded Area $= \int_{-10}^{10} (100 - x^2)\, dx = 4000/3$

(45) $\int_{-1}^{1} (4t^3 - 2t)\, dt = F(1) - F(-1) = 0 - 0 = 0$

$F(t) = \int (4t^3 - 2t)\, dt = \frac{4t^4}{4} - \frac{2t^2}{2} = t^4 - t^2$

$F(1) = (1)^4 - (1)^2 = 0$

$F(-1) = (-1)^4 - (-1)^2 = 0$

(47) $\int_4^9 x\sqrt{x}\,dx = \int_4^9 x\cdot x^{1/2}\,dx = \int_4^9 x^{3/2}\,dx = F(9) - F(4)$

$= 97.2 - 14.8$

$= 82.4$

$F(x) = \int x^{3/2}\,dx = \frac{x^{5/2}}{5/2} = \frac{2}{5}\cdot x^{5/2}$

$F(9) = \frac{2}{5}(9)^{5/2} = 97.2$

$F(4) = \frac{2}{5}(4)^{5/2} = 12.8$

(51) $y = \sin x \qquad [0, 2]$

$$\text{Shaded Area} = \int_0^2 f(x)\,dx = \int_0^2 \sin x\,dx = F(2) - F(0)$$

$$= 0.4161 - (-1)$$

$$= 1.4161$$

$$F(x) = \int \sin x\,dx = -\cos x$$

$$F(2) = -\cos(2) = 0.4161$$

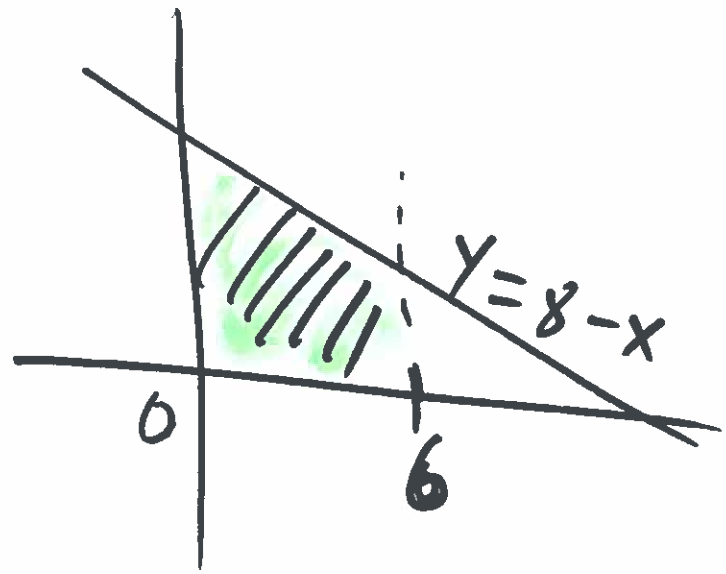
$$F(0) = -\cos(0) = -\underline{1}$$

(53) $y = 8 - x$

$x = 0$

$x = 6$

$y = 0$



$$\text{Shaded Area} = \int_0^6 (8-x)\,dx = F(6) - F(0) = 30 - 0 = 30$$

$$F(x) = \int (8-x)\,dx = 8x - x^2/2$$

$$F(6) = 8(6) - 6^2/2 = 30$$

$$F(0) = 8(0) - 0^2/2 = 0$$

(55) Shaded Area $= \int_0^1 (x - x^3)\,dx = F(1) - F(0)$

$$= \frac{1}{4} - 0 = \frac{1}{4}$$

$$F(x) = \int (x - x^3)\,dx = \frac{x^2}{2} - \frac{x^4}{4}$$

$$F(1) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$F(0) = 0$$

63 $$\int \frac{x^2}{\sqrt{x^3+3}}\,dx = \int \frac{1}{\sqrt{x^3+3}}\,\underbrace{x^2dx}$$

$$= \int \frac{1}{\sqrt{u}} \cdot \frac{1}{3}du$$

$$f(x) = \frac{x^2}{\sqrt{x^3+3}}$$

Let $u = x^3 + 3$

$$\frac{du}{dx} = 3x^2$$

$$du = 3x^2dx$$

$$\frac{1}{3}du = x^2dx$$

$$= \frac{1}{3}\int \frac{1}{u^{1/2}}du$$

$$= \frac{1}{3}\int u^{-1/2}du$$

$$= \frac{1}{3}\left[\frac{u^{1/2}}{1/2}\right] + C$$

$$= \frac{1}{3}\left[2 \cdot u^{1/2}\right] + C$$

$$F(x) = \frac{2}{3}(x^3+3)^{1/2} + C$$

(67) $\int \sin^3 x \cos x \, dx = \int u^3 \cdot du$

$= \frac{u^4}{4} + C$

$F(x) = \frac{(\sin x)^4}{4} + C$

~~Let~~

$f(x) = \sin^3 x \cos x$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \underbrace{\cos x \cdot dx}$

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$$\int \frac{\cos\theta}{\sqrt{1-\sin\theta}}\,d\theta = \int \frac{1}{\sqrt{1-\sin\theta}} \cdot \cos\theta\, d\theta$$

$$= \int \frac{1}{\sqrt{u}} \cdot du$$

$$= \int \frac{1}{u^{1/2}}\, du$$

$$= \int u^{-1/2}\, du$$

$$= \frac{u^{1/2}}{1/2} + C$$

$$= 2\left(u^{1/2}\right) + C$$

$$= 2(\sin\theta)^{1/2} + C$$

Let $u = \sin\theta$

$$\frac{du}{d\theta} = \cos\theta$$

$$du = \cos\theta\, d\theta$$

(75) $\int_0^1 (3x+1)^5\,dx = \int_0^1 u^5 \cdot \frac{1}{3}\,du$

$= \frac{1}{3}\int_0^1 u^5\,du$

$= \frac{1}{3}\left[\frac{u^6}{6}\right]$

$= \frac{1}{18}\left[u^6\right]\Big|_0^1$

$= \frac{1}{18}(1)^6 - \frac{1}{18}(0)^6$

$= \frac{1}{18}$

Let $u = 3x+1$

$\frac{du}{dx} = 3$

$du = 3dx$

$\frac{1}{3}du = dx$