

# Test 2 Review .

Ch. 3 pp 238 - 239

①  $f(x) = x^2 + 5x$   $[-4, 0]$

$$f'(x) = 2x + 5$$

$$\text{set } f'(x) = 0$$

$$2x + 5 = 0$$

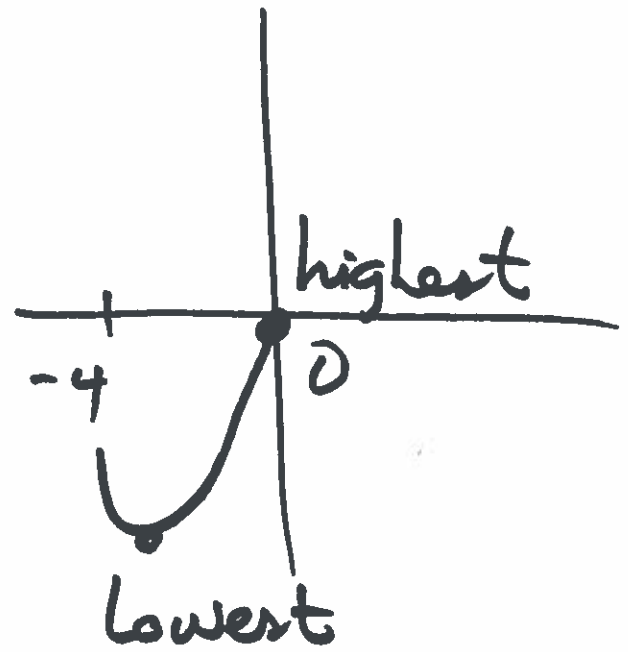
$$x = -2.5$$

$$\text{When } x = -2.5, y = (-2.5)^2 + 5(-2.5) = -6.25$$

$$x = -4, y = -4$$

$$x = 0, y = 0$$

hightest at  $(0, 0)$ ; lowest at  $(-2.5, -6.25)$



$$(7) \quad g(x) = 2x + 5 \cos x \quad [0, 2\pi]$$

$$g'(x) = 2 + 5(-\sin x) = \underbrace{2 - 5 \sin x}$$

$$\text{set } g'(x) = 0$$

$$2 - 5 \sin x = 0$$

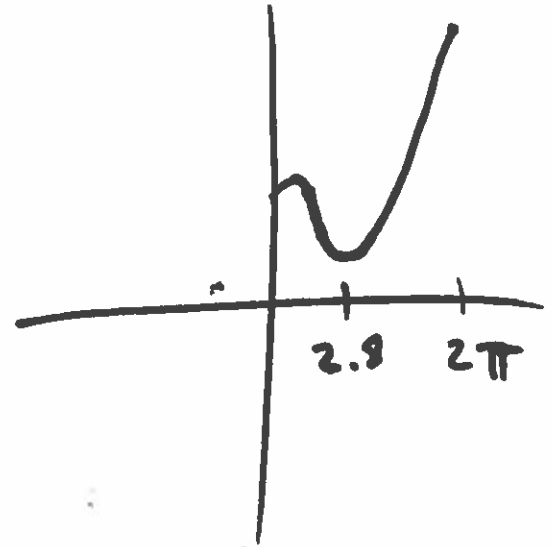
$$-5 \sin x = -2$$

$$\sin x = \frac{-2}{-5} = 0.4$$

$$x = \sin^{-1}(0.4) = \arcsin(0.4) = \text{Asin}(0.4)$$

$$x = 0.4115$$

$$x = 2.73$$



$$X = 0.4115 \quad Y = 2(0.4115) + 5 \cos(0.4115) = 5.4$$

$$X = 2.73 \quad Y = 0.877 \quad \text{Lowest}$$

$$X = 0 \quad Y = 5$$

$$X = 2\pi \quad Y = 17.566 \quad \text{highest.}$$

$$(9) f(x) = 2x^2 - 7 \quad \left[ \begin{matrix} a & b \\ 0 & 4 \end{matrix} \right]$$

Does Rolle's Thm. apply?

a) Condition 1:  $f(x)$  is con't on  $[0, 4]$ ? Yes

b) Condition 2:  $f(a) = f(b)$ ? No

$$f(0) = 2(0)^2 - 7 = -7$$

$$f(4) = 2(4)^2 - 7 = 25$$

Rolle's Thm. does not apply.

$$\textcircled{11} \quad f(x) = \frac{x^2}{1-x^2} \quad [-2, 2]$$

Does Rolle's Theorem apply?

a)  $f(x)$  is continuous on  $[-2, 2]$ ? No

b)  $f(a) = f(b)$ ? Yes

$$f(-2) = \frac{(-2)^2}{1-(-2)^2} = -\frac{4}{3}$$

$$f(2) = -\frac{4}{3}$$

Rolle's Theorem does not apply.

$$(21) f(x) = x^2 + 3x - 12$$

$$f'(x) = 2x + 3$$

$$\text{set } f'(x) = 0$$

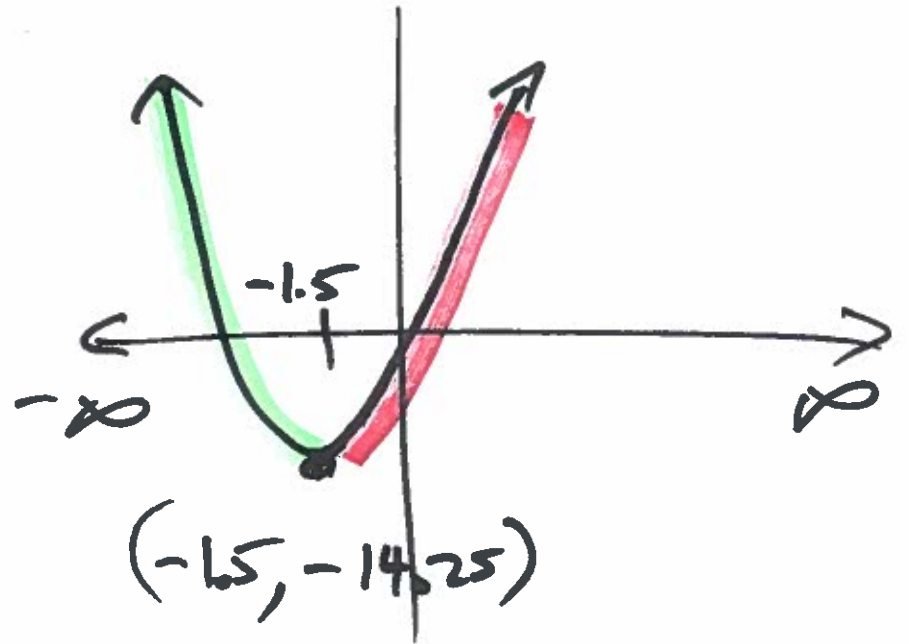
$$2x + 3 = 0$$

$$x = -\frac{3}{2} = -1.5$$

$$\text{When } x = -1.5, y = (-1.5)^2 + 3(-1.5) - 12 = -14.25$$

Graph decreases on  $(-\infty, -1.5)$

Graph increases on  $(-1.5, \infty)$



$$(25) \quad h(x) = \sqrt{x} \cdot (x-3) \quad x > 0$$

$$= x^{1/2} \cdot (x-3)$$

$$h'(x) = F \cdot D_x(S) + S \cdot D_x(F)$$

$$= (x^{1/2})(1) + (x-3)\left(\frac{1}{2}x^{-1/2}\right)$$

$$\text{set } h'(x) = 0$$

$$x^{1/2} + (x-3) \cdot \frac{1}{2} \cdot \frac{1}{x^{1/2}} = 0$$

$$\frac{x^{1/2}}{1} \rightarrow \frac{x-3}{2x^{1/2}} = 0$$

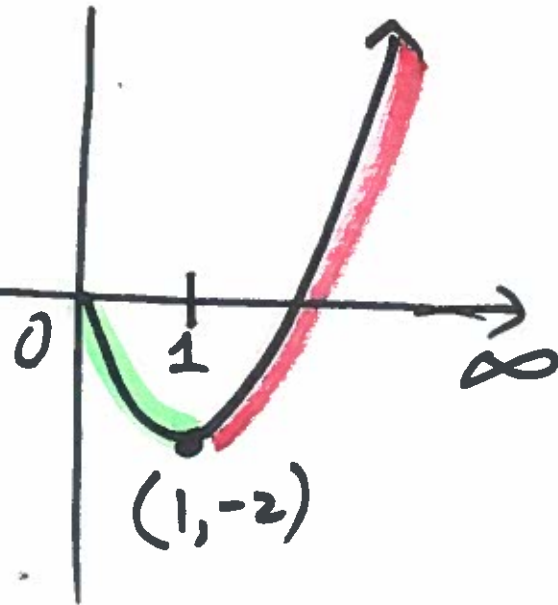
$$\frac{2x + x - 3}{2x^{1/2}} = 0$$

$$\Rightarrow 2x + x - 3 = 0$$

$$3x - 3 = 0$$

$$x = 1$$

$$y = \sqrt{x}(x-3) = \sqrt{1}(1-3) = -2$$



Graph decreases on  $(0, 1)$

Graph increase on  $(1, \infty)$



$$(33) f(x) = (\cos(x) - \sin(x)) \quad (0, 2\pi)$$

$$f'(x) = (-\sin x) - (\cos x)$$

$$\text{set } f'(x) = 0$$

$$-\sin x - \cos x = 0$$

$$-\sin x = \cos x$$

$$\frac{-\sin x}{\cos x} = 1$$

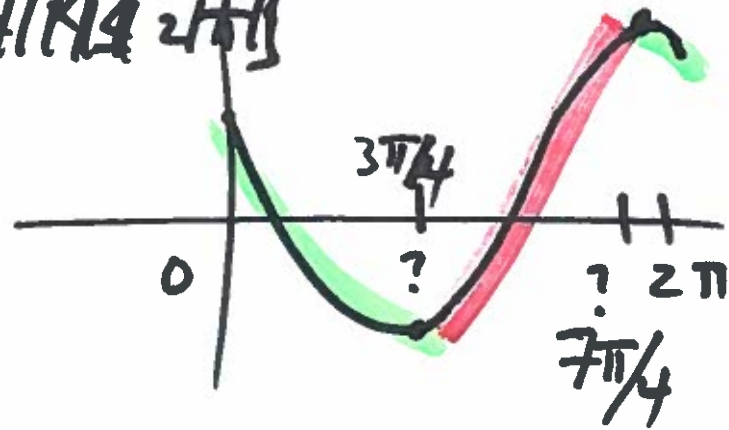
$$-\tan x = 1$$

$$\tan x = -1 \implies$$

$$x = \tan^{-1}(-1)$$

$$x = -\pi/4, 3\pi/4$$

$$x = 7\pi/4, 3\pi/4$$



Graph decreases on  $(0, 3\pi/4)$   $(7\pi/4, 2\pi)$   
Graph increases on  $(3\pi/4, 7\pi/4)$

$$\textcircled{35} \quad f(x) = x^3 - 9x^2$$

$$f'(x) = 3x^2 - 18x$$

$$f''(x) = 6x - 18$$

$$\text{Set } f''(x) = 0$$

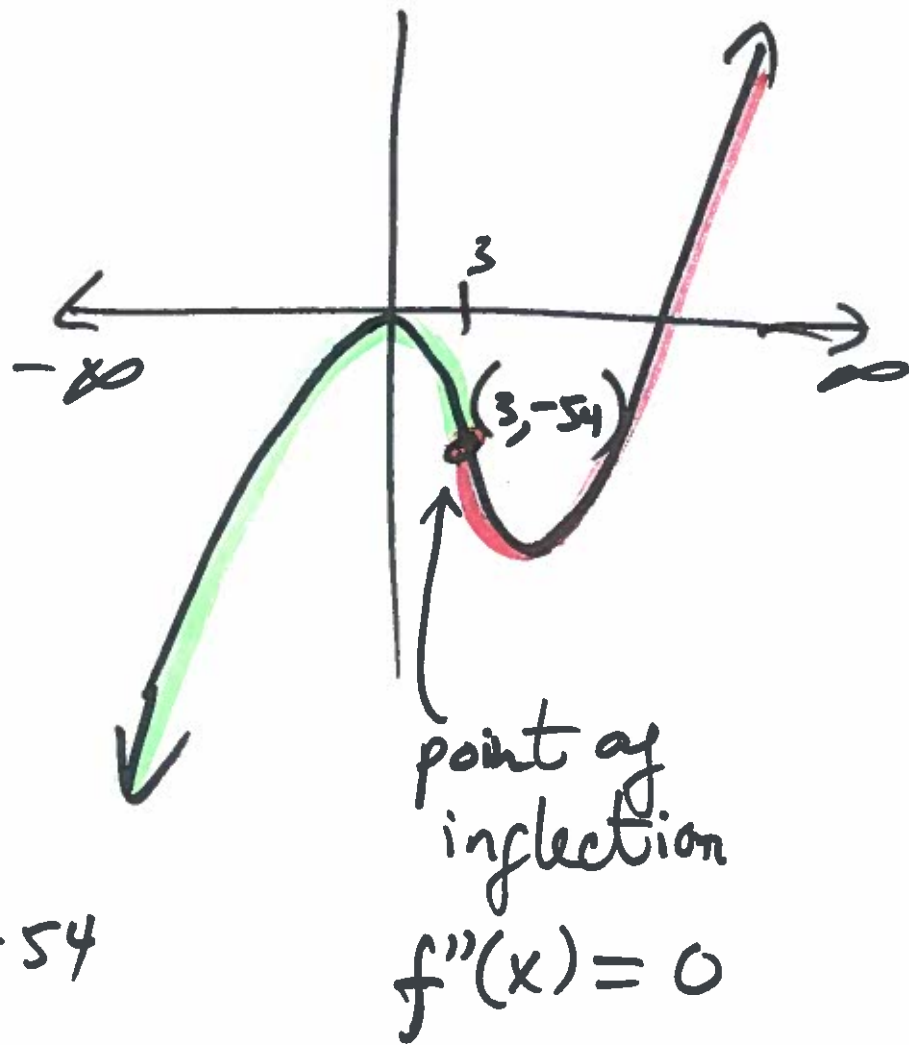
$$6x - 18 = 0$$

$$x = 3$$

$$\text{When } x = 3, \quad y = (3)^3 - 9(3)^2 = -54$$

Graph concaves down on  $(-\infty, 3)$

Graph concaves up on  $(3, \infty)$



$$(39) f(x) = x + \cos x \quad [0, 2\pi]$$

$$f'(x) = 1 + (-\sin x)$$

$$f''(x) = 0 + (-\cos x)$$

$$\text{set } f''(x) = 0$$

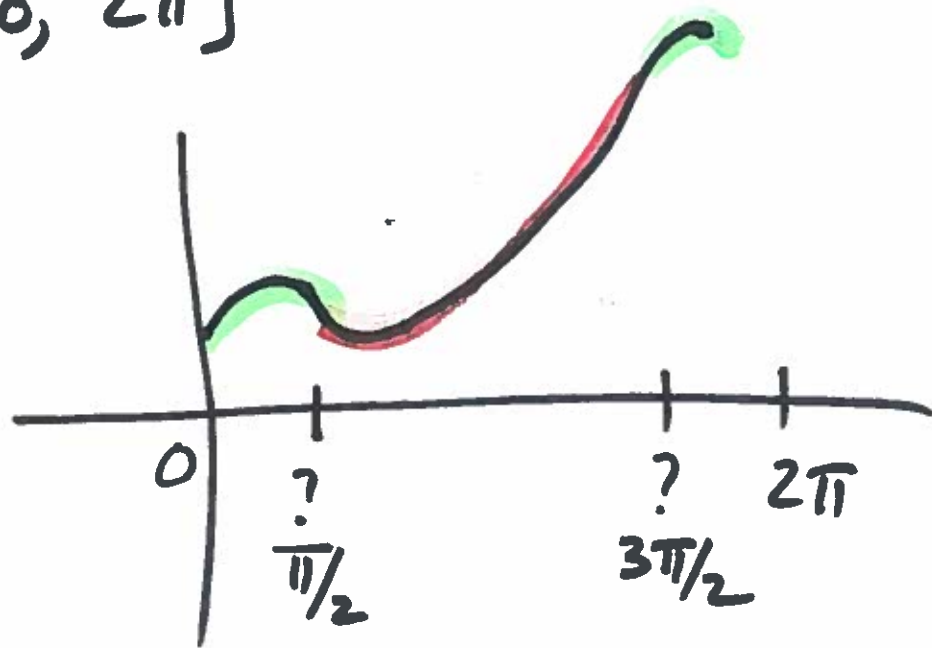
$$-\cos x = 0$$

$$\cos x = 0 \Rightarrow x = \cos^{-1}(0)$$

$$x = \pi/2; 3\pi/2$$

Graph concaves down on  $(0, \pi/2)$ ;  $(\frac{3\pi}{2}, 2\pi)$

Graph concaves up on  $(\pi/2, 3\pi/2)$



$$(43) \quad g(x) = 2x^2 \cdot (1-x^2) = 2x^2 - 2x^4$$

$$g'(x) = 4x - 8x^3$$

$$\text{set } g'(x) = 0$$

$$4x - 8x^3 = 0$$

$$(4x)(1 - 2x^2) = 0$$

$$4x = 0$$

$$x = 0$$

$$1 - 2x^2 = 0$$

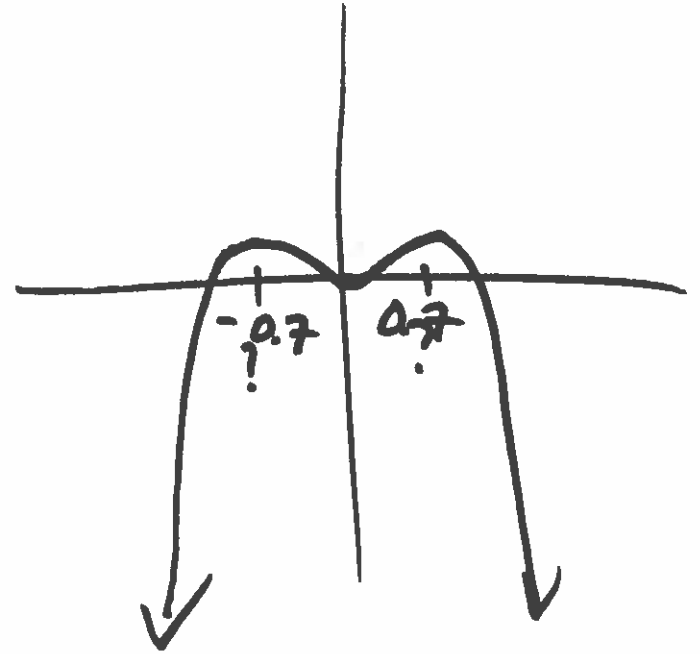
$$1 = 2x^2$$

$$x^2 = \frac{1}{2}$$

$$x = \pm \sqrt{\frac{1}{2}} = \pm 0.707$$

Graph has max. at  $x = \pm 0.707$

Graph has min. at  $x = 0$



$$(53) \lim_{x \rightarrow \infty} \left( 8 + \frac{1}{x} \right) = 8 + \frac{1}{\infty} = 8 + 0 = 8 .$$

$$(57) \lim_{x \rightarrow -\infty} \left( \frac{3x^2}{x + 5} \right)$$

$$= \lim_{x \rightarrow -\infty} \left( \frac{3x^2/x}{x/x + 5/x} \right)$$

$$= \lim_{x \rightarrow -\infty} \left( \frac{3x}{1 + 5/x} \right) = \frac{-\infty}{1 + \frac{5}{-\infty}} = \frac{-\infty}{1} = -\infty$$

$$(85) \quad f(x) = x^3 - 3x - 1$$

$$f'(x) = 3x^2 - 3$$

Use Newton's Program with initial guesses:  $-2$ ;  $-0.8$ ,  $1.1$

X-intercepts are  $-1.532$ ,  $-0.347$ ,  $1.879385$

$$(87) \quad f(x) = x^4 + x^3 - 3x^2 + 2$$

$$f'(x) = 4x^3 + 3x^2 - 6x$$

Use Newton's Program with initial guesses:  $-3$ ;  $-1$

X-intercepts are:  $-2.18227$ ,  $-0.7951977$



Ch. 4 pp. 312 - 313

① Find  $\int (x-6) dx$

$$= \int x' dx - \int 6 dx$$

$$= \frac{x^2}{2} - 6x + C$$

$$\textcircled{5} \int \frac{x^4 + 8}{x^3} dx$$

$$= \int \left( \frac{x^4}{x^3} + \frac{8}{x^3} \right) dx$$

$$= \int (x + 8 \cdot x^{-3}) dx$$

$$= \frac{x^2}{2} + 8 \cdot \left( \frac{x^{-2}}{-2} \right) + C$$

$$= \frac{x^2}{2} + -4x^{-2} + C$$

$$\textcircled{7} \int (2x - 9\sin x) dx$$

$$= 2 \int x dx - 9 \int \sin x dx$$

$$= 2 \cdot \left( \frac{x^2}{2} \right) - 9(-\cos x)$$

$$= x^2 + 9\cos x + C$$

$$\textcircled{25} \quad \sum_{i=1}^{20} 2i = 2(1) + 2(2) + 2(3) + \dots + 2(20)$$
$$= 420$$

$$(27) \sum_{i=1}^{20} (i+1)^2 = (2)^2 + (3)^2 + (4)^2 + \dots + (21)^2 = 3310$$

$$(31) \text{ Shaded Area} = \int_0^3 (8-2x) dx = \sum_{i=1}^{\infty} f(x_i) \Delta x_i = 15$$

$$(33) \text{ Shaded Area} = \int_{-2}^1 (5-x^2) dx = \sum_{i=1}^{\infty} f(x_i) \Delta x_i = 12$$

$$\textcircled{37} \quad f(x) = 2x + 8 \quad [-4, 0]$$

$$\text{Shaded Area} = \int_{-4}^0 f(x) dx$$

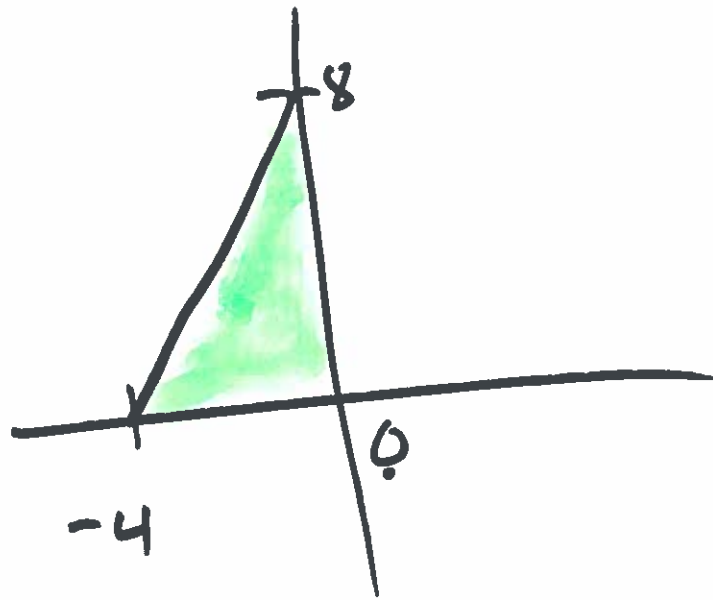
$$= \int_{-4}^0 (2x + 8) dx$$

$$= F(0) - F(-4) = 0 - (-16) = 16$$

$$F(x) = \int (2x + 8) dx = 2 \cdot \left(\frac{x^2}{2}\right) + 8x = x^2 + 8x$$

$$F(0) = (0)^2 + 8(0) = 0$$

$$F(-4) = (-4)^2 + 8(-4) = -16$$



$$(38) \text{ Shaded Area} = \int_{-10}^{10} (100 - x^2) dx = 4000/3$$

$$(45) \int_{-1}^1 (4t^3 - 2t) dt = F(1) - F(-1) = 0 - 0 = 0$$

$$F(t) = \int (4t^3 - 2t) dt = \frac{4t^4}{4} - \frac{2t^2}{2} = t^4 - t^2$$

$$F(1) = (1)^4 - (1)^2 = 0$$

$$F(-1) = (-1)^4 - (-1)^2 = 0$$

$$\begin{aligned} \textcircled{47} \int_4^9 x\sqrt{x} \, dx &= \int_4^9 x \cdot x^{1/2} \, dx = \int_4^9 x^{3/2} \, dx = F(9) - F(4) \\ &= 97.2 - 14.8 \\ &= 82.4 \end{aligned}$$

$$F(x) = \int x^{3/2} \, dx = \frac{x^{5/2}}{5/2} = \frac{2}{5} \cdot x^{5/2}$$

$$F(9) = \frac{2}{5} (9)^{5/2} = 97.2$$

$$F(4) = \frac{2}{5} (4)^{5/2} = 12.8$$



(51)  $y = \sin x$   $[0, 2]$



$$\begin{aligned} \text{Shaded Area} &= \int_0^2 f(x) dx = \int_0^2 \sin x dx = F(2) - F(0) \\ &= 0.4161 - (-1) \\ &= 1.4161 \end{aligned}$$

$$F(x) = \int \sin x dx = -\cos x$$

$$F(2) = -\cos(2) = 0.4161$$

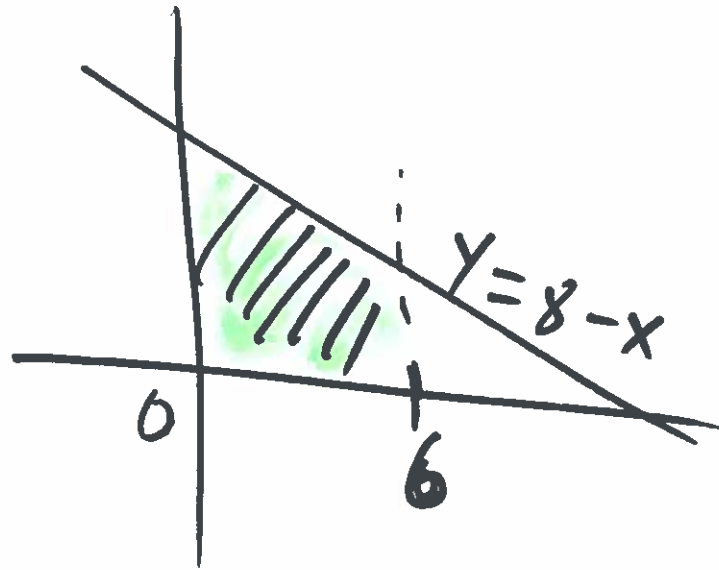
$$F(0) = -\cos(0) = -1$$

$$(53) \quad y = 8 - x$$

$$x = 0$$

$$x = 6$$

$$y = 0$$



$$\text{Shaded Area} = \int_0^6 (8-x) dx = F(6) - F(0) = 30 - 0 = 30$$

$$F(x) = \int (8-x) dx = 8x - \frac{x^2}{2}$$

$$F(6) = 8(6) - \frac{6^2}{2} = 30$$

$$F(0) = 8(0) - \frac{0^2}{2} = 0$$

$$\begin{aligned} \textcircled{55} \text{ Shaded Area} &= \int_0^1 (x - x^3) dx = F(1) - F(0) \\ &= \frac{1}{4} - 0 = \frac{1}{4} \end{aligned}$$

$$F(x) = \int (x - x^3) dx = \frac{x^2}{2} - \frac{x^4}{4}$$

$$F(1) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$F(0) = 0$$

$$(63) \int \frac{x^2}{\sqrt{x^3+3}} dx = \int \frac{1}{\sqrt{x^3+3}} \underbrace{x^2 dx}$$

$$= \int \frac{1}{\sqrt{u}} \cdot \frac{1}{3} du$$

$$= \frac{1}{3} \int \frac{1}{u^{1/2}} du$$

$$= \frac{1}{3} \int u^{-1/2} du$$

$$= \frac{1}{3} \left[ \frac{u^{1/2}}{1/2} \right] + C$$

$$= \frac{1}{3} [2 \cdot u^{1/2}] + C$$

$$= \frac{2}{3} (x^3+3)^{1/2} + C$$

$$f(x) = \frac{x^2}{\sqrt{x^3+3}}$$

$$\text{Let } u = x^3 + 3$$

$$\frac{du}{dx} = 3x^2$$

$$du = 3x^2 dx$$

$$\frac{1}{3} du = \cancel{3} x^2 dx$$

F(x)

$$(67) \int \sin^3 x \cos x dx = \int u^3 \cdot du$$

$$= \frac{u^4}{4} + C$$

~~let~~  
 $f(x) = \sin^3 x \cos x$

Let  $u = \sin x$

$$\frac{du}{dx} = \cos x$$

$$du = \underline{\cos x \cdot dx}$$

$$F(x) = \frac{(\sin x)^4}{4} + C$$

$$(69) \int \frac{\cos \theta}{\sqrt{1-\sin \theta}} d\theta = \int \frac{1}{\sqrt{1-\sin \theta}} \cdot \cos \theta d\theta$$
$$= \int \frac{1}{\sqrt{u}} \cdot du$$

$$\text{Let } u = \sin \theta$$

$$\frac{du}{d\theta} = \cos \theta$$

$$du = \cos \theta d\theta$$

$$= \int \frac{1}{u^{1/2}} du$$

$$= \int u^{-1/2} du$$

$$= \frac{u^{1/2}}{1/2} + C$$

$$= 2(u^{1/2}) + C$$

$$= 2(\sin \theta)^{1/2} + C$$

$$(75) \int_0^1 (3x+1)^5 dx = \int_0^1 u^5 \cdot \frac{1}{3} du$$

$$\text{Let } u = 3x + 1$$

$$\frac{du}{dx} = 3$$

$$du = 3dx$$

$$\frac{1}{3} du = dx$$

$$= \frac{1}{3} \int_0^1 u^5 du$$

$$= \frac{1}{3} \left[ \frac{u^6}{6} \right]$$

$$= \frac{1}{18} [u^6] \Big|_0^1$$

$$= \frac{1}{18} (1)^6 - \frac{1}{18} (0)^6$$

$$= \frac{1}{18}$$