Ch. 13 pp. 960 - 961

(3) $z = f(x, y) = \frac{\sqrt{x}}{y}$

Domain $= \{(x, y) : x \geq 0 \,;\, y \neq 0\}$

Range $= (-\infty, \infty)$

(5) $z = 3 - 2x + y$

Level curve for $c = 0 \Rightarrow 3 - 2x + y = 0$

$c = 2 \Rightarrow 3 - 2x + y = 2$

$c = 4 \Rightarrow 3 - 2x + y = 4$

(11) $$\lim_{(x,y)\to(1,1)} \frac{xy}{x^2+y^2} = \frac{(1)(1)}{(1)^2+(1)^2} = \frac{1}{2}$$

(13) $$\lim_{(x,y)\to(0,0)} \frac{y + xe^{-y^2}}{1+x^2} = \frac{0+0(e^0)}{1+0} = \frac{0}{1} = 0$$

(15) $f(x,y) = 5x^3 + 7y - 3$

$f_x = 15x^2$

$f_y = 7$

(17) $f(x, y) = e^x \cdot \cos y$

$f_x = (\cos y) \cdot e^x$

$f_y = (e^x)(-\sin y)$

(19) ✓

(21) $f(x, y, z) = 2xz^2 + 6xyz - 5x \cdot y^3$

$f_x = (2z^2)(1) + (6yz)(1) - (5y^3)(1)$

$f_y = 0 + (6xz)(1) - 5x(3y^2)$

$f_z = (2x)(2z) + (6xy)(1) + 0$

19) $f(x, y) = y^3 e^{4x}$

$$f_x = \left(y^3\right)\left(e^{4x}\right)(4) = 4y^3 e^{4x}$$

$$f_y = \left(e^{4x}\right)\left(3y^2\right) = 3y^2 e^{4x}$$

23 $f(x,y) = 3x^2 - xy + 2y^3$

$f_x = 6x - y + 0 = 6x - y$

$f_{xx} = 6 - 0 = 6$

$f_{xy} = 0 - 1 = -1$

$f_y = 0 - x + 6y^2 = -x + 6y^2$

$f_{yy} = 0 + 12y = 12y$

$f_{yx} = -1$

39. $w = \frac{xy}{z} = \frac{1}{z}\cdot\cancel{x}\cdot y = xy\cdot z^{-1}$

$x = 2r + t$

$y = rt$

$z = 2r - t$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial r}$$

$$= \left(\frac{1}{z}y\right)(2) + \left(\frac{1}{z}x\right)(t) + \left(-xyz^{-2}\right)(2)$$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial t}$$

(37) $w = \ln(x^2 + y)$

$$x = 2t \qquad dx/dt = 2$$
$$y = 4 - t \qquad dy/dt = -1$$

Find $\dfrac{dw}{dt} = \dfrac{\partial w}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial w}{\partial y}\cdot\dfrac{dy}{dt}$ Chain Rule

$$= \left(\frac{1}{x^2+\cancel{y}}\right)(2x)\cdot(2) + \frac{1}{x^2+\cancel{y}}(\cancel{1})(-1)$$

$$= \frac{4x}{x^2+y} - \frac{\cancel{y}\ 1}{x^2+y}$$

$$= \frac{4(2t)}{(2t)^2+(4-t)} - \frac{\cancel{(4-t)}\ 1}{(2t^2)+(4-t)}$$

37) Direct Method.

$$\mathrm{w} = \ln\left(x^2 + y\right) = \ln\left(\left(2t\right)^2 + 4 - t\right) = \ln\left(4t^2 + 4 - t\right)$$

$$\frac{dw}{dt} = \frac{1}{4t^2 + 4 - t}\left(8t - 1\right) = \frac{8t - 1}{4t^2 + 4 - t}$$

43) $f(x,y) = x^2 y \qquad P(-5,5) \qquad \underset{\sim}{v} = \langle 3, -4 \rangle$

$$D_{\underset{\sim}{u}} f(x,y) = f_x \cdot \cos\theta + f_y \cdot \sin\theta$$

$$= \nabla f \cdot \underset{\sim}{u}$$

$$\underset{\sim}{u} = \frac{\underset{\sim}{v}}{\|\underset{\sim}{v}\|} = \frac{\langle 3, -4 \rangle}{5}$$

$$= \langle 3/5, -4/5 \rangle$$

$$\nabla f = \langle 2xy, x^2 \rangle$$

$$\nabla f(-5,5) = \langle 2(-5)(5), (-5)^2 \rangle = \langle -50, 25 \rangle$$

$$D_{\underset{\sim}{u}} f(-5,5) = \langle -50, 25 \rangle \langle 3/5, -4/5 \rangle =$$

$$= -30 - 20 = -50$$

(45) $w = y^2 + xz$; $P(1,2,2)$; $\underset{\sim}{v} = \langle 2, -1, 2 \rangle$

$$\|\underset{\sim}{v}\| = \sqrt{(2)^2 + (-1)^2 + (2)^2} = 3$$

$$\underset{\sim}{u} = \frac{\underset{\sim}{v}}{\|\underset{\sim}{v}\|} = \frac{\langle 2, -1, 2 \rangle}{3} = \langle 2/3, -1/3, 2/3 \rangle$$

$$\nabla w = \langle w_x, w_y, w_z \rangle = \langle z, 2y, x \rangle$$

$$\nabla w(1,2,2) = \langle 2, 4, 1 \rangle$$

$$D_{\underset{\sim}{u}} w(x,y,z) = \nabla w(1,2,2) \cdot \underset{\sim}{u}$$

$$= \langle 2, 4, 1 \rangle \cdot \langle \tfrac{2}{3}, -\tfrac{1}{3}, \tfrac{2}{3} \rangle$$

$$= 2/3$$

47) $z = x^2 y$; $(2,1)$

$$\nabla z = \langle z_x, z_y \rangle = \langle 2xy, x^2 \rangle$$

$$\nabla z(2,1) = \langle 2(2)(1), (2)^2 \rangle = \langle 4, 4 \rangle$$

$$\|\nabla z(2,1)\| = \|\langle 4,4 \rangle\| = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$\underset{\sim}{u} = \frac{\nabla z(2,1)}{\|\nabla z(2,1)\|} = \frac{\langle 4, 4 \rangle}{4\sqrt{2}} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$D_{\underset{\sim}{u}} z = \nabla z \cdot \underset{\sim}{u}$$

$$= \langle 4, 4 \rangle \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$= \frac{4}{\sqrt{2}} + \frac{4}{\sqrt{2}} = \frac{8}{\sqrt{2}} = \frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 4\sqrt{2}$$

(53) $z = x^2 + y^2 + 2$

Find Tangent Plane at $(\underset{x_0}{1}, \underset{y_0}{3}, \underset{z_0}{12})$

$$F(x, y, z) = x^2 + y^2 + 2 - z = 0$$

$$\nabla F = \langle 2x, 2y, -1 \rangle$$

$$\nabla F(\overset{x}{1}, \overset{y}{3}, \overset{z}{12}) = \langle \underset{<a,}{2}, \underset{b,}{6}, \underset{c>}{-1} \rangle$$

Normal Vector for Tangent Plane

Equ. of Tangent Plane:

$$\underset{a(x - x_0)}{2(x-1)} + \underset{b(y - y_0)}{6(y-3)} + \underset{c(z - z_0)}{(-1)(z-12)} = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

(55) ✓

57 $f(x, y) = x^2 y$; $P(2, 1, 4)$

$z = x^2 y$

$F(x, y, z) = x^2 y - z = 0$

$\nabla F = \langle F_x, F_y, F_z \rangle = \langle 2xy, x^2, -1 \rangle$

$\nabla F(2, 1, 4) = \langle 2(2)(1), (2)^2, -1 \rangle = \langle 4, 4, -1 \rangle$

Normal Vector for Tangent Plane

Equ. of Tangent Plane:

$4(x-2) + 4(y-1) + (-1)(z-4) = 0$

Equation of Line going through normal vector $\langle 4, 4, -1 \rangle$
$a \quad b \quad c$

Line passes through $(2, 1, 4)$

Equ. of Line (Normal Line):

$$x - 2 = 4t \;; \quad y - 1 = 4t \;; \quad z - 4 = -1t$$

$$t = \frac{x-2}{4} \;; \quad t = \frac{y-1}{4} \;; \quad t = \frac{z-4}{-1}$$

$$\frac{x-2}{4} = \frac{y-1}{4} = \frac{z-4}{-1}$$

61 $f(x,y) = -x^2 - 4y^2 + 8x - 8y - 11$

Find extrumum point.

$f_x = -2x + 8$

$f_y = -8y - 8$

set $f_x = 0$

$-2x + 8 = 0$

$x = 4$

$f_y = 0$

$-8y - 8 = 0$

$y = -1$

when $x = 4$, $y = -1$, $z = 9$

$f_{xx} = -2$

$f_{yy} = -8$

$f_{xy} = 0$

$d = f_{xx} \cdot f_{yy} - [f_{xy}]^2 = (-2)(-8) - 0 = 16 > 0$

Since $d > 0$ and $f_{xx} < 0$,

$(4, -1, 9)$ is a relative maximum

Case 1: If $d = f_{xx} f_{yy} - \left(f_{xy} \right)^2 > 0$ and $f_{xx} > 0 \;\Rightarrow\;$ Critical point is a minimum.

Case 2: If $d = f_{xx} f_{yy} - \left(f_{xy} \right)^2 > 0$ and $f_{xx} < 0 \;\Rightarrow\;$ Critical point is a maximum.

Case 3: If $d = f_{xx} f_{yy} - \left(f_{xy} \right)^2 < 0 \;\Rightarrow\;$ Critical point is a saddle point.

(75) Minimize $f(x,y) = x^2+y^2$

Constraint : $x+y-8=0$

$g(x,y) = x+y-8=0$

$\nabla f = \langle 2x, 2y \rangle$

$\nabla g = \langle 1, 1 \rangle$

$\nabla f = \lambda \nabla g$

$\langle 2x, 2y \rangle = \lambda \langle 1, 1 \rangle = \langle \lambda, \lambda \rangle$

(1) $2x = \lambda$; $2y = \lambda$ (2)

$2y = 2x$

$y = x$

(3) $x+y-8=0$

$$x + x - 8 = 0$$

$$2x - 8 = 0$$

$$x = 4$$

$$\Rightarrow y = x = 4$$

$\therefore f(x,y) = x^2 + y^2$ is a minum at $(4,4)$

$$f(4,4) = 32$$

Ch. 14

3. $\int_0^1 \int_0^{1+x} (3x + 2y)\, dy\, dx$

$\left[3x \cdot y + \frac{2y^2}{2} \right]_0^{1+x}$

$\left(3x(1+x) + (1+x)^2\right) - 0$

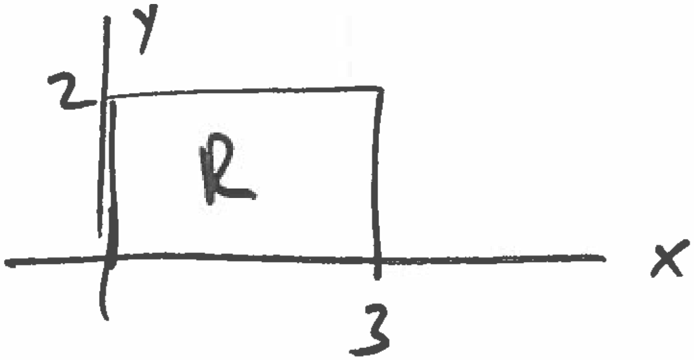
$3x + 3x^2 + 1 + 2x + x^2$

$4x^2 + 5x + 1$

$\int_0^1 (4x^2 + 5x + 1)\, dx = \frac{4}{3}x^3 + \frac{5}{2}x^2 + x \Big|_0^1$

$= 29/6$

(17)

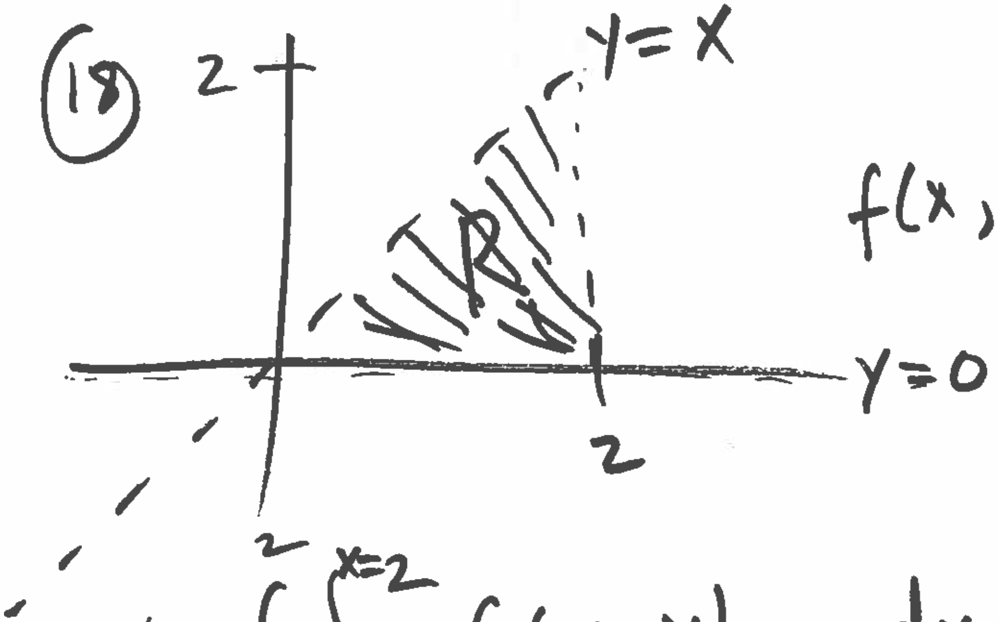


$$f(x,y) = 5 - x$$

$$V = \int_0^3 \int_0^2 f(x,y)\,dy\,dx = \int_0^2 \int_0^3 f(x,y)\,dx\,dy$$

$$V = \int_0^3 \int_0^2 (5-x)\,dy\,dx = \int_0^2 \int_0^3 (5-x)\,dx\,dy = 21$$

(18)

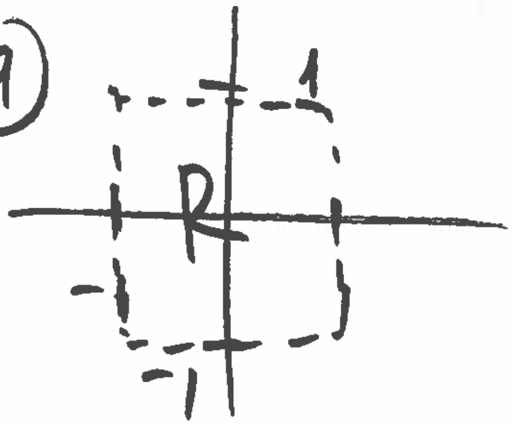


$$f(x,y) = z = 4$$

$$V = \int_0^2 \int_{x=y}^{x=2} f(x,y) \quad dx\,dy$$

$$V = \int_0^2 \int_{y=0}^{y=x} f(x,y) \; dy\,dx = 8$$

(19)

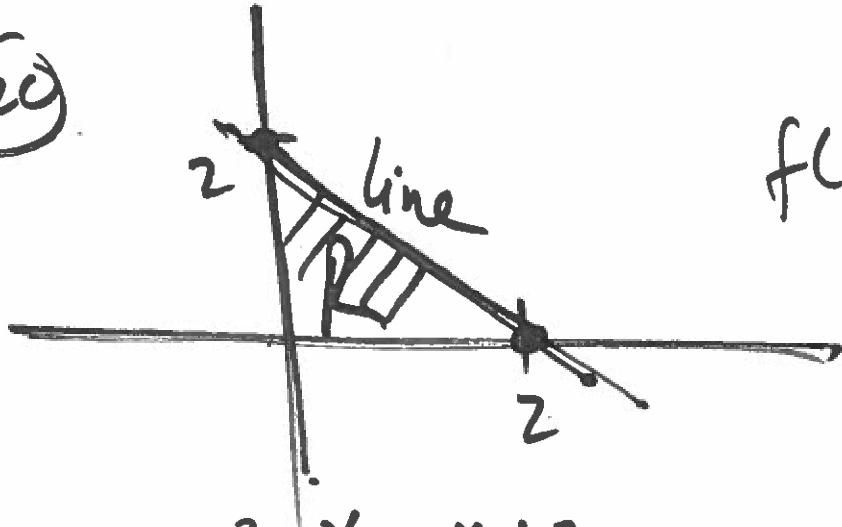


$$f(x,y) = 4 - x^2 - y^2$$

$$V = \int_{-1}^{1}\int_{-1}^{1} f(x,y) \quad dy\,dx$$

$$= \int_{-1}^{1}\int_{-1}^{1} f(x,y)\, dx\,dy = 40/3$$

20



$x+y+z=2$

$f(x,y) = z = 2-x-y$

Slope of line $= \frac{-2}{2} = -1$

$y = mx + b$

$y = -1x + 2$

$x = 2 \cancel{+} y$

$$V = \int_0^2 \int_{y=0}^{y=-x+2} f(x,y) \, dy\,dx$$

$$= \int_0^2 \int_{x=0}^{x=2-y} f(x,y) \, dx\,dy = 4/3$$