Test 3 Review

① $g(x) = \ln\sqrt{2x}$ Find $g'(x)$.

$g(x) = \ln(2x)^{1/2} = \frac{1}{2}\ln(2x)$

$g'(x) = \frac{1}{2}\cdot\left[\frac{1}{2x}\cdot D_x(2x)\right] = \frac{1}{2}\left[\frac{1}{2x}\cdot 2\right] = \frac{1}{2x}$

Recall: $D_x\left[\text{base}^{\text{Exp}}\right] = \text{Exp}\left[\text{base}^{\text{Exp}-1}\right]\cdot D_x(\text{Exp})$

Ex: $D_x\left[(\ln x)^{1/2}\right] = \frac{1}{2}\left[(\ln x)^{-1/2}\cdot D_x(\ln x)\right]$

$= \frac{1}{2}\left[(\ln x)^{-1/2}\cdot\frac{1}{x}\right]$

$= \frac{1}{2x}\cdot(\ln x)^{-1/2}$

② $f(x) = x \cdot \sqrt{\ln x}$ Find $f'(x)$

$f(x) = x \cdot (\ln x)^{1/2}$

$f'(x) = x \cdot D_x\left[(\ln x)^{1/2}\right] + (\ln x)^{1/2} \cdot D_x(x)$

$f'(x) = x \cdot \left[\frac{1}{2x}(\ln x)^{-1/2}\right] + (\ln x)^{1/2} \cdot (1)$

$f'(x) = \frac{1}{2}(\ln x)^{-1/2} + (\ln x)^{1/2}$

③ $y = \ln\sqrt{\frac{x^2+4}{x^2-4}}$ Find y'

$y = \ln\left(\frac{x^2+4}{x^2-4}\right)^{1/2} = \frac{1}{2} \cdot \ln\left(\frac{x^2+4}{x^2-4}\right)$

Note:
$\ln(A/B)$
$= \ln A - \ln B$

$y = \frac{1}{2}\left[\ln(x^2+4) - \ln(x^2-4)\right]$

$y' = \frac{1}{2}\left[\frac{1}{x^2+4}(2x) - \frac{1}{x^2-4}(2x)\right] = \frac{x}{x^2+4} - \frac{x}{x^2-4}$

(4) $y = 2x^2 + \ln x^2$ Find tangent line at $(1,2)$

$y = 2x^2 + 2 \cdot \ln x$

$y' = 2(2x) + 2 \cdot \left(\frac{1}{x}\right) = 4x + 2/x$

Slope of tangent line at $(\underset{x}{1}, \underset{y}{2}) = y' = 4(1) + 2/1 = 6$

Equation of tangent line: $y - y_1 = m(x - x_1)$

$y - 2 = 6(x-1)$

Recall: $\int \frac{1}{x} dx = \ln|x| + C$ $\int \frac{1}{x \pm a} dx = \ln|x \pm a| + c$

$\int \frac{1}{u} du = \ln|u| + C$ $\int \frac{1}{bx \pm a} dx = \frac{1}{b} \ln|bx \pm a| + c$

⑤ Find $\int \frac{1}{7x-2}\,dx = \frac{1}{7}\ln|7x-2| + C$

⑥ Find $I = \int \frac{\sin x}{1+\cos x}\,dx = \int \frac{1}{1+\cos x} \cdot \sin x\,dx$

Use u-substitution method by letting $u = 1+\cos x$

$$\frac{du}{dx} = 0 + {}^{-}\sin x = -\sin x$$

$$du = -\sin x \cdot dx$$

$$(-1)du = (-1)(-\sin x\,dx) = \sin x\,dx$$

$$I = \int \frac{1}{u} \cdot (-1)du = (-1)\int \frac{1}{u}\,du = (-1)\ln|u|$$

$$= (-1)\ln|1+\cos x| + C$$

(7) Find $I = \int_1^4 \frac{2x+1}{2x}\,dx$

Hint: Divide $\frac{2x+1}{2x} = \frac{2x}{2x} + \frac{1}{2x} = 1 + \frac{1}{2x}$

$= 1 + \frac{1}{2}\cdot\frac{1}{x}$

$I = \int \left(1 + \frac{1}{2}\cdot\frac{1}{x}\right)dx$

$I = \int 1\,dx + \frac{1}{2}\cdot\int \frac{1}{x}\,dx = x + \frac{1}{2}\ln|x| \Big|_1^4$

$I = \left(4 + \frac{1}{2}\ln|4|\right) - \left(1 + \frac{1}{2}\ln|1|\right)$ $\boxed{\ln 1 = 0}$

$I = 3 + \frac{1}{2}\ln 4$

(8) Find $I = \int_0^{\pi/3} \sec x \cdot dx$

From Integration Formula List:

$$\int \sec x \cdot dx = \ln|\sec x + \tan x|$$

$$I = \ln|\sec x + \tan x| \Big|_0^{\pi/3}$$

$$I = \ln|\sec \pi/3 + \tan \pi/3| - \ln|\sec 0 + \tan 0|$$

$$I = \ln|\sec \pi/3 + \tan \pi/3| - \ln|1 + 0|$$

$$\boxed{\ln 1 = 0}$$

$$I = \ln|\sec \pi/3 + \tan \pi/3|$$

(9) Let $f(x) = \frac{1}{2}x - 3$ Find inverse function

$$y = \frac{1}{2}x - 3$$

Finding inverse:

$$x = \frac{1}{2}y - 3$$

$$\frac{1}{2}y - 3 = x$$

$$\frac{1}{2}y = x + 3$$

$$2 \cdot \frac{1}{2}y = 2 \cdot x + 3 \cdot 2$$

$$y = 2x + 6$$ Inverse function

$$f^{-1}(x) = 2x + 6$$ Inverse function

(10) $f(x) = \sqrt[3]{x+1}$ Find inverse function

$y = \sqrt[3]{x+1}$

Finding inverse: $x = \sqrt[3]{y+1}$

$\sqrt[3]{y+1} = x$ We have 3^{rd} root; so we need to use 3^{rd} power

$\left(\sqrt[3]{y+1}\right)^3 = (x)^3$

$y+1 = x^3$

$y_{-1} = x^3 - 1$ Inverse function

$f^{-1}(x) = x^3 - 1$ Inverse function

(11) Solve $e^{3x} = 30$ Exponential Equation

$\ln(e^{3x}) = \ln(30)$

$3x = \ln 30$

$x = \frac{\ln 30}{3}$ Answer

$\ln e = 1$

$\ln e^{u} = u$

(12) Solve $-4 + 3e^{-2x} = 6$

$3e^{-2x} = 6+4$

$3e^{-2x} = 10$

$e^{-2x} = 10/3$

$\ln(e^{-2x}) = \ln(10/3)$

$-2x = \ln(10/3)$

$x = \ln(10/3)/-2$ Answer

(13) Solve $\ln\sqrt{x+1} = 2$

Log Equation

$$\ln(x+1)^{1/2} = 2$$

$$\frac{1}{2}\cdot\log_e(x+1) = 2$$

$$2\cdot\left(\frac{1}{2}\log_e(x+1)\right) = 2\cdot 2$$

$$\log_e(x+1) = 4$$

$$\Rightarrow \quad e^4 = x+1$$

$$x = e^4 - 1$$

log Form vs. Exponential Form

$\log_b x = y$

$\Leftrightarrow b^y = x$

(14) Solve $\ln x + \ln(x-3) = 0$

Recall:
$\ln A + \ln B$
$= \ln A \cdot B$

$$\ln\left(x \cdot (x-3)\right) = 0$$

$$\log_e (x^2 - 3x) = 0$$

$$e^0 = x^2 - 3x$$

$$1 = x^2 - 3x$$

$$0 = x^2 - 3x - 1$$

$$x^2 - 3x - 1 = 0$$

Use Quadratic Formula:

$$x = \frac{3+\sqrt{13}}{2} \quad ; \quad \cancel{x = \frac{3-\sqrt{13}}{2}}$$

$$x \approx 3.30 \quad ; \quad \cancel{x = -0.302}$$

Note: x cannot be negative
be ln of negative in undefined

(15) $g(x) = x^2 \cdot e^x$ Find $g'(x)$

$$g'(x) = x^2 \cdot D_x(e^x) + e^x \cdot D_x(x^2) \quad \text{Product Rule}$$

$$g'(x) = \cancel{x^2 e^x} + e^x \cdot 2x = x^2 \cdot e^x + 2xe^x$$

Note: $D_x(e^x) = e^x$

(16) $g(x) = x^2/e^x$ Find $g'(x)$

$$g'(x) = \frac{e^x \cdot D_x(x^2) - x^2 \cdot D_x(e^x)}{(e^x)^2} \quad \text{Quotient Rule}$$

$$g'(x) = \frac{e^x \cdot 2x - x^2 \cdot e^x}{e^{2x}}$$

(17) $f(x) = e^{6x}$ Find tangent line at $(0,1)$

Chain Rule: $D_x(e^u) = e^u \cdot D_x(u)$

$f'(x) = e^{6x} \cdot D_x(6x) = e^{6x} \cdot 6$

Slope of tangent line at $\underset{x\ y}{(0,1)} = f'(x) = e^{6\cdot(0)} \cdot 6$

$= e^0 \cdot 6 = 1 \cdot 6 = 6$

Equation of tangent line:

$y - y_1 = m(x - x_1)$

$y - 1 = 6(x - 0)$

(18) Find $I = \int x \cdot e^{1-x^2} dx$ Hint: let $u = 1-x^2$

$\frac{du}{dx} = 0 - 2x = -2x$

$du = -2x \cdot dx$

$-\frac{1}{2} du = -\frac{1}{2}(-2x dx)$

$-\frac{1}{2} du = x \cdot dx$

$I = \int e^{1-x^2} \cdot x \, dx$

$I = \int e^u \cdot -\frac{1}{2} \cdot du$

$I = -\frac{1}{2} \int e^u \, du$

$I = -\frac{1}{2} e^u$

$I = -\frac{1}{2} e^{1-x^2} + C$

(19) Find $I = \int \dfrac{e^{4x} - e^{2x} + 1}{e^x}\,dx$

$$I = \int\left(\frac{e^{4x}}{e^x} - \frac{e^{2x}}{e^x} + \frac{1}{e^x}\right)dx$$

$$I = \int e^{3x} - e^x + e^{-1x}\Big)\,dx$$

$$I = \frac{1}{3}e^{3x} - e^x + \frac{1}{-1}e^{-x}$$

$$I = \frac{1}{3}e^{3x} - e^x - e^{-x} + C$$

Recall

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

(20) Find $I = \int_0^1 x \cdot e^{-3x^2} dx$ Hint: Let $u = -3x^2$

$I = \int e^{-3x^2} \cdot x \cdot dx$

$\frac{du}{dx} = -3 \cdot 2x = -6x$

$du = -6x \cdot dx$

$I = \int e^u \cdot -\frac{1}{6} \cdot du$

$-\frac{1}{6} du = -\frac{1}{6}(-6x dx)$

$-\frac{1}{6} du = x\,dx$

$I = -\frac{1}{6} \int e^u du$

$I = -\frac{1}{6} e^u = -\frac{1}{6} e^{-3x^2} \Big|_0^1$

$I = \left(-\frac{1}{6} e^{-3(1)^2}\right) - \left(-\frac{1}{6} e^{-3(0)^2}\right) = -\frac{1}{6} e^{-3} + \frac{1}{6}$

(21) $I = \int_1^3 \frac{e^x}{e^x - 1}\, dx$

Hint: Let $u = e^x - 1$

$\frac{du}{dx} = e^x - 0 = e^x$

$du = e^x \cdot dx$

$I = \int \frac{1}{e^x - 1} \cdot e^x dx$

$I = \int \frac{1}{u} \cdot du$

$I = \ln|u|$

$I = \ln|e^x - 1| \Big|_1^3$

$I = \ln|e^3 - 1| - \ln|e^1 - 1|$

(22) $f(x) = 3^{x-1}$. Find $f'(x)$

Recall: $D_x(a^u) = (\ln a) \cdot a^u \cdot D_x(u)$

$f'(x) = (\ln 3) \cdot 3^{x-1} \cdot D_x(x-1) = (\ln 3) 3^{x-1} \cdot (1)$

$= (\ln 3) \cdot 3^{x-1}$

(23) $f(x) = 5^{3x}$

$f'(x) = (\ln 5) \cdot 5^{3x} \cdot D_x(3x)$

$f'(x) = (\ln 5) \cdot 5^{3x} \cdot 3 = 3 \cdot (\ln 5) \cdot 5^{3x}$

(24) $Y = \arctan(2x^2-3)$ Find y'.

Recall: $D_x(\tan^{-1} u) = \dfrac{1}{1+u^2} \cdot D_x(u)$

$$y' = \frac{1}{1+(2x^2-3)^2} \cdot D_x(2x^2-3)$$

$$y' = \frac{1}{1+(2x^2-3)^2} \cdot (4x) = \frac{4x}{1+(2x^2-3)^2}$$

(25) $Y = x \cdot \operatorname{arcsec} x$ Find y'

$$y' = x \cdot D_x(\sec^{-1} x) + \sec^{-1} x \cdot D_x(x)$$

$$y' = x \cdot \left[\frac{1}{|x|\sqrt{x^2+1}}\right] + \sec^{-1}(x) \cdot (1)$$

(26) $I = \int \frac{1}{3+25x^2}\,dx$

Formula: $\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

Note: $3 + 25x^2 = 25\left(\frac{3}{25} + \frac{25x^2}{25}\right)$

$= 25\left(\frac{3}{25} + x^2\right)$

$I = \int \frac{1}{25\left(\frac{3}{25}+x^2\right)}\,dx = \frac{1}{25}\int \frac{1}{\underbrace{\frac{3}{25}}_{a^2} + \underbrace{x^2}_{u^2}}\,dx$

$a = \sqrt{\frac{3}{25}} = \frac{\sqrt{3}}{5}$ $\qquad u = x$

$I = \frac{1}{25}\left[\frac{1}{\sqrt{3}/5}\tan^{-1}\left(\frac{x}{\sqrt{3}/5}\right)\right] + C$

(27) Find $I = \int \frac{1}{x\sqrt{9x^2-49}}\,dx$

Formula: $\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\frac{|u|}{a} + C$

$$\sqrt{9x^2-49} = \sqrt{9\left(\frac{9x^2}{9} - 49/9\right)} = \sqrt{9\cdot(x^2 - 49/9)}$$
$$= 3\sqrt{x^2 - 49/9}$$

$$I = \int \frac{1}{x\cdot 3\sqrt{x^2 - 49/9}}\,dx = \frac{1}{3}\int \frac{1}{x\cdot\sqrt{\underbrace{x^2}_{u^2} - \underbrace{49/9}_{a^2}}}\,dx$$

$u^2 \downarrow u = x$ $\quad a^2 \downarrow a = \sqrt{\frac{49}{9}} = \frac{7}{3}$

$$I = \frac{1}{3}\left[\frac{1}{7/3}\sec^{-1}\left(\frac{|x|}{7/3}\right)\right] + C$$

(28) $Y = \operatorname{sech}(4x-1)$ Find y'

Recall: $D_x(\operatorname{sech} u) = -\operatorname{sech} u \cdot \tanh u \cdot D_x(u)$

$y' = -\operatorname{sech}(4x-1) \cdot \tanh(4x-1) \cdot D_x(4x-1)$

$y' = -\operatorname{sech}(4x-1) \cdot \tanh(4x-1) \cdot 4$

(29) $Y = \coth(8x^2)$ Find y'

Recall: $D_x(\coth u) = -(\operatorname{csch} u)^2 \cdot D_x(u)$

$y' = -(\operatorname{csch} 8x^2)^2 \cdot D_x(8x^2)$

$y' = -(\operatorname{csch} 8x^2)^2 \cdot (16x)$

(30) $y = \sinh^{-1}(4x)$ Find y'

Recall: $D_x(\sinh^{-1} u) = \dfrac{1}{\sqrt{u^2+1}} D_x(u)$

$$y' = \frac{1}{\sqrt{(4x)^2+1}} D_x(4x)$$

$$y' = \frac{1}{\sqrt{16x^2+1}} \cdot 4$$

31) $y = x^{2x+1}$

$\ln y = \ln x^{2x+1}$

$\ln y = (2x+1) \cdot \ln x$

$\frac{1}{y} \cdot y' = F \cdot D_x(S) + S \cdot D_x(F)$

$\frac{1}{y} y' = (2x+1)\left(\frac{1}{x}\right) + \ln x \cdot (2x)$

$y' = y \cdot \left[\frac{2x+1}{x} + (\ln x)(2x) \right]$