

Test 3 Review

PP. 393-394

(7)  $g(x) = \ln \sqrt{2x}$

Find  $g'(x)$

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$$g(x) = \ln (2x)^{1/2} = \frac{1}{2} \cdot \ln(2x)$$

$$\begin{aligned} g'(x) &= \frac{1}{2} \left[ \frac{1}{2x} \cdot D_x(2x) \right] \\ &= \frac{1}{2} \left[ \frac{1}{2x} \cdot 2 \right] \\ &= \frac{1}{2x} \end{aligned}$$

Formula $D_x(\ln u)$ $= \frac{1}{u} \cdot u'$
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Recall:

$$\ln x^{1/2} = \frac{1}{2} \ln x$$

$$(\ln x)^{1/2} = (\ln x)^{1/2}$$

$$D_x(\text{base}^{\text{exp}}) = \text{exp} \cdot (\text{base}^{\text{exp}-1}) \cdot D_x(\text{base})$$

$$(9) f(x) = x \cdot \sqrt{\ln x}$$

Find  $f'(x)$

$$f(x) = x \cdot (\ln x)^{1/2}$$

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$$\begin{aligned} f'(x) &= F \cdot D_x(S) + S \cdot D_x(F) \\ &= (x) D_x[(\ln x)^{1/2}] + [(\ln x)^{1/2} \cdot D_x(x)] \\ &= (x) \left[ \frac{1}{2} (\ln x)^{-1/2} \cdot \left(\frac{1}{x}\right) \right] + [(\ln x)^{1/2} \cdot (1)] \\ &= \frac{1}{2} (\ln x)^{-1/2} + (\ln x)^{1/2} \end{aligned}$$

(11)  $y = \ln \sqrt{\frac{x^2+4}{x^2-4}}$  Find  $y'$

$$y = \ln \left( \frac{x^2+4}{x^2-4} \right)^{1/2} = \frac{1}{2} \cdot \ln \left( \frac{x^2+4}{x^2-4} \right)$$
$$= \frac{1}{2} \left[ \ln(x^2+4) - \ln(x^2-4) \right]$$

Recall:  $D_x(\ln u) = \frac{1}{u} \cdot u'$

$$y' = \frac{1}{2} \left[ \frac{1}{x^2+4} (2x) - \frac{1}{x^2-4} (2x) \right]$$

$$y' = \frac{x}{x^2+4} - \frac{x}{x^2-4}$$

(14)  $y = 2x^2 + \ln x^2$  Find tangent line at  $(1, 2)$

$$y' = 2 \cdot (2x) + \frac{1}{x^2} \cdot Dx(x^2)$$

$$y' = 4x + \frac{1}{x^2} (2x)$$

$$y' = 4x + \frac{2}{x}$$

Recall:  
 $Dx(\ln u) = \frac{1}{u} \cdot u'$

Slope of tangent line at  $(1, 2)$   
 $= y'(at x = 1)$

$$= 4(1) + \frac{2}{1} = 6$$

Equ. of tangent line:  $y - y_1 = m(x - x_1)$

$$y - 2 = 6(x - 1)$$

Answer

Recall:

$$\int \frac{1}{u} du = \ln|u| + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \frac{1}{x \pm a} dx = \ln|x \pm a| + C$$

$$\int \frac{1}{bx \pm a} dx = \frac{1}{b} \ln|bx \pm a| + C$$

15)  
Find  $I = \int \frac{1}{7x-2} dx$

Recall:  $\int \frac{1}{bx-a} dx = \frac{1}{b} \ln |bx-a| + C$

$$I = \frac{1}{7} \ln |7x-2| + C$$

$$\textcircled{17} I = \int \frac{\sin x}{1 + \cos x} dx = \int \frac{1}{1 + \cos x} \cdot \sin x dx$$

Hint: let  $u = 1 + \cos x$

$$\frac{du}{dx} = -\sin x$$

$$du = -\sin x dx$$

$$-1 \cdot du = \sin x dx$$

$$I = \int \frac{1}{u} \cdot (-1) \cdot du = (-1) \cdot \int \frac{1}{u} du$$

$$= (-1) \cdot \ln|u| + C$$

$$= (-1) \ln|1 + \cos x| + C$$



$$\textcircled{19} I = \int_1^4 \frac{2x+1}{2x} dx$$

$$I = \int \left(1 + \frac{1}{2x}\right) dx$$

$$I = \int 1 dx + \int \frac{1}{2} \cdot \frac{1}{x} dx$$

$$I = 1x + \frac{1}{2} \cdot \ln|x| \Big|_1^4$$

$$I = \left(4 + \frac{1}{2} \ln 4\right) - \left(1 + \frac{1}{2} \ln 1\right)$$

$$I = 3 + \frac{1}{2} \ln 4$$

Recall $\ln 1 = 0$
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(21) Find  $I = \int_0^{\pi/3} \sec \theta d\theta$

Recall:  $\int \sec \theta d\theta = \ln |\sec \theta + \tan \theta| + C$

$$I = \ln \left| \sec \frac{\pi}{3} + \tan \frac{\pi}{3} \right| - \ln \left| \sec(0) + \tan(0) \right|$$

$$I = \ln \left| \sec \frac{\pi}{3} + \tan \frac{\pi}{3} \right|$$

Recall:

$$\sec(0) = 1$$

$$\tan(0) = 0$$

$$\textcircled{23} \quad f(x) = \frac{1}{2}x - 3$$

$$y = \frac{1}{2}x - 3$$

Find inverse function

$$x = \frac{1}{2}y - 3$$

$$\frac{1}{2}y - 3 = x$$

$$\frac{1}{2}y = x + 3$$

$$y = 2x + 6 \quad \text{Inverse}$$

$$f^{-1}(x) = 2x + 6$$

$$\textcircled{27} f(x) = \sqrt[3]{x+1}$$

$$y = \sqrt[3]{x+1}$$

Find inverse

$$x = \sqrt[3]{y+1}$$

$$\sqrt[3]{y+1} = x$$

$$\left(\sqrt[3]{y+1}\right)^3 = (x)^3$$

$$y+1 = x^3$$

$$y = x^3 - 1 \quad \text{Inverse}$$

$$f^{-1}(x) = x^3 - 1$$

(33) Solve  $e^{3x} = 30$

$$\ln e^{3x} = \ln 30$$

$$3x \cdot \ln e = \ln 30$$

$$3x = \ln 30$$

$$x = \frac{\ln 30}{3} \text{ Ans.}$$

Recall:

$$\ln e = 1$$

(34) Solve  $-4 + 3e^{-2x} = 6$

$$3e^{-2x} = 10$$

$$e^{-2x} = \frac{10}{3}$$

$$\ln e^{-2x} = \ln\left(\frac{10}{3}\right)$$

$$-2x \cdot \ln e = \ln\left(\frac{10}{3}\right)$$

$$-2x = \ln\left(\frac{10}{3}\right)$$

$$x = \frac{\ln\left(\frac{10}{3}\right)}{-2} \quad \text{Ans.}$$

Add 4

Divide by 3

Recall  
 $\ln e = 1$

Recall:

$$\ln e^x = x$$

$$e^{\ln x} = x$$

(35) Solve  $\ln \sqrt{x+1} = 2$

$$e^{\ln \sqrt{x+1}} = e^2$$

$$\sqrt{x+1} = e^2$$

$$(\sqrt{x+1})^2 = (e^2)^2$$

$$x+1 = e^4$$

$$x = e^4 - 1 \quad \text{Answer}$$

$$\textcircled{36} \quad \ln x + \ln(x-3) = 0$$

$$\ln(x \cdot (x-3)) = 0$$

$$\ln(x^2 - 3x) = 0$$

$$e^{\ln(x^2 - 3x)} = e^0$$

Recall:  
 $e^0 = 1$

$$x^2 - 3x = 1$$

$$x^2 - 3x - 1 = 0 \quad \text{Quadratic Equ.}$$

To solve, use Quadratic Formula.

$$x = \frac{3 + \sqrt{13}}{2}$$

$$x = 3.30$$

~~$$x = \frac{3 - \sqrt{13}}{2}$$~~

~~$$x = -0.302$$~~

$\ln(\text{neg})$   
 $= \text{undefined}$



\* (37)  $g(t) = t^2 \cdot e^t$  Find  $g'(t)$   
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$$g'(t) = F \cdot D_t(s) + s \cdot D_t(F)$$
$$= (t^2) \cdot D_t(e^t) + (e^t) \cdot D_t(t^2)$$

Recall:  $D_x(e^{u}) = e^{u} \cdot u'$

$$g'(t) = (t^2)(e^t) + (e^t)(2t)$$

$$(41) \quad g(x) = \frac{x^2}{e^x} \quad \text{Find } g'(x)$$

$$\begin{aligned} g'(x) &= \frac{D \cdot D_x(N) - (N) \cdot D_x(D)}{D^2} \\ &= \frac{(e^x) \cdot D_x(x^2) - (x^2) D_x(e^x)}{(e^x)^2} \\ &= \frac{(e^x)(2x) + (x^2)(e^x)}{e^{2x}} \end{aligned}$$

Ans.

(43)  $f(x) = e^{6x}$  Find tangent line at  $(0, 1)$

$$\text{Recall: } D_x(e^u) = e^u \cdot u'$$

$$f'(x) = e^{6x} \cdot D_x(6x) = e^{6x} \cdot 6$$

$$\begin{aligned} \text{Slope of tangent line} &= f'(\text{at } x=0) \\ &= e^{6(0)} \cdot 6 = e^0 \cdot 6 = 1 \cdot 6 = 6 \end{aligned}$$

$$\text{Equ. of tangent line: } y - 1 = 6(x - 0)$$

(47) Find  $I = \int x e^{1-x^2} dx$

Hint: let  $u = 1 - x^2$

$$\frac{du}{dx} = -2x$$

$$du = -2x \cdot dx$$

$$\frac{-1}{2} du = x dx$$

$$\begin{aligned} I &= \int e^u \cdot \left(-\frac{1}{2}\right) du = \left(-\frac{1}{2}\right) \int e^u du \\ &= \left(-\frac{1}{2}\right) [e^u + C] \\ &= \left(-\frac{1}{2}\right) [e^{1-x^2}] + C \end{aligned}$$

Answer

Recall:

$$\int e^u du = e^u + C$$

$$\#49 \quad I = \int \frac{e^{4x} - e^{2x} + 1}{e^x} dx$$

$$= \int (e^{3x} - e^x + \frac{1}{e^x}) dx$$

$$= \int (e^{3x} - e^x + e^{-x}) dx$$

$$= \frac{1}{3} e^{3x} - e^x + \frac{1}{-1} e^{-x} + C$$

$$= \frac{1}{3} e^{3x} - e^x - e^{-x} + C$$

Recall  
∫

$$(51) I = \int_0^1 x e^{-3x^2} dx$$

$$\text{Let } u = -3x^2$$

$$\frac{du}{dx} = -6x$$

$$du = -6x dx$$

$$-\frac{1}{6} du = dx$$

$$\begin{aligned} I &= \int e^u \cdot \left(-\frac{1}{6}\right) du = \left(-\frac{1}{6}\right) e^u + C \\ &= \left(-\frac{1}{6}\right) e^{-3x^2} \Big|_0^1 \\ &= \left(-\frac{1}{6}\right) e^{-3} - \left(-\frac{1}{6}\right) e^0 \end{aligned}$$

$$\textcircled{53} \quad I = \int_1^3 \frac{e^x}{e^x - 1} dx$$

$$\text{let } u = e^x - 1$$

$$\frac{du}{dx} = e^x \Rightarrow du = e^x dx$$

$$\begin{aligned} I &= \int \frac{1}{u} \cdot du = \ln |u| \\ &= \ln |e^x - 1| \Big|_1^3 \\ &= \ln(e^3 - 1) - \ln(e - 1) \end{aligned}$$

$$\textcircled{59} f(x) = 3^{x-1}$$

Find  $f'(x)$

$$f'(x) = (\ln 3) \cdot 3^{x-1} \cdot (1)$$

$$f'(x) = (\ln 3) \cdot 3^{x-1}$$

$$\boxed{\begin{aligned} \cancel{D_x(a^u)} \quad D_x(a^u) \\ = \ln a \cdot a^u \cdot u' \end{aligned}}$$

$$\textcircled{60} f(x) = 5^{3x}$$

$$f'(x) = (\ln 5) \cdot 5^{3x} \cdot D_x(3x)$$

$$f'(x) = (\ln 5) \cdot 5^{3x} \cdot 3$$



$$\textcircled{61} \quad y = x^{2x+1}$$

$$\ln y = \ln x^{2x+1}$$

$$\ln y = (2x+1) \cdot \ln x$$

$$\frac{1}{y} \cdot y' = F \cdot D_x(S) + S \cdot D_x(F)$$

$$\frac{1}{y} y' = (2x+1) \left(\frac{1}{x}\right) + \ln x \cdot (2x)$$

$$y' = y \cdot \left[ \frac{2x+1}{x} + (\ln x)(2x) \right]$$

$$(72) \quad y = \arctan(2x^2 - 3)$$

$$\text{Recall: } D_x(\tan^{-1} u) = \frac{1}{1+u^2} \cdot u'$$

$$\text{Let } u = 2x^2 - 3$$

$$u' = \frac{du}{dx} = 4x$$

$$y = \arctan(u)$$

$$y' = \frac{1}{1+u^2} u' = \frac{1}{1+(2x^2-3)^2} (4x) \quad \text{Ans.}$$

73)  $y = x \cdot \arcsin x$  Find  $y'$   
F. S

$$y' = F \cdot D_x(S) + S \cdot D_x(F)$$

$$\text{Recall: } D_x(\sec^{-1} u) = \frac{1}{|u| \sqrt{u^2 - 1}} \cdot u'$$

$$y' = (x) \cdot \left( \frac{1}{|x| \sqrt{x^2 - 1}} \right) + (\sec^{-1} x)(1)$$

$$(78) \quad I = \int \frac{1}{3+25x^2} dx$$

$$\text{Recall: } \int \frac{1}{a^2+u^2} du = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\text{Let } a^2 = 3$$

$$a = \sqrt{3}$$

$$\text{Let } u^2 = 25x^2$$

$$u = 5x$$

$$\frac{du}{dx} = 5$$

$$du = 5dx$$

$$\frac{1}{5} du = dx$$

$$I = \int \frac{1}{a^2+u^2} \cdot \frac{1}{5} du$$

$$I = \frac{1}{5} \left[ \frac{1}{a} \tan^{-1} \left( \frac{y}{a} \right) \right] + C$$

#78

$$I = \frac{1}{5} \left[ \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{5x}{\sqrt{3}} \right) \right] + C$$

Answer

(80) Find  $I = \int \frac{1}{x \sqrt{9x^2 - 49}} dx$

Recall:  $\int \frac{1}{u \sqrt{u^2 - a^2}} du = \frac{1}{a} \sec^{-1} \frac{|u|}{a} + C$

Let  $u^2 = 9x^2$

$a^2 = 49$

$u = 3x$

$a = 7$

$\frac{du}{dx} = 3$

$du = 3 dx$

$\frac{1}{3} du = dx$

Also:  $u = 3x$

$x = \frac{1}{3} u$

$$I = \int \frac{1}{\frac{1}{3}u \sqrt{u^2 - a^2}} \cdot \frac{1}{3} du$$

$$= \frac{1}{a} \sec^{-1} \left| \frac{u}{a} \right| + C$$

$$= \frac{1}{7} \sec^{-1} \left| \frac{3x}{7} \right| + C$$

#80

$$\textcircled{85} \quad y = \operatorname{sech}(4x-1) \quad \text{Find } y'$$

$$\text{Recall: } D_x(\operatorname{sech} u) = -(\operatorname{sech} u)(\tanh u) \cdot u'$$

$$\text{Let } u = 4x - 1.$$

$$u' = 4$$

$$y' = -(\operatorname{sech} u)(\tanh u) \cdot (4)$$

$$y' = -(\operatorname{sech}(4x-1))(\tanh(4x-1)) \cdot 4$$



$$\textcircled{87} \quad y = \coth(8x^2)$$

$$\text{Recall: } D_x(\coth u) = -(\operatorname{csch}^2 u) \cdot u'$$

$$\text{let } u = 8x^2$$

$$u' = 16x$$

$$y' = -(\operatorname{csch}^2 u) \cdot (16x)$$

$$y' = -(\operatorname{csch}^2(8x^2)) \cdot (16x)$$

Answer

$$\textcircled{89} \quad y = \sinh^{-1}(4x)$$

$$\text{Recall: } D_x(\sinh^{-1} u) = \frac{1}{\sqrt{u^2 + 1}} u'$$

$$\text{Let } u = 4x$$

$$u' = 4$$

$$y' = \frac{1}{\sqrt{u^2 + 1}} \cdot (4) = \frac{1}{\sqrt{16x^2 + 1}} (4) \quad \text{Ans.}$$