Test 3 Review

① $16x^2 + 16y^2 - 16x + 24y - 3 = 0$

Find center, vertex, Focus

$$(16x^2 - 16x) + (16y^2 + 24y) = 3$$

$$16\left(\frac{16x^2}{16} - \frac{16x}{16}\right) + 16\left(\frac{16y^2}{16} + \frac{24y}{16}\right) = 3$$

$$16\left(x^2 - 1x + \tfrac{1}{4}\right) + 16\left(y^2 + \tfrac{3}{2}y + \tfrac{9}{16}\right) = 3 + 16(\tfrac{1}{4}) + 16(\tfrac{9}{16})$$

$-\frac{1}{2} + -\frac{1}{2} = -1$ $\qquad$ $\frac{3}{4} + \frac{3}{4} = \frac{3}{2}$

$(-\frac{1}{2})(-\frac{1}{2}) = \frac{1}{4}$ $\qquad$ $(\frac{3}{4})(\frac{3}{4}) = 9/16$

$$16(x - \tfrac{1}{2})(x - \tfrac{1}{2}) + 16(x + \tfrac{3}{4})(x + \tfrac{3}{4}) = 16$$

$$16(x - \tfrac{1}{2})^2 + 16(x + \tfrac{3}{4})^2 = 16$$

$$\frac{(x-\frac{1}{2})^2}{1} + \frac{(x+\frac{3}{4})^2}{1} = 1$$ Divide by 16

Note: Conic Section is a circle with center at $(\frac{1}{2}, -\frac{3}{4})$

② $3x^2 - 2y^2 + 24x + 12y + 24 = 0$ Find center focus, vertex

$(3x^2 + 24x) + (-2y^2 + 12y) = -24$

$3\left(\frac{3x^2}{3} + \frac{24x}{3}\right) + -2\left(\frac{-2y^2}{-2} + \frac{12y}{-2}\right) = -24$

$3(x^2 + \underbrace{8x}_{4+4=8} +16) - 2(y^2 \underbrace{-6y}_{-3+-3=-6} +9) = -24 + 3\cdot 16 + -2\cdot 9$

$4\cdot 4 = 16$

$-3\cdot -3 = 9$

$$3(x+4)(x+4) - 2(y-3)(y-3) = 6$$

$$\frac{3(x+4)^2}{6} - \frac{2(y-3)^2}{6} = 1$$ Divide by 6

$$\frac{1(x+4)^2}{2} - \frac{1(y-3)^2}{3} = 1$$

$$\frac{(x+4)^2}{2} - \frac{(y-3)^2}{3} = 1$$ Hyperpola)(

Center $= (h = -4,\ k = 3)$

$a^2 = 2$ $\quad$ $b^2 = 3$ $\quad$ $c^2 = a^2 + b^2$

$a = \sqrt{2}$ $\quad$ $b = \sqrt{3}$ $\quad$ $c^2 = 2 + 3 = 5$

$c = \sqrt{5}$

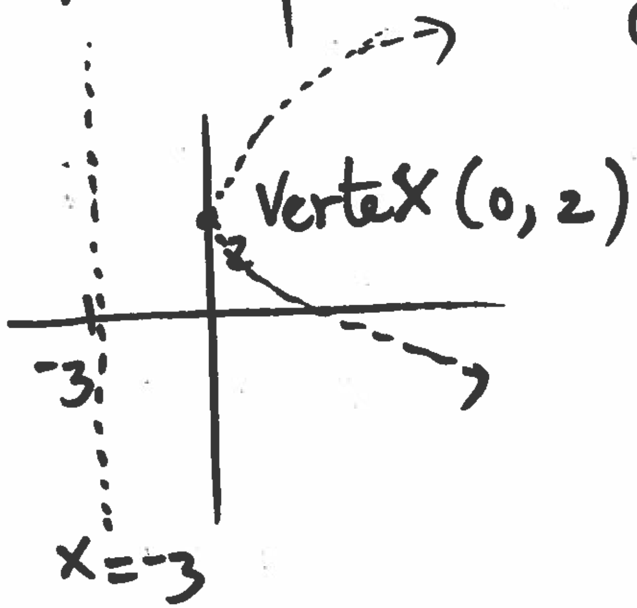
Focus $= (h \pm c, k)$

$= (-4 \pm \sqrt{5}, 3)$

Vertex $= (h \pm a, k) = (-4 \pm \sqrt{2}, 3)$

e: Eccentricity $= c/a = \sqrt{5}/\sqrt{2}$

(3) Parabola has Vertex at $(0, 2)$ and directrix $x = -3$
Find equation of parabola.



Parabola opens to the right.

$(y-k)^2 = 4p(x-h)$

p = distance from vertex to directrix

$p = 3$

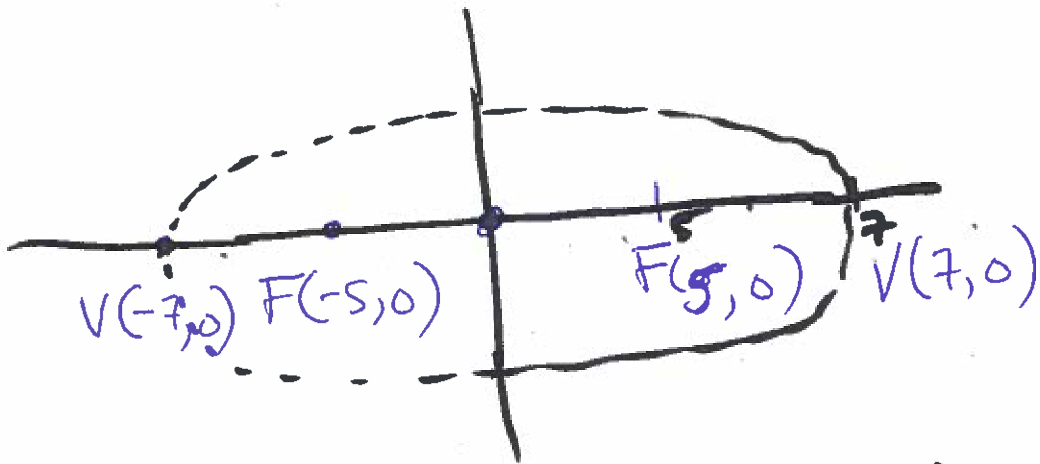
$h = 0$; $k = 2$

Equ. of parabola: $(y-2)^2 = 4(3)(x-0)$

$(y-2)^2 = 12x$

④ Ellipse has center $(0, 0)$, Focus $(5,0)$, vertex $(7,0)$.
Find equation of ellipse.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



a = distance from center to vertex = 7
c = distance from center to focus = 5

$a^2 = b^2 + c^2$

$7^2 = b^2 + 5^2$

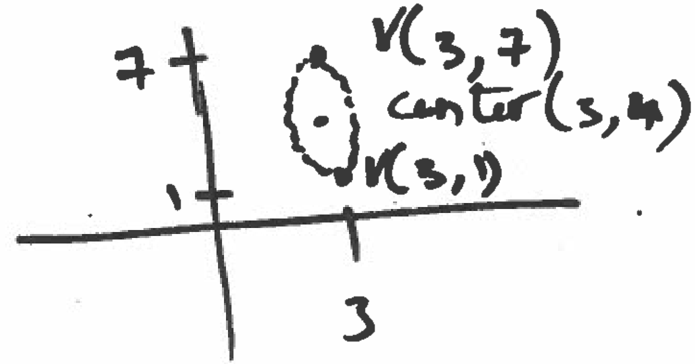
$b^2 = 24$

Center = $(h=0, k=0)$

Equ. of Ellipse:

$$\frac{(x-0)^2}{49} + \frac{(y-0)^2}{24} = 1$$

(5) Ellipse has vertices $(3,1)$ and $(3,7)$ and $e = c/a = 2/3$. Find equ. of ellipse.



center $= (h=3,\ k=4)$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

a = distance from center to vertex = 3

$$e = \frac{c}{a}$$

$$\frac{2}{3} = \frac{c}{3} \Rightarrow c = 2$$

$$a^2 = b^2 + c^2$$

$$3^2 = b^2 + 2^2$$

$$b^2 = 5$$

Equ. for Ellipse:

$$\frac{(x-3)^2}{5} + \frac{(y-4)^2}{9} = 1$$

(6) Parametric Equations: $x = 1 + 8t$ (1)

$y = 3 - 4t$. (2)

Find corresponding rectangular equation.

(1) $x = 1 + 8t$

$8t = x - 1$

$t = \frac{x-1}{8}$

(2) $y = 3 - 4t$

$\underbrace{y = 3 - 4\left(\frac{x-1}{8}\right)}_{\text{Rectangular Equ.}}$

(7) Parametric Equations: $x = e^t - 1$ (1)

$y = e^{3t}$ (2)

Find corresponding rectangular equation.

(1) $x = e^t - 1$

$e^t = x + 1$

(2) $y = e^{3t} = (e^t)^3$

$\underbrace{y = (x+1)^3}_{\text{Rectangular Equ.}}$

(8) Parametric Equations: $x = 6\cos\theta$ (1)

$y = 6\sin\theta$ (2)

Find corresponding rectangular equation.

We will use : $\cos^2\theta + \sin^2\theta = 1$

(1) $x = 6\cos\theta$

$\cos\theta = \frac{x}{6}$

(2) $y = 6\sin\theta$

$\sin\theta = y/6$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\underbrace{\left(\frac{x}{6}\right)^2 + \left(\frac{y}{6}\right)^2 = 1}_{\text{Rectangular Equ.}}$$

(9) Parametric Equations: $x = 2 + \sec\theta$ (1)

$y = 3 + \tan\theta$

Find corresponding rectangular equation.

We will use: $1 + \tan^2\theta = \sec^2\theta$

(1) $x = 2 + \sec\theta$

$\sec\theta = x - 2$

(2) $y = 3 + \tan\theta$

$\tan\theta = y - 3$

$$\underline{1} + \tan^2\theta = \sec^2\theta$$

$$\underbrace{\underline{1} + (y-3)^2 = (x-2)^2}_{\text{Rectangular Equ.}}$$

(10) Rectangular Equ: $y = 4x + 3$

Find corresponding parametric Equs.

Let $x = t$; so $y = 4x + 3 = 4t + 3$

Parametric Equs. : $x = t$
$y = 4t + 3$

(11) Parametric Equs: $x = 2 + 5t$
$y = 1 - 4t$

Find $y' = \frac{dy}{dx}$

Chain Rule : $y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-4}{5}$

$$y'' = \frac{\frac{d}{dt}[y']}{dx/dt} = \frac{\frac{d}{dt}[-4/5]}{5} = \frac{0}{5} = 0$$

(12) Parametric Equs: $x = 1/t$

$y = 2t + 3$

Note:

$$\frac{dx}{dt} = -1/t^2 = -t^{-2}$$

Find y', and y''.

$$y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2}{-1/t^2} = 2 \cdot -\frac{t^2}{1} = -2t^2$$

$$y'' = \frac{\frac{d}{dt}[y']}{dx/dt} = \frac{\frac{d}{dt}[-2t^2]}{-1/t^2} = \frac{-4t}{-1/t^2} = -4t \cdot -t^2$$

$$y'' = +4t^3$$

$$y'' = 4(1/x)^3$$

(13) Parametric Equs: $x = 5 + \cos\theta$ Find y', y''

$y = 3 + 4\sin\theta$

$$y' = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{0 + 4\cos\theta}{0 + -\sin\theta} = \frac{4\cos\theta}{-\sin\theta} = -4\cot\theta$$

$$y'' = \frac{\frac{d}{d\theta}[y']}{dx/d\theta} = \frac{\frac{d}{d\theta}[-4\cot\theta]}{-\sin\theta} = \frac{-4[-\csc^2\theta]}{-\cancel{4}\sin\theta}$$

$$= \frac{-4\csc^2\theta}{\sin\theta}$$

(14) Polar Point $\left(\underset{r}{5}, \underset{\theta}{3\pi/2}\right)$

Find corresponding rectangular point.

$$x = r\cos\theta \quad = 5\cos(3\pi/2) = 0$$
$$y = r\sin\theta \quad = 5\sin(3\pi/2) = -5$$

Rectangular Point = $(x = 0,\ y = -5)$

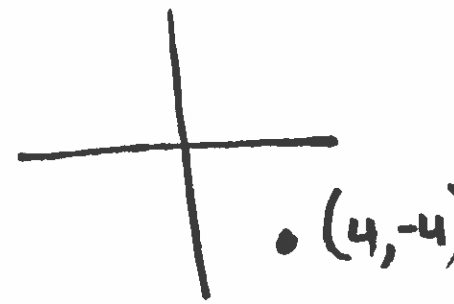
(15) Polar Point $(\sqrt{3}, 1.56)$
$\quad\quad\quad\quad\quad\quad\; r \quad\;\; \theta$

Find correspondind rectangular point:

$x = r\cos\theta = \sqrt{3} \cdot \cos(1.56) =$ ~~0.179[illegible]67~~ 0.018699

$y = r\sin\theta = \sqrt{3}\sin(1.56) = 1.731949$

(16) Rectangular Point $(x = 4, y = {}^{-}4)$.
Find corresponding Polar point.



Note: $(4, -4)$ is in Quadrant IV.

$r^2 = x^2 + y^2$

$r^2 = 16 + 16 = 32$

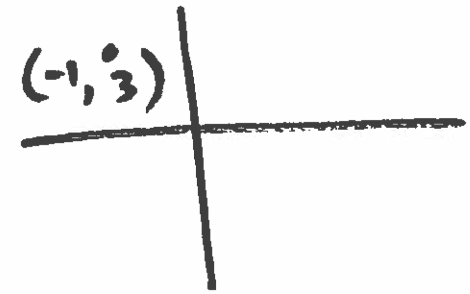
$r = \sqrt{32}$

$\theta = \tan^{-1}(y/x) + 2\pi$

$\theta = \tan^{-1}(-1) + 2\pi = 7\pi/4$

$(r, \theta) = (\sqrt{32}, 7\pi/4)$

(17) Rectangular Point $(-1, 3)$.
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad x, y$



Find corresponding polar point.

Note: $(-1, 3)$ is in Quadrant II.

$r^2 = x^2 + y^2$

$r^2 = (-1)^2 + (3)^2 = 10$

$r = \sqrt{10}$

$\theta = \tan^{-1}(y/x) + \pi$

$\theta = \tan^{-1}(3/-1) + \pi = 1.89254$

$(r, \theta) = (\sqrt{10}, 1.892546)$

(18) Rectanglular Equ. $x^2 + y^2 = 25$

Find corresponding polar equation.

$$x^2 + y^2 = 25$$

$$(r\cos\theta)^2 + (r\sin\theta)^2 = 25$$

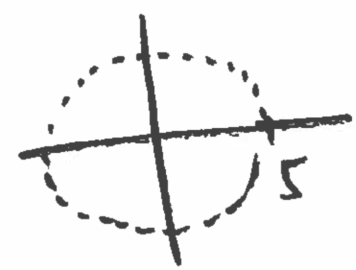
$$r^2 \cdot \cos^2\theta + r^2 \cdot \sin^2\theta = 25$$

$$r^2(\cos^2\theta + \sin^2\theta) = 25$$

$$r^2(1) = 25$$

$$r^2 = 25$$

$$\underbrace{r = 5}_{\text{Polar Equ.}}$$



(19) Rectangular Equ. : $x^2=4y$

Find corresponding polar equ.

$$x^2=4y$$

$$(r\cos\theta)^2 = 4(r\sin\theta)$$

$$r^2\cos^2\theta = 4r\sin\theta$$

$$r\cos^2\theta = 4\sin\theta$$ Divide by r.

$$r = \frac{4\sin\theta}{\cos^2\theta}$$

Polar Equ.

(20) Polar Equ. $r = 3\cos\theta$

Find corresponding rectangular equ.

$$r = 3\cos\theta$$

$$r \cdot r = 3 \cdot r\cos\theta$$

$$r^2 = 3r\cos\theta$$

$$\underbrace{x^2 + y^2 = 3 \cdot x}_{\text{Rectangular Equ.}}$$

(21) $r = -2\sec\theta \cdot \tan\theta$ Polar Equ.

Find corresponding rectangular equ.

$r = -2\sec\theta \cdot \tan\theta$

$r = -2 \cdot \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta} = -\frac{2\sin\theta}{\cos^2\theta}$

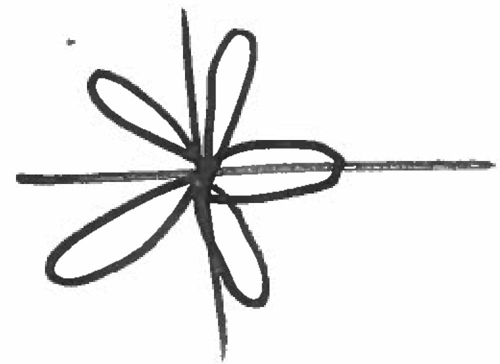
$r = \frac{-2\sin\theta}{\cos^2\theta} = \frac{-2\sin\theta \cdot r^2}{\cos^2\theta \cdot r^2} = \frac{-2r^2\sin\theta}{r^2\cos^2\theta} = \frac{-2 \cdot r \cdot r\sin\theta}{r^2\cos^2\theta}$

$r = \frac{-2y \cdot r}{x^2}$

$\underbrace{1 = \frac{-2y}{x^2}}_{\text{Rectangular Equ.}}$ Divide by r

(22) Find area of one petal.

$r = 3\cos 5\theta$



To find where each petal starts and ends, set $r=0$.

$r = 3\cos 5\theta$

$0 = 3\cos 5\theta$

$\cos 5\theta = 0$

Note: $\cos \pi/2 = 0$

$\cos 3\pi/2 = 0$

$\cos 5\pi/2 = 0$

So $5\theta = \pi/2, 3\pi/2, 5\pi/2, \ldots$

$\theta = \pi/10, 3\pi/10, 5\pi/10$

Now graph $r = 3\cos 5\theta \qquad \pi/10 \le \theta \le 3\pi/10$

Area of one petal $= \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$

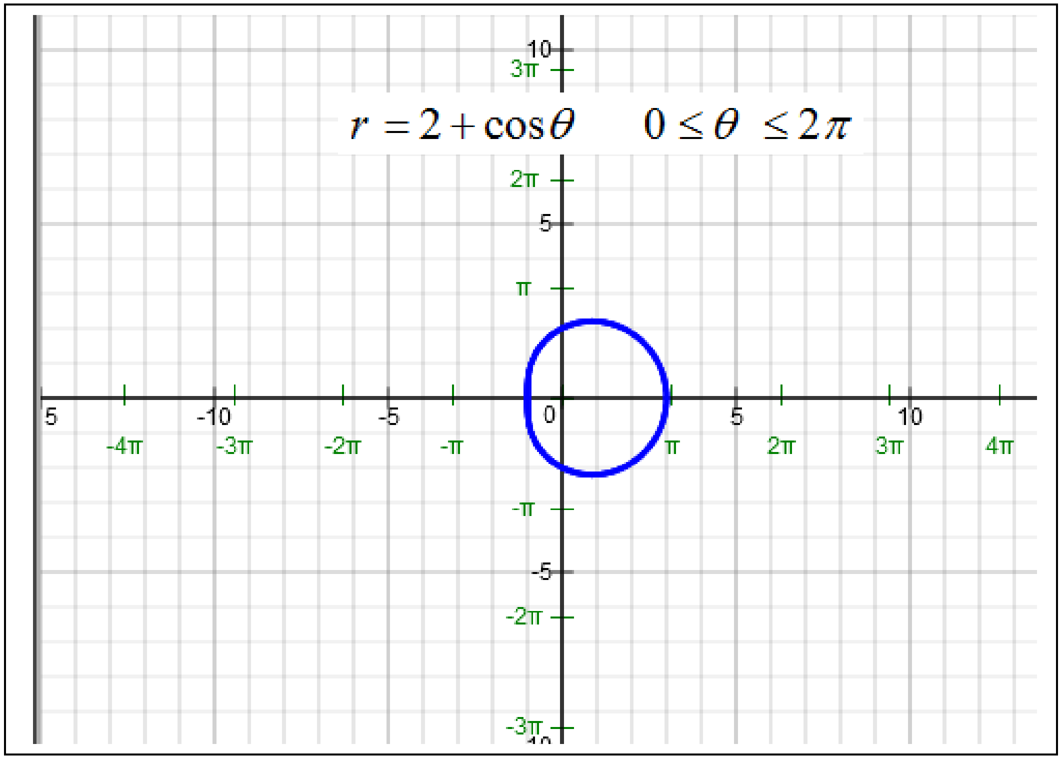
$$= \frac{1}{2}\int_{\pi/10}^{3\pi} (3\cos 5\theta)^2 \, d\theta$$

$$= 1.4137166$$

23) Find the area of the interior of $r = 2 + \cos\theta$.

Note: Graph of $r = 2 + \cos\theta$ never touches the pole. So we cannot set r equal to 0.

To figure out where the graph of $r = 2 + \cos\theta$ starts and ends we can try $\theta = 0$ and $\theta = 2\pi$.



Area of interior $= \dfrac{1}{2}\displaystyle\int_0^{2\pi} r^2 d\theta = \dfrac{1}{2}\displaystyle\int_0^{2\pi} (2 + \cos\theta)^2 \, d\theta = 14.137166925$

24) Polar equation $r = \dfrac{6}{1-\sin\theta}$.

Find the eccentricity and the distance from the pole to the directrix. Graph $r = \dfrac{6}{1-\sin\theta}$ and the directrix.

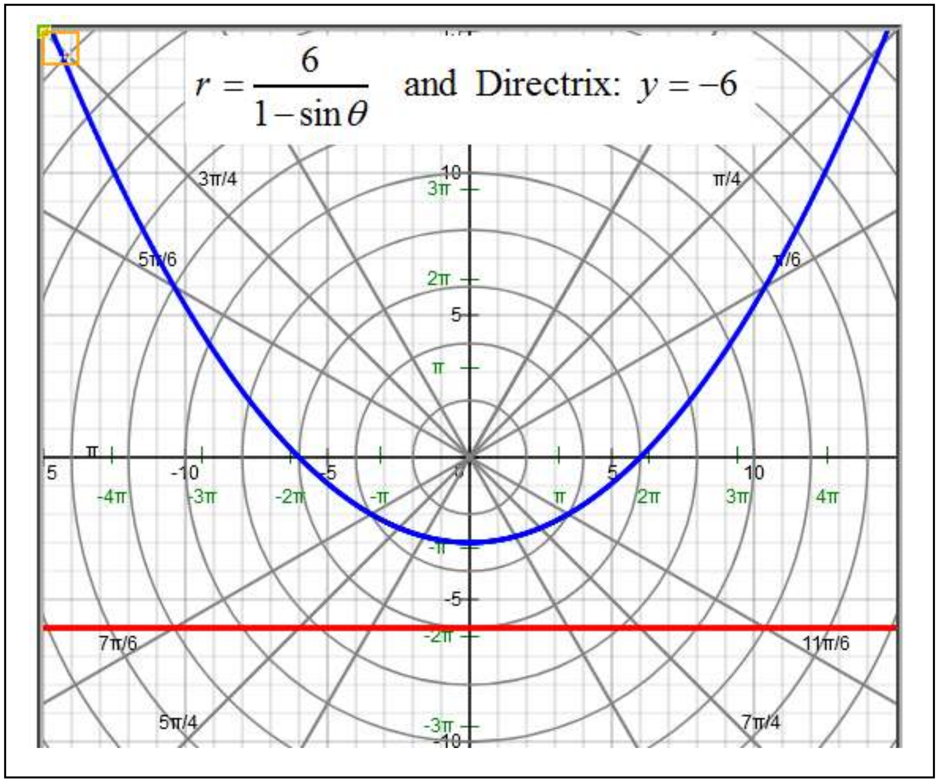
$r = \dfrac{6}{1-\sin\theta} = \dfrac{6}{1-1\sin\theta}$ has the form $r = \dfrac{ed}{1-e\sin\theta}$

Hence, $e = 1$ and $ed = 6$.

Since $e = 1$ and $ed = 6$, $d = 6$.

distance from the pole to the directrix = $|d| = 6$

Directrix: $y = -6$



25) Polar equation $r = \dfrac{6}{3+2\cos\theta}$.

Find the eccentricity and the distance from the pole to the directrix. Graph $r = \dfrac{6}{3+2\cos\theta}$ and the directrix.

$r = \dfrac{6}{3+2\cos\theta}$ has the form $r = \dfrac{ed}{1+e\cos\theta}$.

Note: $r = \dfrac{6}{3+2\cos\theta} = r = \dfrac{6/3}{3/3+\dfrac{2\cos\theta}{3}} = r = \dfrac{2}{1+(2/3)\cos\theta} = \dfrac{ed}{1+e\cos\theta}$

Hence, $e = 2/3$ and $ed = 2$.

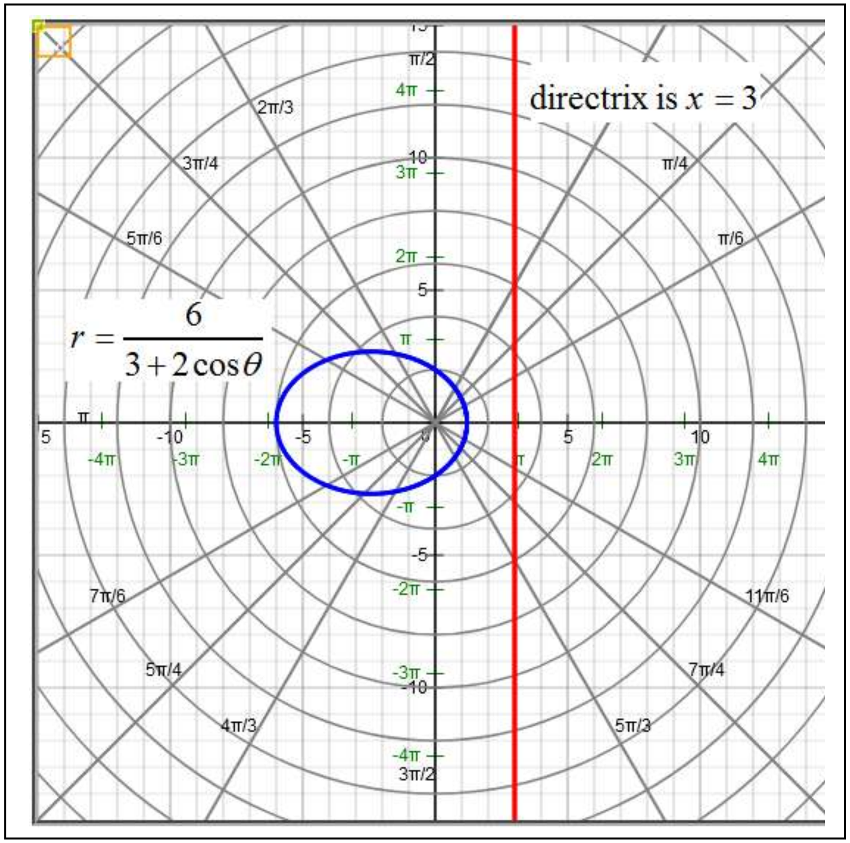
$ed = 2$

$\dfrac{2}{3}d = 2$

$d = 3$

Hence, distance from the pole to the directrix = $|d| = 3$;

and directrix is $x = 3$.



26) Polar equation $r = \dfrac{4}{2-3\sin\theta}$.

Find the eccentricity and the distance from the pole to the directrix. Graph $r = \dfrac{4}{2-3\sin\theta}$ and the directrix.

$r = \dfrac{4}{2-3\sin\theta}$ has the form $r = \dfrac{ed}{1-e\sin\theta}$.

Note: $r = \dfrac{4}{2-3\sin\theta} = r = \dfrac{4/2}{2/2 - \dfrac{3\sin\theta}{2}} = r = \dfrac{2}{1-(3/2)\cos\theta} = \dfrac{ed}{1-e\sin\theta}$

Hence, $e = 3/2$ and $ed = 2$.

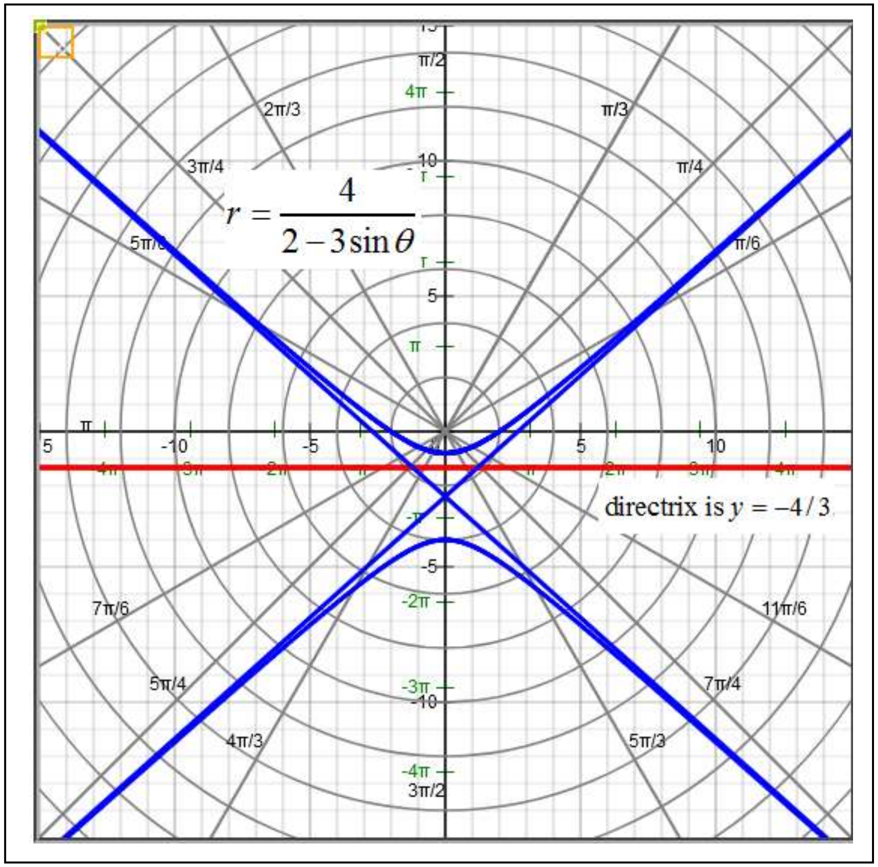
$ed = 2$

$\dfrac{3}{2}d = 2$

$d = 4/3$

Hence, distance from the pole to the directrix = $|d| = 4/3$;

and directrix is $y = -4/3$.

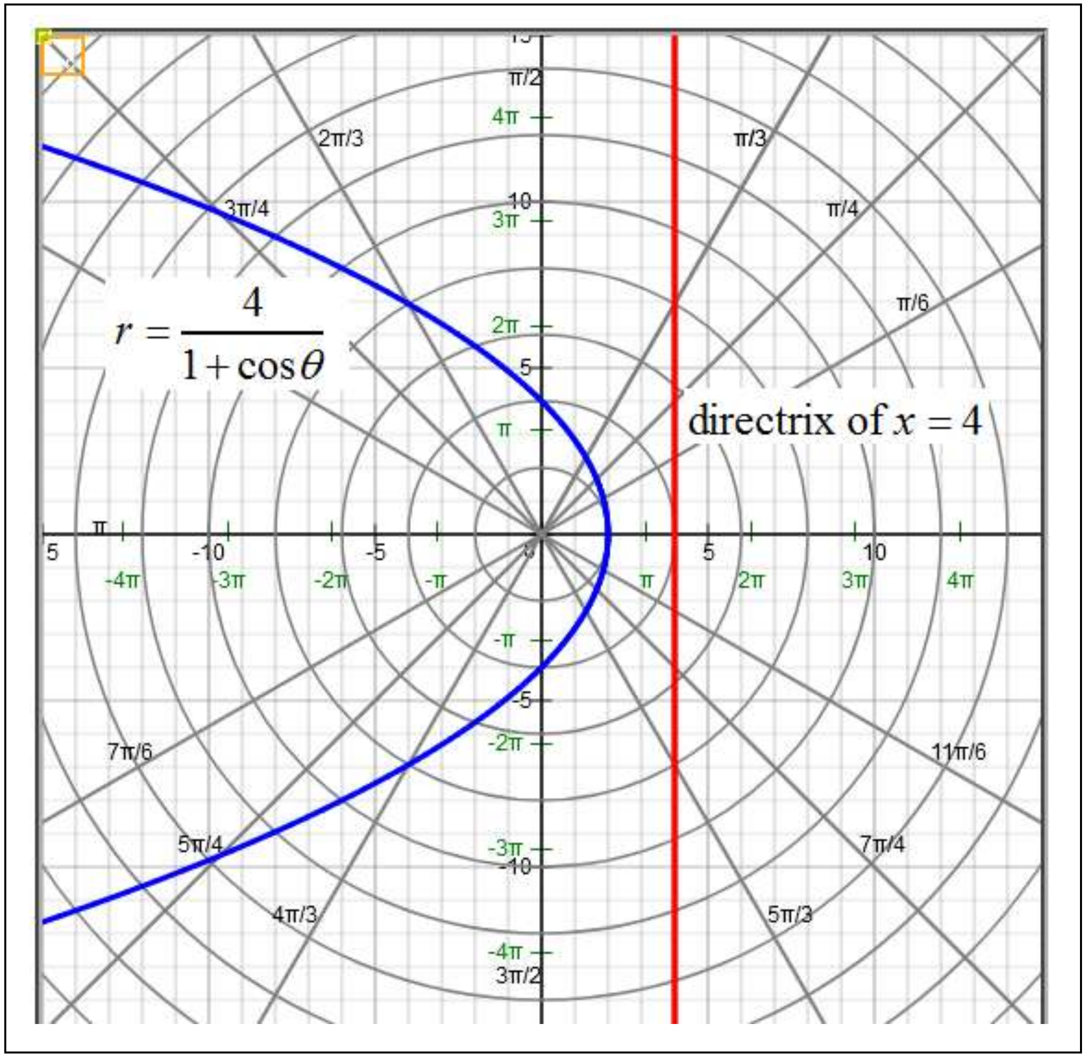


27) Find the equation of a parabola with $e = 1$ and directrix of $x = 4$.

Note: The directrix $x = 4$ is to the right of the of pole.

So we will use the equation $r = \dfrac{ed}{1+e\cos\theta}$

Hence, $r = \dfrac{ed}{1+e\cos\theta} = \dfrac{(1)(4)}{1+1\cos\theta} = \dfrac{4}{1+\cos\theta}$



28) Find the equation of a hyperbola with $e = 3$ and directrix of $y = 3$.

Note: The directrix $y = 3$ is above the of pole.

So we will use the equation $r = \dfrac{ed}{1+e\sin\theta}$

Hence, $r = \dfrac{ed}{1+e\sin\theta} = \dfrac{(3)(3)}{1+3\sin\theta} = \dfrac{9}{1+3\sin\theta}$

$r = \dfrac{9}{1+3\sin\theta}$

directrix of $y = 3$