Test 3 Review: pp. 742-743

(7) $16x^2 + 16y^2 - 16x + 24y - 3 = 0$

$$(16x^2 - 16x \quad) + (16y^2 + 24y \quad) = 3$$

$$16\left(x^2 - 1x + \frac{1}{4}\right) + 16\left(y^2 + \frac{24}{16}y + \frac{9}{16}\right) = 3 + 16\left(\frac{1}{4}\right) + 16\left(\frac{9}{16}\right)$$

$$-\tfrac{1}{2} + -\tfrac{1}{2} = -1$$

$$\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right) = \frac{1}{4}$$

$$\frac{3}{2}$$

$$\frac{3}{4} + \frac{3}{4} = \frac{3}{2}$$

$$\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = 9/16$$

$$16\left(x - \frac{1}{2}\right)^2 + 16\left(y + \frac{3}{4}\right)^2 = 16$$

Divide by 16

$$\frac{\left(x - \tfrac{1}{2}\right)^2}{1} + \frac{\left(y + \tfrac{3}{4}\right)^2}{1} = 1$$

#7 Conic is Ellipse (Circle)

Center $(h = 1/2, \ k = -3/4)$

Focus $= (h \pm c, k) = (h \pm 0, k) = (\frac{1}{2}, \frac{-3}{4})$

$a^2 = 1 \qquad b^2 = 1$

$a^2 = b^2 + c^2 \Longrightarrow 1 = 1 + c^2 \Longrightarrow c = 0$

Vertex: $(h \pm a, k) \doteq (\frac{1}{2} \pm 1, \frac{-3}{4})$

e = eccentricity $= \frac{c}{a} = \frac{0}{1} = 0$

#9 $3x^2 - 2y^2 + 24x + 12y + 24 = 0$

$(3x^2 + 24x) + (-2y^2 + 12y) = -24$

$3(x^2 + 8x + 16) \quad -2(y^2 - 6y + 9) = -24$

$+3(16)$

$+(-2)(9)$

$4 + 4 = 8$

$(4)(4) = 16$

$-3 + -3 = -6$

$(-3)(-3) = 9$

$3(x+4)^2 - 2(y-3)^2 = 6$

Divide by 6

$$\frac{(x+4)^2}{2} - \frac{(y-3)^2}{3} = 1$$

Conic: Hyperbola; branches opens left and right

$a^2 = 2$ $\qquad$ $b^2 = 3$ $\qquad$ #9

$\cancel{a^2 = b^2 + c^2}$

$\cancel{3 = 2 + c^2}$

$\cancel{c^2 = 1}$

$\cancel{c = 1}$

$c^2 = a^2 + b^2$

$c^2 = 2 + 3$

$c = \sqrt{5}$

$a = \sqrt{2} = 1.4$

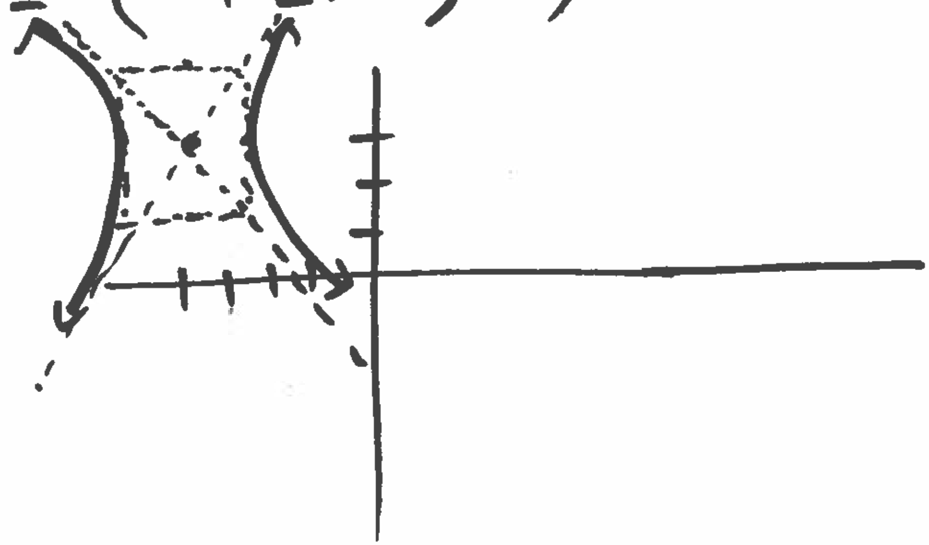
$b = \sqrt{3} = 1.7$

Center $(h = -4 \ , \ k = +3)$

Focus: $(h \pm c, k) = (-4 \pm \sqrt{5}, 3)$

Vertex: $(h \pm a, k) = (-4 \pm \sqrt{2}, 3)$

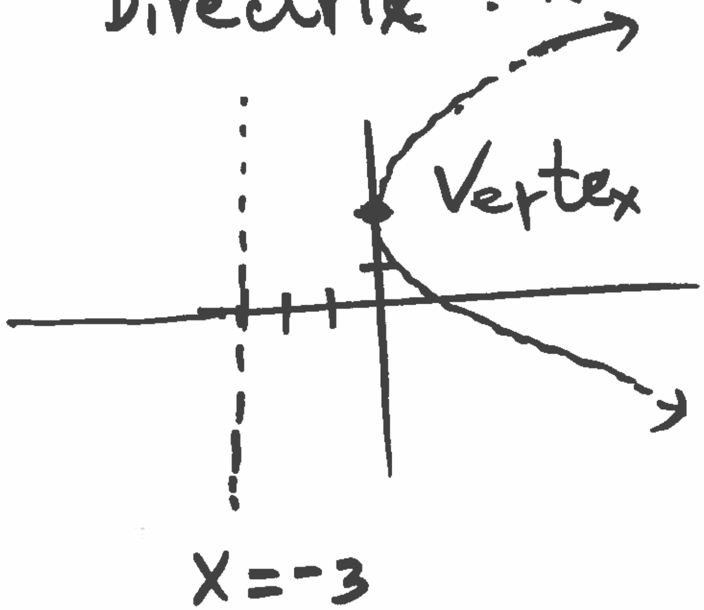
$e = \frac{c}{a} = \frac{\sqrt{5}}{2}$



(15) Vertex $= (0, 2)$

Directrix : $x = -3$

Find equation.



$V = (h, k) = (0, 2)$

p = distance from vertex to directrix

$$(y-k)^2 = 4p(x-h)$$

$$(y-2)^2 = 4(3)(x-0)$$

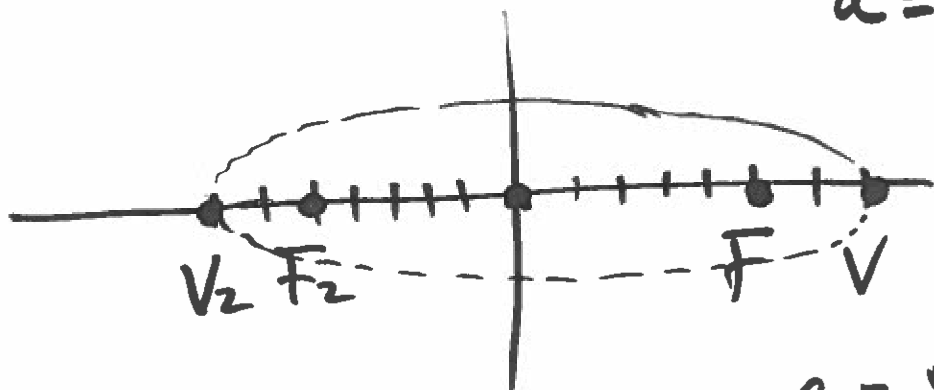
$$(y-2)^2 = 12x$$

(17) Center $(0, 0)$

$F(5, 0)$

$V(7, 0)$

Find equation of ellipse



$a = 7 =$ distance from center to vertex

$c = 5 =$ distance from center to focus

For ellipse:

$$a^2 = b^2 + c^2$$

$$7^2 = b^2 + 5^2$$

$$b^2 = 24$$

$$b = \sqrt{24}$$

#17

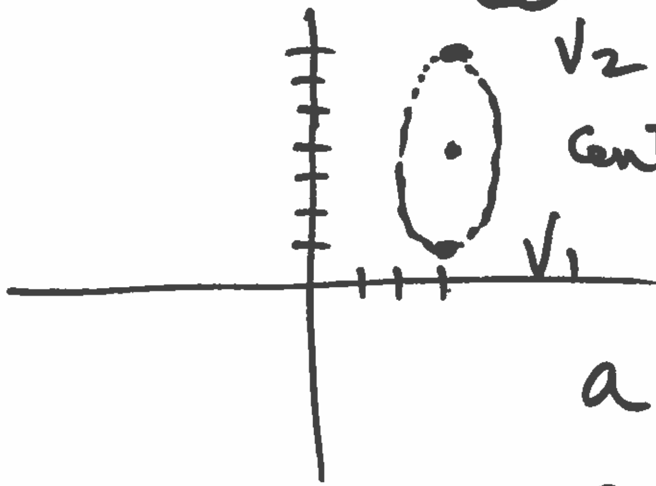
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-0)^2}{49} + \frac{(y-0)^2}{24} = 1$$ Answer

(19) Vertices : $(3,1)$ $(3,7)$

Find equation of ellipse

$e = 2/3 = \frac{c}{a}$



center ~~(3, 4)~~ $(3,4) = (h,k)$

$a = 3$

$\frac{c}{a} = \frac{2}{3} \Rightarrow c = 2$

$a^2 = b^2 + c^2$
$9 = b^2 + 4$
$b^2 = 5$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \Leftrightarrow \frac{(x-3)^2}{5} + \frac{(y-4)^2}{9} = 1$$

Given: Parametric Equations.

(27) $x = 1 + 8t$ ①

$y = 3 - 4t$ ②

Find rectangular equ.

① $x = 1 + 8t$

$x - 1 = 8t$

$t = \dfrac{x-1}{8}$

② $y = 3 - 4t$

$y = 3 - 4\left(\dfrac{x-1}{8}\right)$ Rectangula Equ.

Given: Parametric equations

(29) $x = e^{t} - 1$ ①

$y = e^{3t}$ ②

Find Rectangular Equ.

① $x = e^{t} - 1$

$e^{t} = x + 1$

② $y = e^{3t} = (e^{t})^3 = (x+1)^3$

$y = (x+1)^3$ Rect. Equ.

(31) $x = 6\cos\theta$ (1) Find Rect. Equ.

$y = 6\sin\theta$ (2)

(1) $\cos\theta = x/6$

(2) $\sin\theta = y/6$

From Trig. : $\cos^2\theta + \sin^2\theta = 1$

$$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{6}\right)^2 = 1$$

$$\frac{x^2}{36} + \frac{y^2}{36} = 1$$

Given: Parametric Equations
Find Rectangular Equation

(33) $x = 2 + \sec\theta$ (1)

$y = 3 + \tan\theta$ (2)

(1) $\sec\theta = x - 2$

(2) $\tan\theta = y - 3$

From Trig: $1 + \tan^2\theta = \sec^2\theta$

$1 + (y-3)^2 = (x-2)^2$ Rect. Equ.

Given: Rectangular equation

35 $y = 4x + 3$ Find Parametric Equ.

Let $x = t$

$y = 4t + 3$

Let $x = t + 4$

$y = 4(t+4) + 3$

(39) $x = 2 + 5t \quad , \quad y = 1 - 4t \qquad$ Find $y' = \frac{dy}{dx}$

Chain Rule : $y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$$= \cancel{\frac{5}{-4}} = \frac{-4}{5}$$

$$y'' = \frac{\frac{d}{dt}[y']}{dx/dt} = \frac{\frac{d}{dt}[-4/5]}{5} = \frac{0}{5} = 0$$

(41) $x = \frac{1}{t}$ (1)

$y = 2t + 3$ (2)

(1) $t = 1/x$

(2) $y = 2\left(\frac{1}{x}\right) + 3 = \frac{2}{x} + 3$

$$y' = \frac{-2}{x^2}$$

$$y'' = \frac{-4}{x^3}$$

(43) $x = 5 + \cos\theta$

$y = 3 + 4\sin\theta$

Find $y' = \dfrac{dy}{dx}$

Chain Rule : $y' = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{4\cos\theta}{-\sin\theta} = -4\cot\theta$

$$y'' = \frac{\frac{d}{d\theta}[y']}{dx/d\theta} = \frac{-4\left[-(\csc^2\theta)\right]}{-\sin\theta} = \frac{4\csc^2\theta}{-\sin\theta}$$

(49) ✓

(51) ✓

([illegible])

Given: Polar Point

Find Rectangular Point

(59) $\begin{array}{cc} r & \theta \\ (5, & 3\pi/2) \end{array}$

$x = r\cos\theta$

$y = r\sin\theta$

$x = 5\cos 3\pi/2 = 5(0) = 0$

$y = 5\sin 3\pi/2 = 5(-1) = -5$

$(0, -5)$ Rectangllar

Given: Polor Point

Find Rectangular Point

(61) $\overset{r}{(\sqrt{3},}\ \overset{\theta}{1.56)}$

$x = r\cos\theta = \sqrt{3}\cos(1.56)$

$y = r\sin\theta = \sqrt{3}\sin(1.56)$

(63) $(\overset{x}{4}, \overset{y}{-4})$ Find Polar Point

$x^2 + y^2 = r^2$

$\tan\theta = y/x$

$(4)^2 + (-4)^2 = r^2 \Rightarrow r^2 = 32 \Rightarrow r = \sqrt{32}$

$\tan\theta = -4/4 = -1$

$\theta = \tan^{-1}(-1) = 3\pi/4$

Polar: $(\sqrt{32}, 3\pi/4)$

(65) $\overset{x,\,y}{(-1, 3)}$

Find Polar Point

$$x^2 + y^2 = r^2$$
$$(-1)^2 + (3)^2 = r^2$$
$$r^2 = 10$$
$$r = \sqrt{10}$$

$$\tan\theta = y/x$$
$$\theta = \tan^{-1}\left(\frac{3}{-1}\right) = -1.25$$

$$(r, \theta) = (\sqrt{10}, -1.25)$$

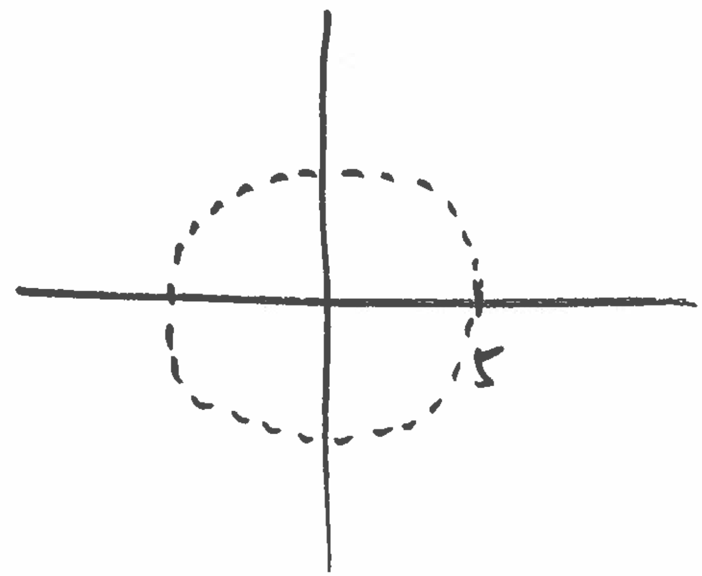
~~67~~ ✓

Given: Rectangular Equation
Find Polar Equation

67 $x^2 + y^2 = 25$

$r^2 = 25$

$r = 5$ Polar Equ.



Given: Rectangular Equation
Find Polar Equation

71) $x^2 = 4y$

$$(r\cos\theta)^2 = 4(r\sin\theta)$$

$$r^2 \cdot \cos^2\theta = 4r\sin\theta$$

$$r\cos^2\theta = 4\sin\theta$$

$$r = \frac{4\sin\theta}{\cos^2\theta}$$ Polar Equ.

Given: Polar Equation

Find Rectangular

(73) $r = 3\cos\theta$

$\cancel{r^2 = 9\cos^2}$

$r \cdot r = 3 \cdot r\cos\theta$

$r^2 = 3 \cdot r\cos\theta$

$x^2 + y^2 = 3x$ Rect. Equ.

(77) ✓

(79) Graph $r = \dfrac{3}{\cos(\theta - \pi/4)} = 3/\cos(\theta - \pi/4)$

(81) Graph $r = 4\cos 2\theta \cdot \sec\theta = 4\cos(2\theta)\sec(\theta)$

Given: Polar Equ.
Find rectangular

(77) $r = -2\sec\theta \tan\theta$

$$r = -2 \cdot \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos}$$

$$r = -\frac{2\sin\theta}{\cos^2\theta}.$$

$$r = \frac{-2 \cdot \sin\theta \cdot r^2}{r^2\cos^2\theta} = \frac{-2r \cdot r\sin\theta}{x^2}$$

$$r = \frac{-2r \cdot y}{x^2}$$

$$1 = \frac{-2y}{x^2} \Leftrightarrow x^2 = -2y$$ Rectangular.

97 $r = 3\cos 5\theta$ Find area of one petal

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta =$$

To find where each petal starts and ends:

set $r = 0$ because each petal starts and ends at the pole.

$$3\cos 5\theta = 0$$

$$\cos 5\theta = 0$$

$$\cos(\pi/2) = 0 \; ; \; \cos(3\pi/2) = 0 ; \; \ldots$$

$$5\theta = \pi/2 \; ; \; 3\pi/2 ; \; 5\pi/2 ; \; \ldots$$

$$\theta = \pi/10, \; 3\pi/10, \; \ldots$$

(97) $$A = \frac{1}{2}\int_{\pi/10}^{3\pi/10} [3\cos 5\theta]^2 \, d\theta$$

$$A = \frac{1}{2}(2.827433301) = 1.4137166$$

99) $r = 2 + \cos\theta$ Find the area of the interior.

$$A = \frac{1}{2}\int_0^{2\pi} r^2\,d\theta = \frac{1}{2}\int_0^{2\pi} (2+\cos\theta)^2\,d\theta$$

$$= 14.137166925$$

Find the eccentricity and the distance from pole to the directrix.

(113) $r = \dfrac{6}{1 - \sin\theta} = \dfrac{ed}{1 - e\sin\theta}$

$e = 1$ eccentricity

$ed = 6$

$1d = 6$

$d = 6$ distance from pole to directrix = |d| = 6

Graph: $r = 6/(1-\sin(\theta))$

Find the eccentricity and the distance from pole to the directrix.

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$$r = \frac{6}{3 + 2\cos\theta}$$

$$r = \frac{2}{1 + \frac{2}{3}\cos\theta} = \frac{ed}{1 + e\cos\theta}$$

$e = 2/3$ eccentricity

$ed = 2$

$\frac{2}{3}d = 2$

$d = 3$ distance from pole to directrix = $|d| = 3$

Graph : $r = 6/(3 + 2\cos(\theta))$

Find the eccentricity and the distance from pole to the directrix.

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$$r = \frac{4}{2 - 3\sin\theta}$$

$$r = \frac{2}{1 - \frac{3}{2}\sin\theta}$$

$$e = \frac{3}{2}$$

eccentricity

$$ed = 2$$

$$\frac{3}{2}d = 2$$

$$d = 2 \cdot \frac{2}{3} = 4/3$$

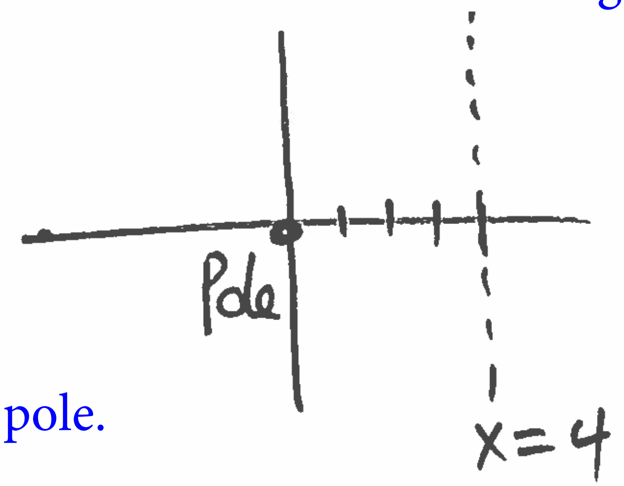
distance from pole to directrix = |d| = 4/3

Find polar equation of a parabola when e and directrix are given.

(119) Parabola

$e = 1$ eccentricity

$x = 4$ directrix



Note: directrix is to the right of the pole.
So the following equation is used.

$$r = \frac{ed}{1 + e\cos\theta}$$

$d = 4$

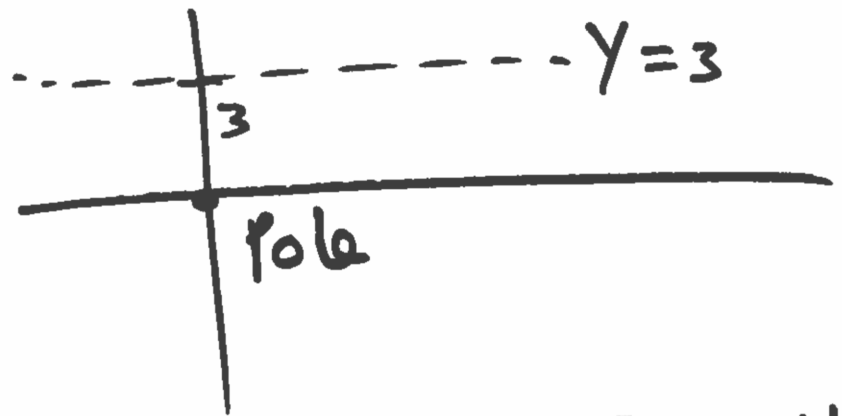
$$r = \frac{(1)(4)}{1 + 1\cos\theta} = \frac{4}{1+\cos\theta}$$

Find polar equation of a hyperbola when e and directrix are given.

(121) Hyperbola

$e = 3$

Dir: $Y = 3$



Note: directrix is above the pole.
So the following equation is used.

Directrix is above the pole and horizonta.

$d = 3$

$$r = \frac{ed}{1 + e\sin\theta}$$

$$r = \frac{3(3)}{1 + 3\sin\theta} = \frac{9}{1 + 3\sin\theta}$$