

Calculator and Inverse Trigonometry Functions ($\tan^{-1} x, \sin^{-1} x, \cos^{-1} x$)

Find x using calculator:

For $\sin(x) = c$, then: $x = \sin^{-1}(c)$, $x = \pi - \sin^{-1}(c)$, $x = \sin^{-1}(c) + 2\pi$, $x = 3\pi - \sin^{-1}(c)$

For $\cos(x) = c$, then: $x = \cos^{-1}(c)$, $x = 2\pi - \cos^{-1}(c)$, $x = \cos^{-1}(c) + 2\pi$, $x = 4\pi - \cos^{-1}(c)$

For $\tan(x) = c$, then: $x = \tan^{-1}(c)$, $x = \pi + \tan^{-1}(c)$, $x = \tan^{-1}(c) + 2\pi$, $x = 3\pi + \tan^{-1}(c)$

For $\sec(x) = c$, then: $x = \sec^{-1}(c)$, $x = 2\pi - \sec^{-1}(c)$, $x = \sec^{-1}(c) + 2\pi$, $x = 4\pi - \sec^{-1}(c)$

For $\csc(x) = c$, then: $x = \csc^{-1}(c)$, $x = \pi - \csc^{-1}(c)$, $x = \csc^{-1}(c) + 2\pi$, $x = 3\pi - \csc^{-1}(c)$

For $\cot(x) = c$, then: $x = \cot^{-1}(c)$, $x = \pi + \cot^{-1}(c)$, $x = \cot^{-1}(c) + 2\pi$, $x = 3\pi + \cot^{-1}(c)$

Examples

1) $\tan(x)$ and $\tan^{-1}(x)$

Let $\tan(x) = 1$ Find x .

Note: $\tan(x)$ is positive in Quadrants I and III.

Using Calculator: $x = \tan^{-1}(1) = \pi/4$ (Quadrant I)

Also, $\tan(\pi + \pi/4) = \tan(5\pi/4) = 1$;

hence $x = 5\pi/4$ (Quadrant III)

Therefore,

$$x = \pi/4, \pi/4+2\pi, \pi/4+4\pi, \pi/4+6\pi, \dots$$

$$x = 5\pi/4, 5\pi/4+2\pi, 5\pi/4+4\pi, 5\pi/4+6\pi, \dots$$

2) $\tan(x)$ and $\tan^{-1}(x)$

Let $\tan(x) = -\sqrt{3}$. Find x .

Note: $\tan(x)$ is negative in Quadrants II and IV.

Using Calculator: $x = \tan^{-1}(-\sqrt{3}) = -1.0471975511965976 \text{ rad} = -\pi/3 \text{ rad}$ (Quadrant IV)

Since $\tan(x)$ is also negative in Quadrants II, $\tan(\pi + -\pi/3) = \tan(2\pi/3) = -\sqrt{3}$

hence $x = \pi + -\pi/3 = 2\pi/3$ (Quadrant II)

Therefore,

$$x = -\pi/3, \quad -\pi/3 + 2\pi = 5\pi/3, \quad -\pi/3 + 4\pi = 11\pi/3, \quad -\pi/3 + 6\pi = 17\pi/3, \dots$$

$$x = 2\pi/3, \quad 2\pi/3 + 2\pi = 8\pi/3, \quad 2\pi/3 + 4\pi = 14\pi/3, \quad 2\pi/3 + 6\pi = 20\pi/3, \dots$$

3) $\sin(x)$ and $\sin^{-1}(x)$

Let $\sin(x) = 1/2$. Find x .

Note: $\sin(x)$ is positive in Quadrants I and II.

Using Calculator: $x = \sin^{-1}(1/2) = \pi/6$ rad (Quadrant I)

Also, $\tan(\pi - \pi/6) = \tan(5\pi/6) = 1/2$;

hence $x = \pi - \pi/6$ rad = $5\pi/6$ rad (Quadrant II)

Therefore,

$$x = \pi/6, \quad \pi/6 + 2\pi = 13\pi/6, \quad \pi/6 + 4\pi = 25\pi/6, \quad \pi/6 + 6\pi = 37\pi/6, \dots$$

$$x = 5\pi/6, \quad 5\pi/6 + 2\pi = 17\pi/6, \quad 5\pi/6 + 4\pi = 29\pi/6, \quad 5\pi/6 + 6\pi = 41\pi/6, \dots$$

4) $\sin(x)$ and $\sin^{-1}(x)$

Let $\sin(x) = -\sqrt{3}/2$. Find x .

Note: $\sin(x)$ is negative in Quadrants III and IV.

Using Calculator: $x = \sin^{-1}(-\sqrt{3}/2) = -1.0471975511965976 = -\pi/3$ rad (Quadrant IV)

Also, $x = \sin(\pi - (-\pi/3)) = \sin(4\pi/3) = -\sqrt{3}/2$;

hence, $x = \pi - (-\pi/3)$ rad = $4\pi/3$ rad (Quadrant III)

Therefore,

$$x = -\pi/3, \quad -\pi/3 + 2\pi = 5\pi/3, \quad -\pi/3 + 4\pi = 11\pi/3, \quad -\pi/3 + 6\pi = 17\pi/3, \dots$$

$$x = 4\pi/3, \quad 4\pi/3 + 2\pi = 10\pi/3, \quad 4\pi/3 + 4\pi = 16\pi/3, \quad 4\pi/3 + 6\pi = 22\pi/3, \dots$$

5) $\cos(x)$ and $\cos^{-1}(x)$

Let $\cos(x) = 1/2$. Find x .

Note: $\cos(x)$ is positive in Quadrants I and IV

Using Calculator: $x = \cos^{-1}(1/2) = \pi/3$ rad (Quadrant I)

Also, $x = \cos(2\pi - \pi/3) = \cos(5\pi/3) = 1/2$;

hence, $x = 2\pi - \pi/3$ rad = $5\pi/3$ rad (Quadrant IV)

Therefore,

$$x = \pi/3, \quad \pi/3 + 2\pi = 7\pi/3, \quad \pi/3 + 4\pi = 13\pi/3, \quad \pi/3 + 6\pi = 19\pi/3, \dots$$

$$x = 5\pi/3, \quad 5\pi/3 + 2\pi = 11\pi/3, \quad 5\pi/3 + 4\pi = 17\pi/3, \quad 5\pi/3 + 6\pi = 23\pi/3, \dots$$

6) $\cos(x)$ and $\cos^{-1}(x)$

Let $\cos(x) = -1/2$. Find x .

Note: $\cos(x)$ is negative in Quadrants II and III

Using Calculator: $x = \cos^{-1}(-1/2) = 2.0943951023931957 = 2\pi/3$ rad (Quadrant II)

Also, $\cos(2\pi - 2\pi/3) = \cos(4\pi/3) = -1/2$;

hence $x = 2\pi - 2\pi/3$ rad = $4\pi/3$ (Quadrant III)

Therefore,

$$x = 2\pi/3, \quad 2\pi/3 + 2\pi = 8\pi/3, \quad 2\pi/3 + 4\pi = 14\pi/3, \quad 2\pi/3 + 6\pi = 20\pi/3, \dots$$

$$x = 4\pi/3, \quad 4\pi/3 + 2\pi = 10\pi/3, \quad 4\pi/3 + 4\pi = 16\pi/3, \quad 4\pi/3 + 6\pi = 22\pi/3, \dots$$

7) $\sec(x)$ and $\sec^{-1}(x)$

Let $\sec(x) = 2$. Find x .

Using Calculator: $x = \sec^{-1}(2) = 1.0471975511965979 \text{ rad} = \pi/3 \text{ rad}$

Also, $\sec(2\pi - \pi/3) = \sec(5\pi/3) = 2$; hence $x = 2\pi - \pi/3 \text{ rad} = 5\pi/3 \text{ rad}$

Therefore,

$$x = \pi/3, \quad \pi/3 + 2\pi = 7\pi/3, \quad \pi/3 + 4\pi = 13\pi/3, \quad \pi/3 + 6\pi = 19\pi/3, \dots$$

$$x = 5\pi/3, \quad 5\pi/3 + 2\pi = 11\pi/3, \quad 5\pi/3 + 4\pi = 17\pi/3, \quad 5\pi/3 + 6\pi = 23\pi/3, \dots$$

8) $\sec(x)$ and $\sec^{-1}(x)$

Let $\sec(x) = -2$. Find x .

Using Calculator: $x = \sec^{-1}(-2) = 2.0943951023931957 = 2\pi/3$ rad

Also, $\sec(2\pi - 2\pi/3) = \sec(4\pi/3) = -2$; hence $x = 2\pi - 2\pi/3$ rad $= 4\pi/3$

Therefore,

$$x = 2\pi/3, \quad 2\pi/3 + 2\pi = 8\pi/3, \quad 2\pi/3 + 4\pi = 14\pi/3, \quad 2\pi/3 + 6\pi = 20\pi/3, \dots$$

$$x = 4\pi/3, \quad 4\pi/3 + 2\pi = 10\pi/3, \quad 4\pi/3 + 4\pi = 16\pi/3, \quad 4\pi/3 + 6\pi = 22\pi/3, \dots$$

9) $\csc(x)$ and $\csc^{-1}(x)$

Let $\csc(x) = 2$. Find x .

Using Calculator: $x = \csc^{-1}(2) = 0.5235987755982989 \text{ rad} = \pi/6 \text{ rad}$

Also, $\csc(\pi - \pi/6) = \csc(5\pi/6) = 2$; hence, $x = \pi - \pi/6 \text{ rad} = 5\pi/6 \text{ rad}$

Therefore,

$$x = \pi/6, \quad \pi/6 + 2\pi = 13\pi/6, \quad \pi/6 + 4\pi = 25\pi/6, \quad \pi/6 + 6\pi = 37\pi/6, \dots$$

$$x = 5\pi/6, \quad 5\pi/6 + 2\pi = 17\pi/6, \quad 5\pi/6 + 4\pi = 29\pi/6, \quad 5\pi/6 + 6\pi = 41\pi/6, \dots$$

10) $\csc(x)$ and $\csc^{-1}(x)$

Let $\csc(x) = -2/\sqrt{3}$. Find x .

Using Calculator: $x = \csc^{-1}\left(-2/\sqrt{3}\right) = -1.0471975511965976 = -\pi/3$ rad

Also, $\csc\left(\pi - \left(-\pi/3\right)\right) = \csc\left(4\pi/3\right) = -2/\sqrt{3}$; hence $x = \pi - \left(-\pi/3\right)$ rad = $4\pi/3$ rad

Therefore,

$$x = -\pi/3, \quad -\pi/3 + 2\pi = 5\pi/3, \quad -\pi/3 + 4\pi = 11\pi/3, \quad -\pi/3 + 6\pi = 17\pi/3, \dots$$

$$x = 4\pi/3, \quad 4\pi/3 + 2\pi = 10\pi/3, \quad 4\pi/3 + 4\pi = 16\pi/3, \quad 4\pi/3 + 6\pi = 22\pi/3, \dots$$

11) $\cot(x)$ and $\cot^{-1}(x)$

Let $\cot(x) = 1$. Find x .

Using Calculator: $x = \cot^{-1}(1) = \pi/4$;

Also, $\cot(\pi + \pi/4) = \cot(5\pi/4) = 1$; hence $x = \pi + \pi/4 = 5\pi/4$

Therefore,

$$x = \pi/4, \pi/4 + 2\pi, \pi/4 + 4\pi, \pi/4 + 6\pi, \dots$$

$$x = 5\pi/4, 5\pi/4 + 2\pi, 5\pi/4 + 4\pi, 5\pi/4 + 6\pi, \dots$$

12) $\cot(x)$ and $\cot^{-1}(x)$

Let $\cot(x) = -1/\sqrt{3}$. Find x .

Using Calculator: $x = \cot^{-1}(-1/\sqrt{3}) = 2.0943951023931957 \text{ rad} = 2\pi/3 \text{ rad}$

Also, $\cot(\pi + 2\pi/3) = \cot(5\pi/3) = -1/\sqrt{3}$; hence, $x = \pi + 2\pi/3 = 5\pi/3$

Therefore,

$$x = 2\pi/3, \quad 2\pi/3 + 2\pi = 8\pi/3, \quad 2\pi/3 + 4\pi = 14\pi/3, \quad 2\pi/3 + 6\pi = 20\pi/3, \dots$$

$$x = 5\pi/3, \quad 5\pi/3 + 2\pi = 11\pi/3, \quad 5\pi/3 + 4\pi = 17\pi/3, \quad 5\pi/3 + 6\pi = 23\pi/3, \dots$$