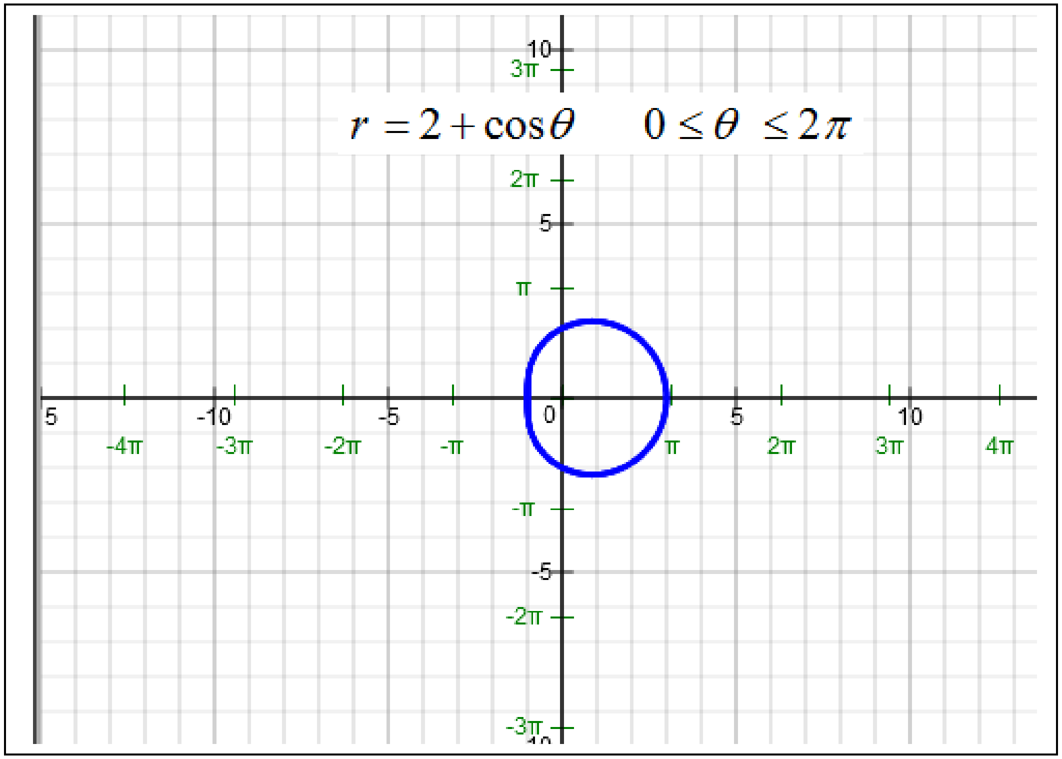
23) Find the area of the interior of $r = 2 + \cos\theta$.

Note: Graph of $r = 2 + \cos\theta$ never touches the pole. So we cannot set r equal to 0.

To figure out where the graph of $r = 2 + \cos\theta$ starts and ends we can try $\theta = 0$ and $\theta = 2\pi$.



$$\text{Area of interior} = \frac{1}{2}\int_0^{2\pi} r^2 d\theta = \frac{1}{2}\int_0^{2\pi} (2+\cos\theta)^2\, d\theta = 14.137166925$$

24) Polar equation $r = \dfrac{6}{1-\sin\theta}$.

Find the eccentricity and the distance from the pole to the directrix. Graph $r = \dfrac{6}{1-\sin\theta}$ and the directrix.

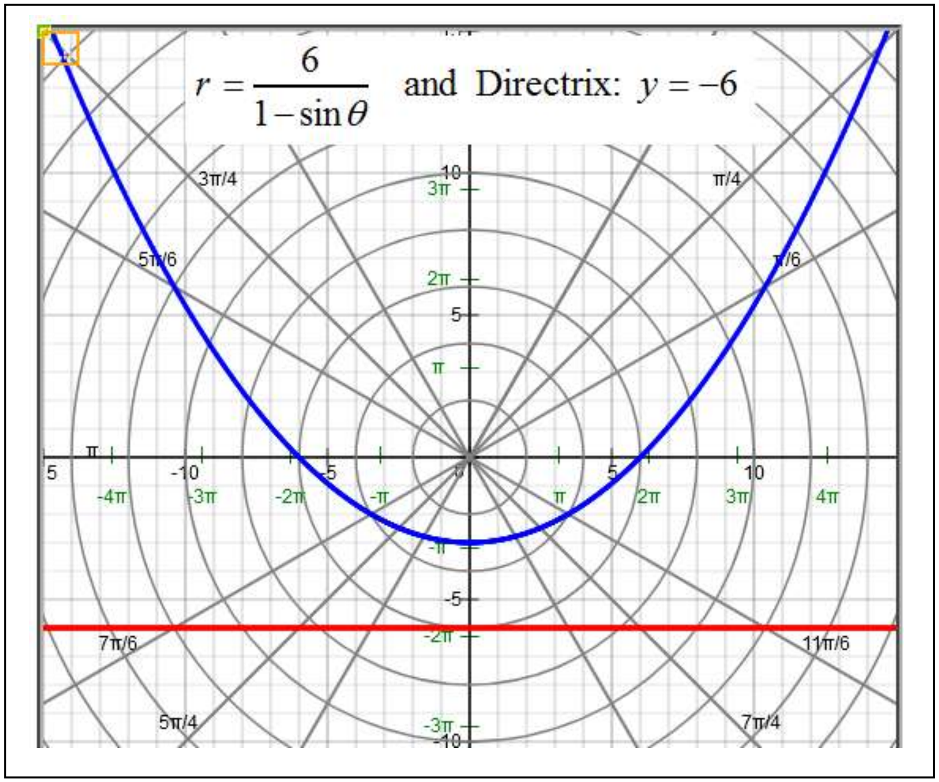
$r = \dfrac{6}{1-\sin\theta} = \dfrac{6}{1-1\sin\theta}$ has the form $r = \dfrac{ed}{1-e\sin\theta}$

Hence, $e=1$ and $ed=6$.

Since $e=1$ and $ed=6$, $d=6$.

distance from the pole to the directrix = $|d| = 6$

Directrix: $y=-6$



25) Polar equation $r = \dfrac{6}{3 + 2\cos\theta}$.

Find the eccentricity and the distance from the pole to the directrix. Graph $r = \dfrac{6}{3 + 2\cos\theta}$ and the directrix.

$r = \dfrac{6}{3 + 2\cos\theta}$ has the form $r = \dfrac{ed}{1 + e\cos\theta}$.

Note: $r = \dfrac{6}{3 + 2\cos\theta} = r = \dfrac{6/3}{3/3 + \dfrac{2\cos\theta}{3}} = r = \dfrac{2}{1 + (2/3)\cos\theta} = \dfrac{ed}{1 + e\cos\theta}$

Hence, $e = 2/3$ and $ed = 2$.

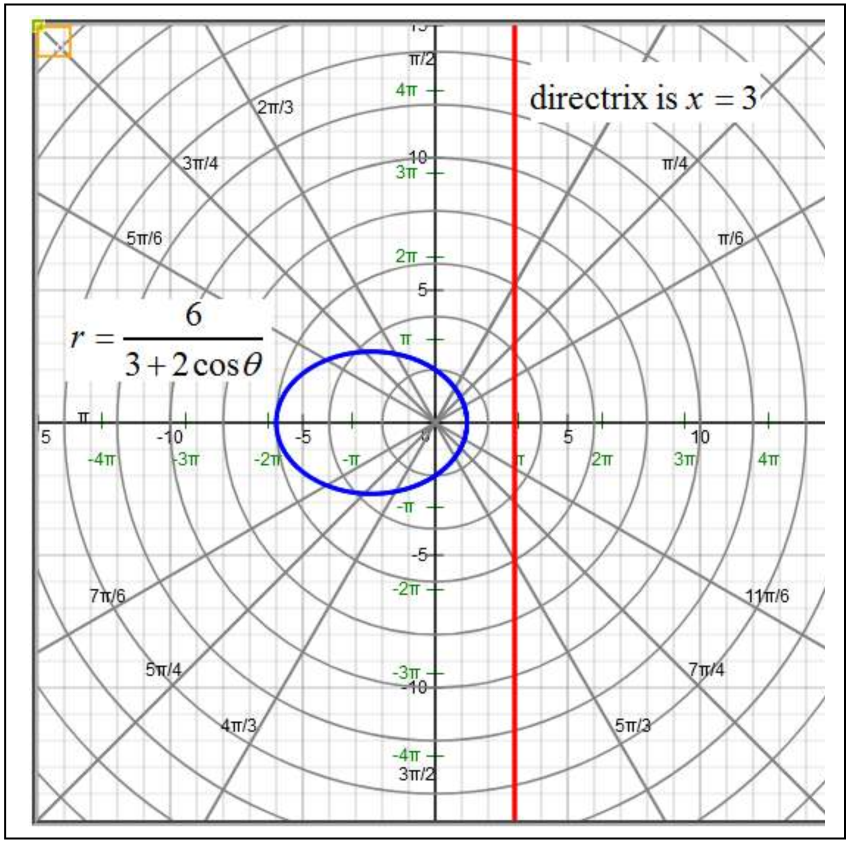
$ed = 2$

$\dfrac{2}{3}d = 2$

$d = 3$

Hence, distance from the pole to the directrix = $|d| = 3$;

and directrix is $x = 3$.



26) Polar equation $r = \dfrac{4}{2-3\sin\theta}$.

Find the eccentricity and the distance from the pole to the directrix. Graph $r = \dfrac{4}{2-3\sin\theta}$ and the directrix.

$r = \dfrac{4}{2-3\sin\theta}$ has the form $r = \dfrac{ed}{1-e\sin\theta}$.

Note: $r = \dfrac{4}{2-3\sin\theta} = r = \dfrac{4/2}{2/2-\dfrac{3\sin\theta}{2}} = r = \dfrac{2}{1-(3/2)\cos\theta} = \dfrac{ed}{1-e\sin\theta}$

Hence, $e = 3/2$ and $ed = 2$.

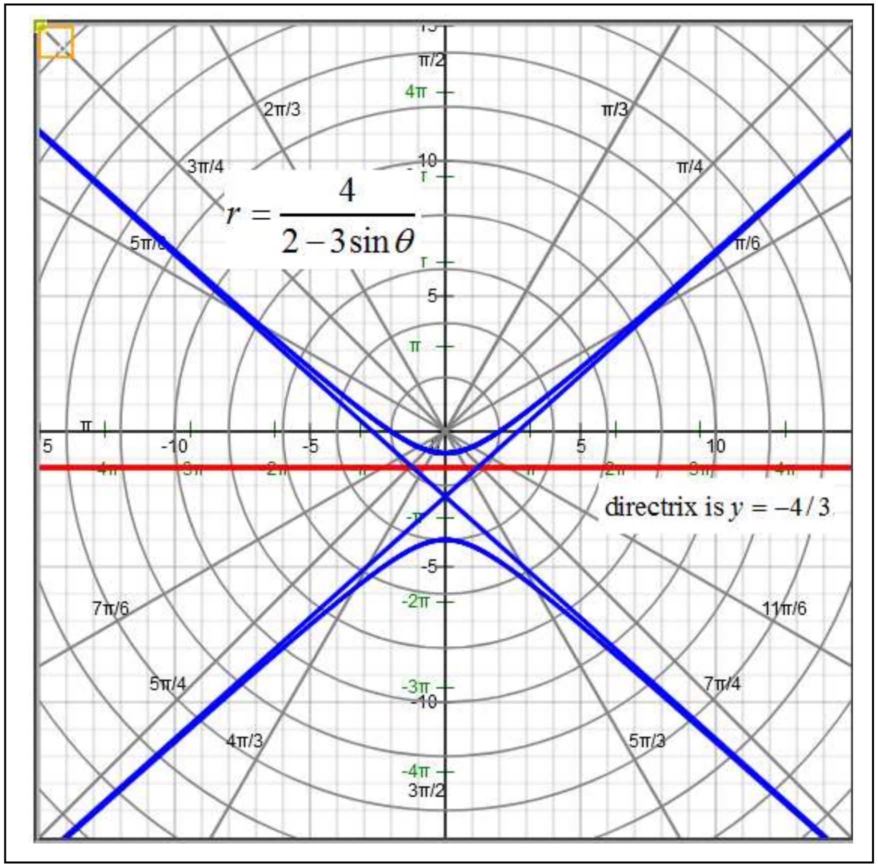
$ed = 2$

$\dfrac{3}{2}d = 2$

$d = 4/3$

Hence, distance from the pole to the directrix = $|d| = 4/3$;

and directrix is $y = -4/3$.

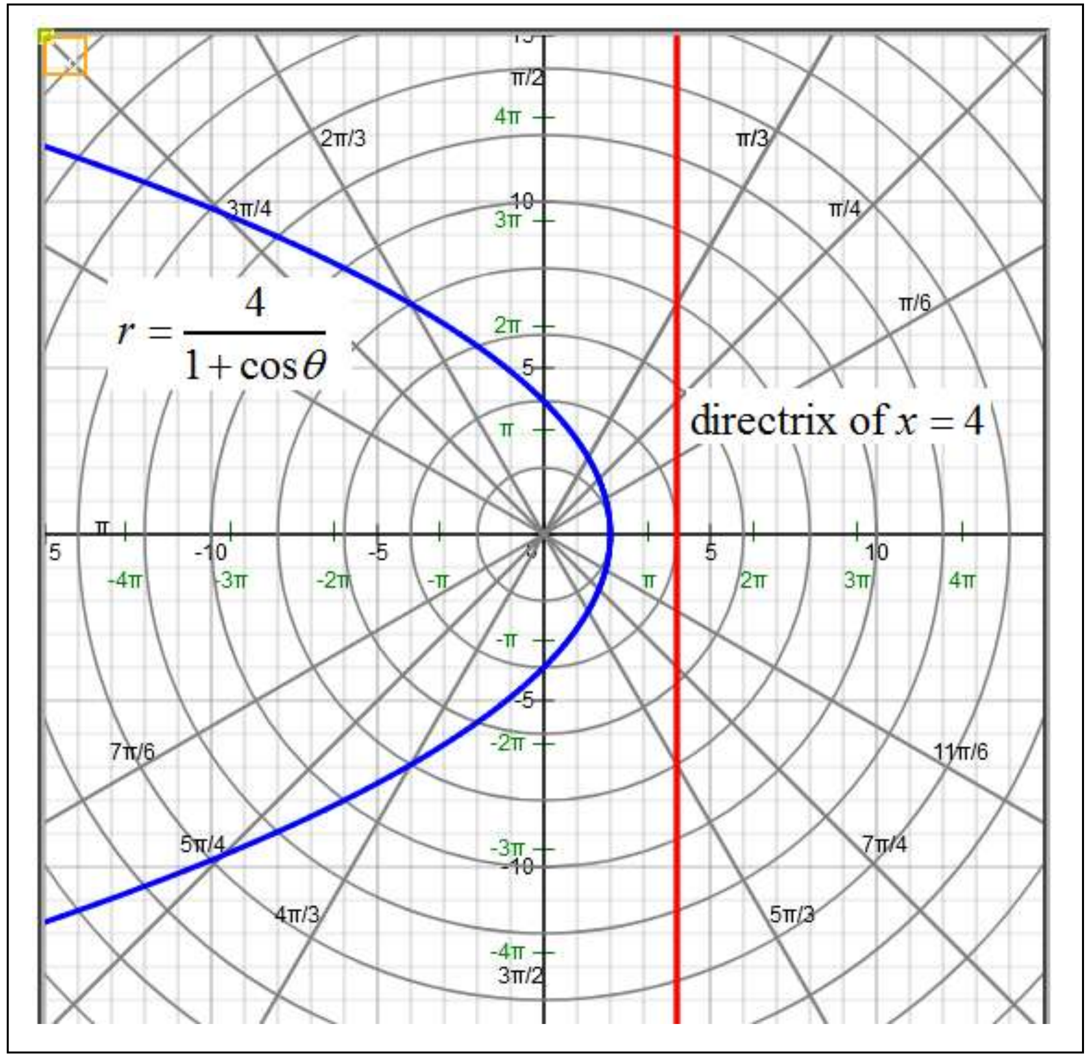


27) Find the equation of a parabola with $e = 1$ and directrix of $x = 4$.

Note: The directrix $x = 4$ is to the right of the of pole.

So we will use the equation $r = \dfrac{ed}{1 + e\cos\theta}$

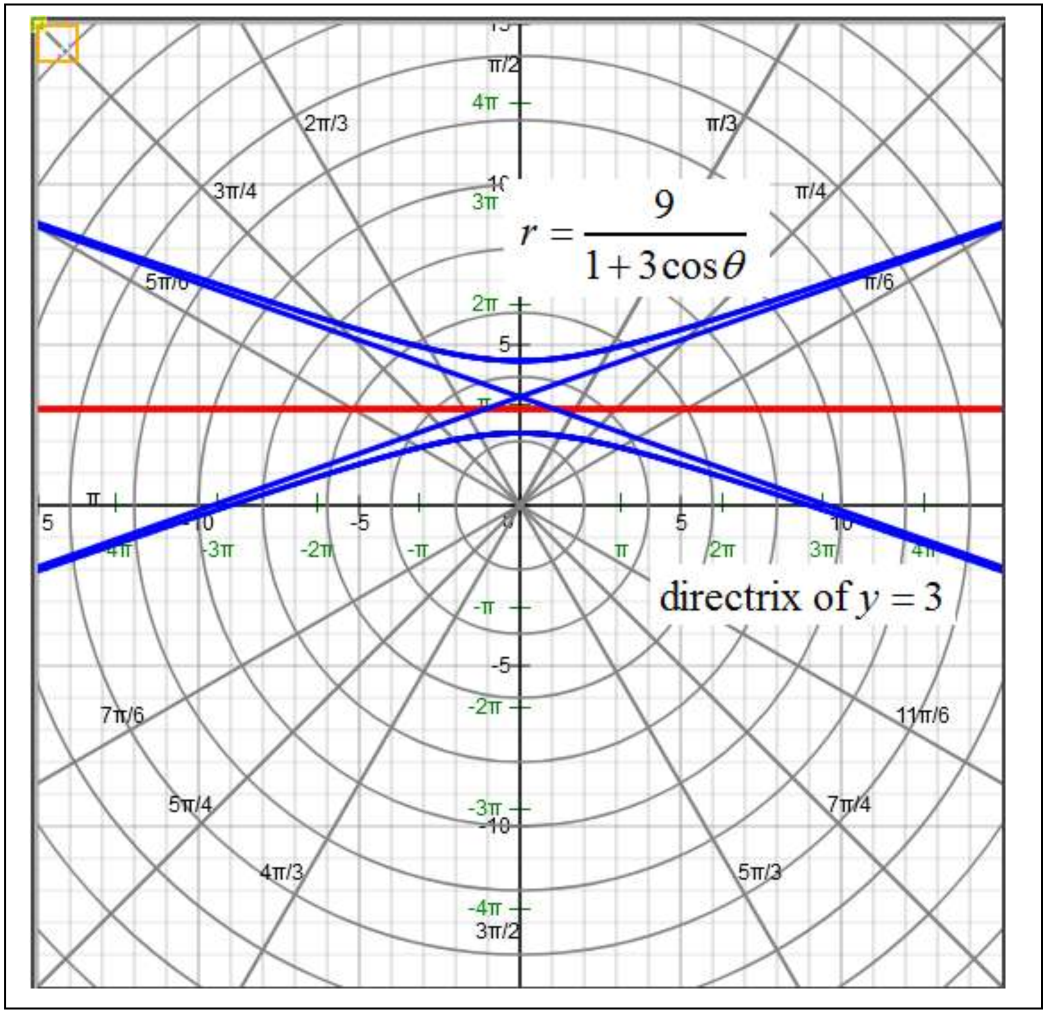
Hence, $r = \dfrac{ed}{1 + e\cos\theta} = \dfrac{(1)(4)}{1 + 1\cos\theta} = \dfrac{4}{1 + \cos\theta}$



28) Find the equation of a hyperbola with $e = 3$ and directrix of $y = 3$.

Note: The directrix $y = 3$ is above the of pole.

So we will use the equation $r = \dfrac{ed}{1 + e\sin\theta}$



Hence, $r = \dfrac{ed}{1 + e\sin\theta} = \dfrac{(3)(3)}{1 + 3\sin\theta} = \dfrac{9}{1 + 3\sin\theta}$

$$r = \frac{9}{1 + 3\sin\theta}$$

directrix of $y = 3$