Test 3 Review

For lamina with non-homogeneous density,

mass of lamina: $m = \iint_R \rho(x, y)\,dA = \underline{\ \ ?\ \ \ \ \ \ }$

moment of mass with respect to x-axis: $M_x = \iint_R y\rho(x, y)\,dA$

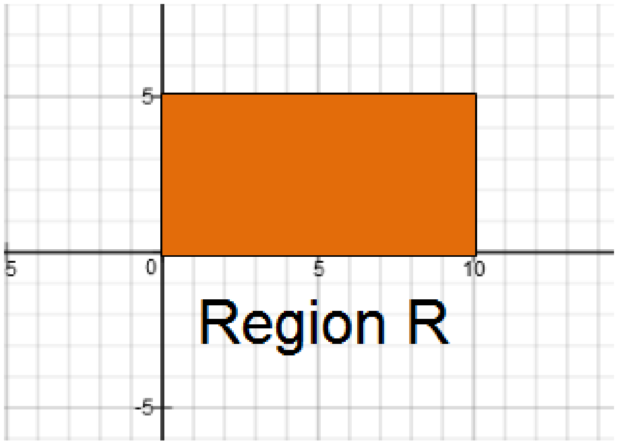
moment of mass with respect to y-axis: $M_y = \iint_R x\rho(x, y)\,dA$

Center of Mass: $\bar{x} = \dfrac{M_y}{m} \qquad \bar{y} = \dfrac{M_x}{m}$

Hence, $\bar{x} = \dfrac{M_y}{m} = \dfrac{\iint_R x\rho(x, y)\,dA}{\iint_R \rho(x, y)\,dA} \qquad \bar{y} = \dfrac{M_x}{m} = \dfrac{\iint_R y\rho(x, y)\,dA}{\iint_R \rho(x, y)\,dA}$

12) Find the mass and center of mass of the lamina corresponding to region R

with the given desisty function $\rho(x, y) = 8xy$.



$$m = \iint_R \rho(x, y)dA = \int_0^5 \int_0^{10} 8xydxdy$$

Evaluate $\int_0^{10} 8xydx = 8y\int_0^{10} xdx = 8y\left[\frac{x^2}{2}\right]_0^{10} = 400y$

Hence, $m = \int_0^5 \int_0^{10} 8xydxdy = \int_0^5 400ydy = 400\left[\frac{y^2}{2}\right]_0^5 = 5000$

$$M_x = \text{moment of mass with respect to } x\text{-axis} = \iint_R y\rho(x,y)\,dA = \int_0^5\int_0^{10} y\left(8xy\right)dxdy = \int_0^5\int_0^{10} 8xy^2 dxdy$$

$$\text{Evaluate } \int_0^{10} 8xy^2 dx = 8y^2\int_0^{10} xdx = 8y^2\left[\frac{1}{2}x^2\right]_0^{10} = 400y^2$$

$$\text{Hence, } \int_0^5\int_0^{10} 8xy^2 dxdy = \int_0^5 400y^2 dy = 400\left[\frac{1}{3}y^3\right]_0^5 = 50000/3$$

$$M_y = \text{moment of mass with respect to } y\text{-axis} = \iint_R x\rho(x,y)\,dA = \int_0^5\int_0^{10} x\left(8xy\right)dxdy = \int_0^5\int_0^{10} 8x^2 ydxdy$$

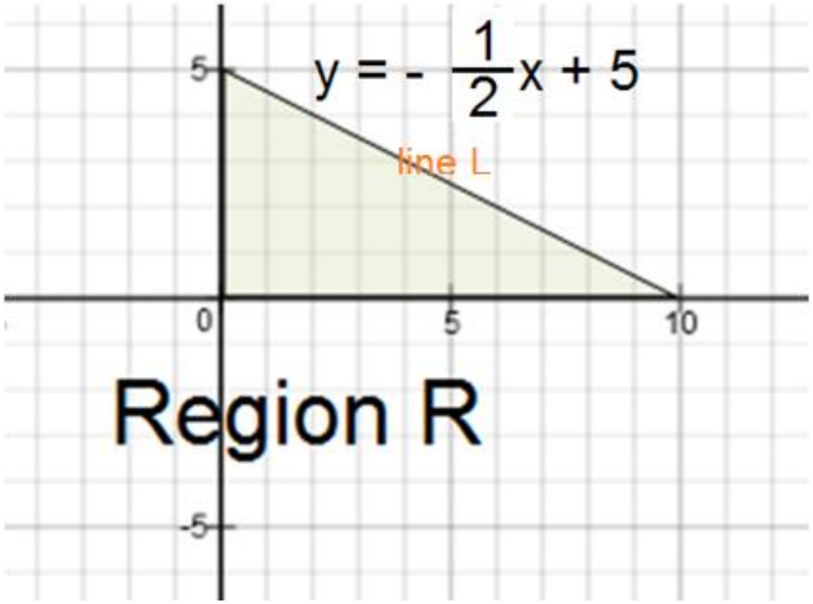
$$\text{Evaluate } \int_0^{10} 8x^2 ydx = 8y\int_0^{10} x^2 dx = 8y\left[\frac{1}{3}x^3\right]_0^{10} = \frac{8000}{3}y$$

$$\text{Hence, } \int_0^5\int_0^{10} 8xy^2 dxdy = \int_0^5 \frac{8000}{3} ydy = \frac{8000}{3}\left[\frac{1}{2}y^2\right]_0^5 = 100000/3$$

$$\text{Center of Mass: } \bar{x} = \frac{M_y}{m} = \frac{100000/3}{5000} = \frac{20}{3} \qquad \bar{y} = \frac{M_x}{m} = \frac{50000/3}{5000} = \frac{10}{3}$$

13) Find the mass of the lamina corresponding to region R

with the given desisty function $\rho(x, y) = 14xy$.



How to find line L:

Line L has $(0,5)$ and $(10,0)$. So $m = \text{slope}=\frac{0-5}{10-0} = -\frac{1}{2}$

and b = y-intercept = 5

Hence, equation for line L is $y = mx + b = -\frac{1}{2}x + 5$

$$m = \iint_R \rho(x,y)\,dA = \iint_R 14xy\,dy\,dx = \int_0^{10}\int_{y=0}^{y=-0.5x+5} 14xy\,dy\,dx$$

Evaluate $\displaystyle\int_{y=0}^{y=-0.5x+5} 14xy\,dy = 14x\int_0^{-0.5x+5} y\,dy = 14x\left[\frac{y^2}{2}\right]_0^{-0.5x+5}$

$$= 14x\left[\frac{(-0.5x+5)^2}{2}\right] - 4kx\left[\frac{(0)}{2}\right] = 14x\left[\frac{(-0.5x+5)^2}{2}\right]$$

Hence, $\displaystyle\int_0^{10}\int_{y=0}^{y=-0.5x+5} 14xy\,dy\,dx = \int_0^{10} 14x\left[\frac{(-0.5x+5)^2}{2}\right]dx$

$$= \int_0^{10} 7x(-0.5x+5)^2\,dx = 4375/3$$

$$M_x = \text{moment about the } x\text{-axis} = \iint_R y\rho(x,y)\,dA = \iint_R y\left(14xy\right)dydx = \int_0^{10}\int_{y=0}^{y=-0.5x+5} 14xy^2\,dydx$$

$$\text{Evaluate } \int_{y=0}^{y=-0.5x+5} 14xy^2\,dy = 14x\int_0^{-0.5x+5} y^2\,dy = 14x\left[\frac{1}{3}y^3\right]_0^{-0.5x+5} = 14x\left[\frac{1}{3}\left(-0.5x+5\right)^3\right]$$

$$M_x = \int_0^{10} 14x\left[\frac{1}{3}\left(-0.5x+5\right)^3\right]dx = 8750/3$$

$$M_y = \text{moment about the } y\text{-axis} = \iint_R x\rho(x,y)\,dA = \iint_R x\left(14xy\right)dydx = \int_0^{10}\int_{y=0}^{y=-0.5x+5} 14x^2y\,dydx$$

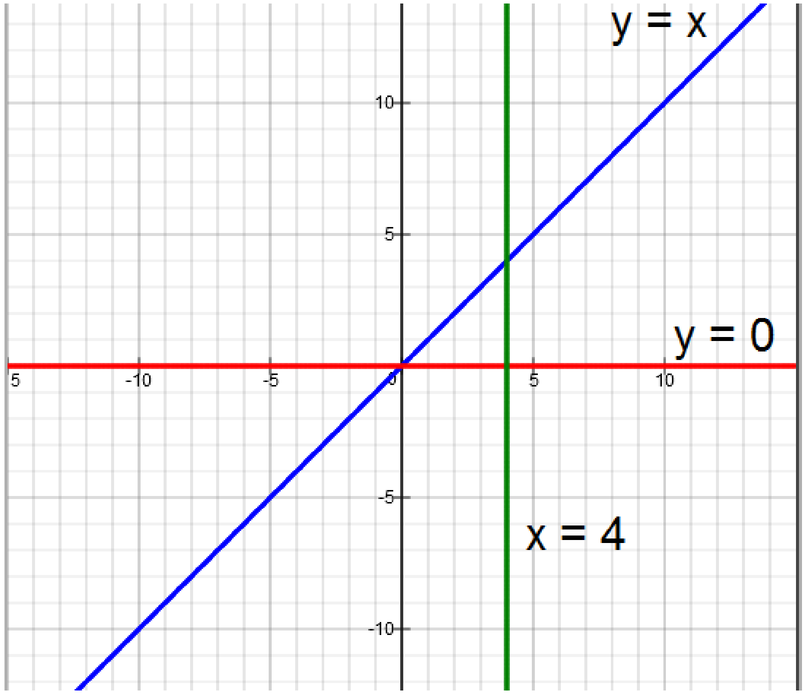
$$\text{Evaluate } \int_{y=0}^{y=-0.5x+5} 14x^2y\,dy = 14x^2\int_0^{-0.5x+5} y\,dy = 14x^2\left[\frac{1}{2}y^2\right]_0^{-0.5x+5} = 14x^2\left[\frac{1}{2}\left(-0.5x+5\right)^2\right]$$

$$M_y = \int_0^{10} 14x^2\left[\frac{1}{2}\left(-0.5x+5\right)^2\right]dx = 17500/3$$

$$\text{Center of Mass: } \bar{x} = \frac{M_y}{m} = \frac{17500/3}{4375/3} = 4; \qquad \bar{y} = \frac{M_x}{m} = \frac{8750/3}{4375/3} = 2$$

14) Find the mass and the center of mass of the lamina bounded by the following graphs.

$y = x; \quad y = 0; \quad x = 4; \qquad$ Density Function: $\rho(x, y) = 8x.$



$$m = \text{mass of lamina} = \iint_R \rho(x,y)\,dydx = \int_0^4 \int_{y=0}^{y=x} 8x\,dydx$$

$$\text{Evaluate inside integral: } \int_{y=0}^{y=x} 8x\,dy = \left[8xy\right]_0^x = 8x^2$$

$$m = \int_0^4 \int_{y=0}^{y=x} 8x\,dydx = \int_0^4 8x^2\,dx = \left[8 \cdot \frac{x^3}{3}\right]_0^4 = \frac{512}{3}$$

$$M_x = \text{moment of mass with respect to } x\text{-axis} = \iint_R y\rho(x,y)\,dA = \int_0^4 \int_{y=0}^{y=x} 8xy\,dydx$$

$$\text{Evaluate inside integral: } \int_{y=0}^{y=x} 8xy\,dy = \left[8x \cdot \frac{y^2}{2}\right]_0^x = 4x^3$$

$$M_x = \int_0^4 \int_{y=0}^{y=x} 8xy\,dydx = \int_0^4 4x^3\,dx = \left[4 \cdot \frac{x^4}{4}\right]_0^4 = 256$$

M_y = moment of mass with respect to y-axis = $\iint_R x\rho(x,y)\,dA = \int_0^4 \int_{y=0}^{y=x} 8xx\,dy\,dx$

Evaluate inside integral: $\int_{y=0}^{y=x} 8xx\,dy = \int_{y=0}^{y=x} 8x^2\,dy = \left[8x^2 y\right]_0^x = 8x^3$

$$M_y = \int_0^4 \int_{y=0}^{y=x} 8xx\,dy\,dx = \int_0^4 8x^3\,dx = 8\left[\frac{x^4}{4}\right]_0^4 = 512$$

Center of Mass: $\bar{x} = \frac{M_y}{m} = \frac{512}{\left(\frac{512}{3}\right)} = 3$ $\bar{y} = \frac{M_x}{m} = \frac{256}{\left(\frac{512}{3}\right)} = 1.5$